Develop the flux linkage and voltage equations for a round rotor machine.

As shown in the figure, for a round rotor machine, add another damper winding G to the q-axis of the rotor.

Then, the flux and voltage vectors become:

\[
\begin{bmatrix}
\psi_{abc} \\
\psi_{FDGQ}
\end{bmatrix} =
\begin{bmatrix}
\psi_a \\
\psi_b \\
\psi_c \\
\psi_F \\
\psi_D \\
\psi_G \\
\psi_Q
\end{bmatrix}, \quad
\begin{bmatrix}
e_{abc} \\
e_{FDGQ}
\end{bmatrix} =
\begin{bmatrix}
e_a \\
e_b \\
e_c \\
e_F \\
e_D \\
e_G \\
e_Q
\end{bmatrix}
\]

For Park’s transformation, use \( P = \frac{\sqrt{2}}{3} \)

\[
\begin{bmatrix}
1/ \sqrt{2} & 1/ \sqrt{2} & 1/ \sqrt{2} \\
\cos \theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\
-\sin \theta & -\sin(\theta - 2\pi/3) & -\sin(\theta + 2\pi/3)
\end{bmatrix}
\]

For a salient pole machine, we already developed the following matrices and equations in class.
\[
L_{SS} = \begin{bmatrix}
L_s + L_m \cos 2\theta & -M_s - L_m \cos 2 \left( \theta + \frac{\pi}{6} \right) & -M_s - L_m \cos 2 \left( \theta + \frac{5\pi}{6} \right) \\
-M_s - L_m \cos 2 \left( \theta + \frac{\pi}{6} \right) & L_s + L_m \cos 2 \left( \theta - \frac{2\pi}{3} \right) & -M_s - L_m \cos 2 \left( \theta - \frac{\pi}{2} \right) \\
-M_s - L_m \cos 2 \left( \theta + \frac{5\pi}{6} \right) & -M_s - L_m \cos 2 \left( \theta - \frac{\pi}{2} \right) & L_s + L_m \cos 2 \left( \theta + \frac{2\pi}{3} \right)
\end{bmatrix}
\]

\[
L_{SR} = \begin{bmatrix}
M_F \cos \theta & M_D \cos \theta & -M_Q \sin \theta \\
M_F \cos \left( \theta - \frac{2\pi}{3} \right) & M_D \cos \left( \theta - \frac{2\pi}{3} \right) & -M_Q \sin \left( \theta - \frac{2\pi}{3} \right) \\
M_F \cos \left( \theta + \frac{2\pi}{3} \right) & M_D \cos \left( \theta + \frac{2\pi}{3} \right) & -M_Q \sin \left( \theta + \frac{2\pi}{3} \right)
\end{bmatrix}
\]

\[
L_{RR} = \begin{bmatrix}
L_F & M_R & 0 \\
M_R & L_D & 0 \\
0 & 0 & L_Q
\end{bmatrix}
\]

\[
\begin{align*}
\psi_0 &= \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -i_0 \end{bmatrix} \\
\psi_d &= \begin{bmatrix} 0 & L_d & kM_F & kM_D & 0 & 0 \end{bmatrix} \begin{bmatrix} -i_d \end{bmatrix} \\
\psi_r &= \begin{bmatrix} 0 & 0 & L_r & M_R & 0 & 0 \end{bmatrix} \begin{bmatrix} -i_r \end{bmatrix} \\
\psi_d &= \begin{bmatrix} 0 & kM_D & M_R & L_D & 0 & 0 \end{bmatrix} \begin{bmatrix} i_d \end{bmatrix} \\
\psi_q &= \begin{bmatrix} 0 & 0 & 0 & 0 & L_q & kM_Q \end{bmatrix} \begin{bmatrix} i_q \end{bmatrix} \\
\psi_Q &= \begin{bmatrix} 0 & 0 & 0 & 0 & kM_Q & L_Q \end{bmatrix} \begin{bmatrix} i_Q \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
e_0 &= \begin{bmatrix} R_a & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -i_0 \end{bmatrix} + \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_0 \end{bmatrix} \\
e_d &= \begin{bmatrix} 0 & R_a & 0 & 0 & -\omega_L q & -\omega_L kM_Q \end{bmatrix} \begin{bmatrix} -i_d \end{bmatrix} + \begin{bmatrix} 0 & L_d & kM_F & kM_D & 0 & 0 \end{bmatrix} \begin{bmatrix} i_d \end{bmatrix} \\
e_r &= \begin{bmatrix} 0 & 0 & R_r & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_r \end{bmatrix} + \begin{bmatrix} 0 & kM_F & L_r & M_R & 0 & 0 \end{bmatrix} \begin{bmatrix} i_f \end{bmatrix} \\
e_d &= \begin{bmatrix} 0 & 0 & 0 & R_D & 0 & 0 \end{bmatrix} \begin{bmatrix} i_d \end{bmatrix} + \begin{bmatrix} 0 & kM_D & M_R & L_D & 0 & 0 \end{bmatrix} \begin{bmatrix} d i_d \end{bmatrix} \\
e_q &= \begin{bmatrix} 0 & \omega_L d & \omega_L kM_F & \omega_L kM_D & R_a & 0 \end{bmatrix} \begin{bmatrix} i_q \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & L_q & kM_Q \end{bmatrix} \begin{bmatrix} d i_q \end{bmatrix} \\
e_Q &= \begin{bmatrix} 0 & 0 & 0 & 0 & R_q & 0 \end{bmatrix} \begin{bmatrix} i_Q \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
L_0 &= L_s - 2M_s \\
L_d &= L_s + M_s + \frac{3}{2} L_m \\
L_q &= L_s + M_s - \frac{3}{2} L_m \\
k &= \sqrt{3}/2
\end{align*}
\]

Re-develop the above matrices or equations for a round rotor machine. You may define whatever new notations, e.g. \( i_{ag}, M_G, L_G \), etc. for the new variables involving damper winding G.

**Solutions:**

Grading rules:

- 20 points for each of 5 major matrices.
- Subtract 1-3 points in case of minor errors
- Subtract 5 points in case of large errors.
- Subtract 5-10 points in case of incomplete attempt or understanding on the problem