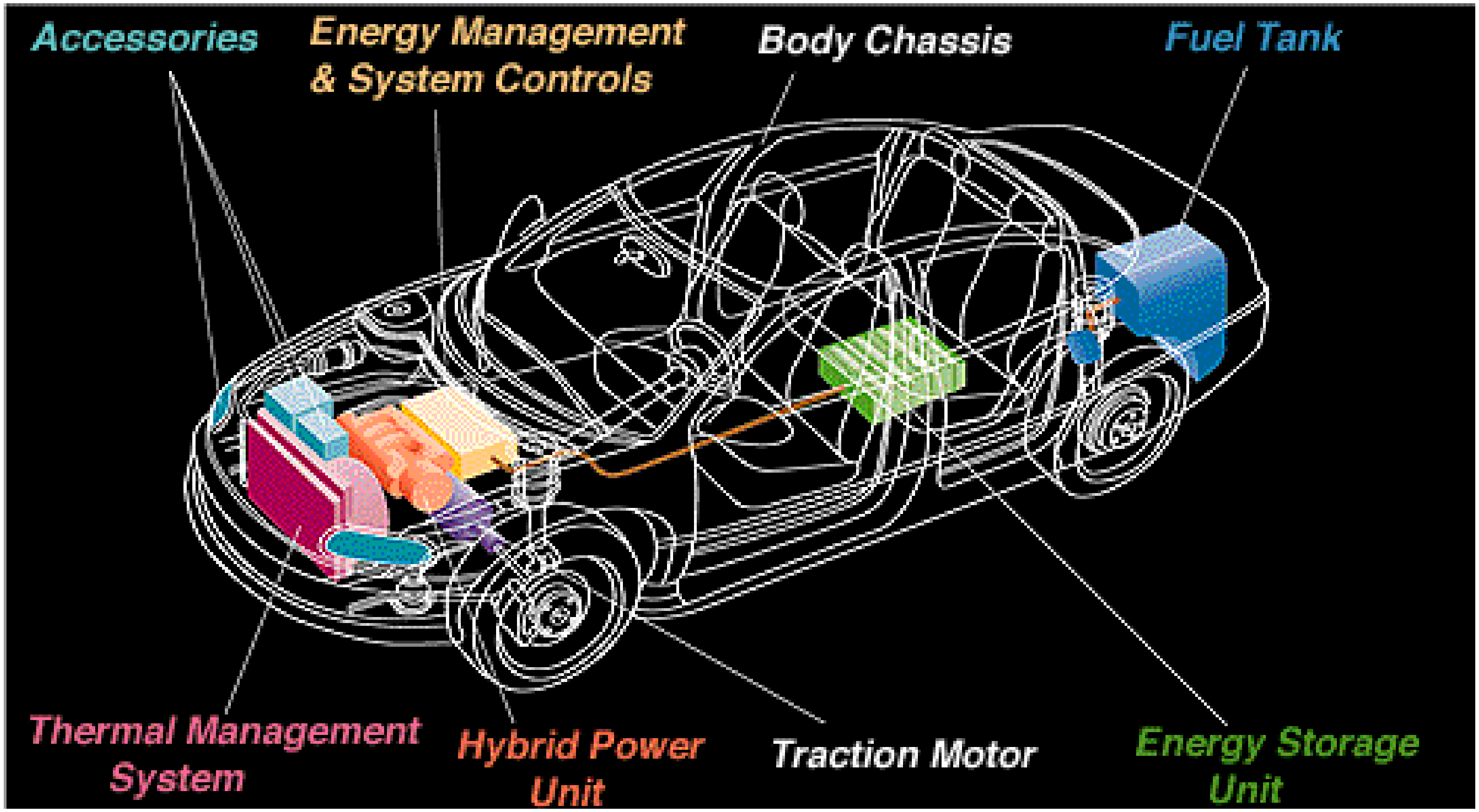
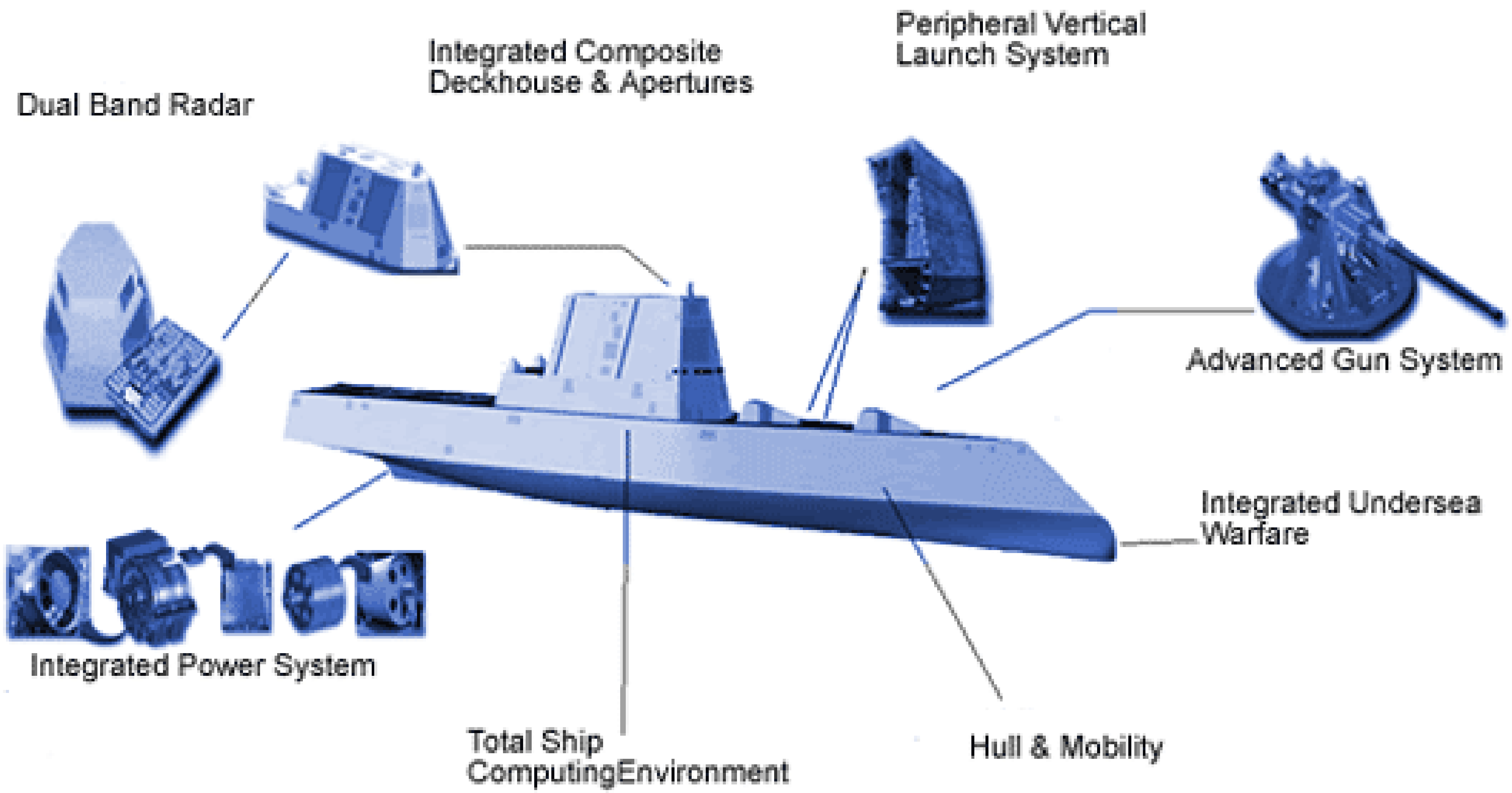
Electric Machines

Leila Parsa
Rensselaer Polytechnic Institute

Hybrid Vehicles



Electric Ship Applications

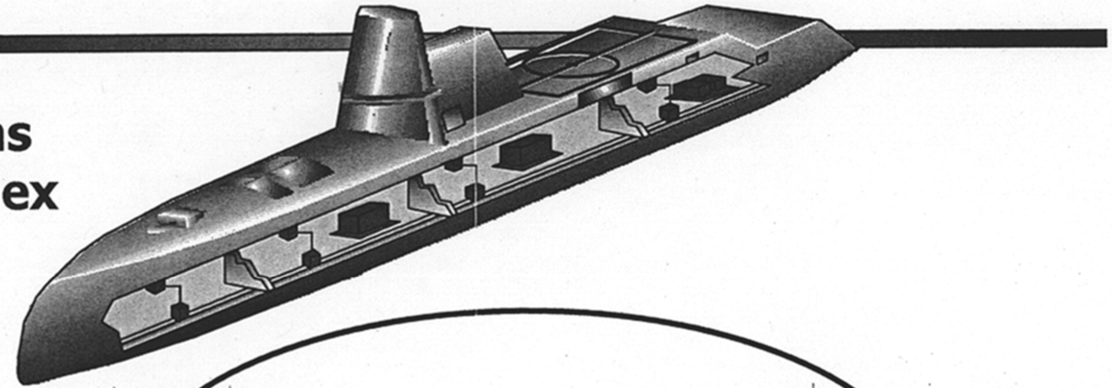


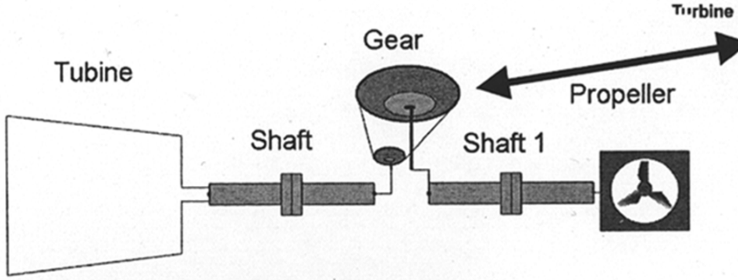
Surface & Subsurface Platforms

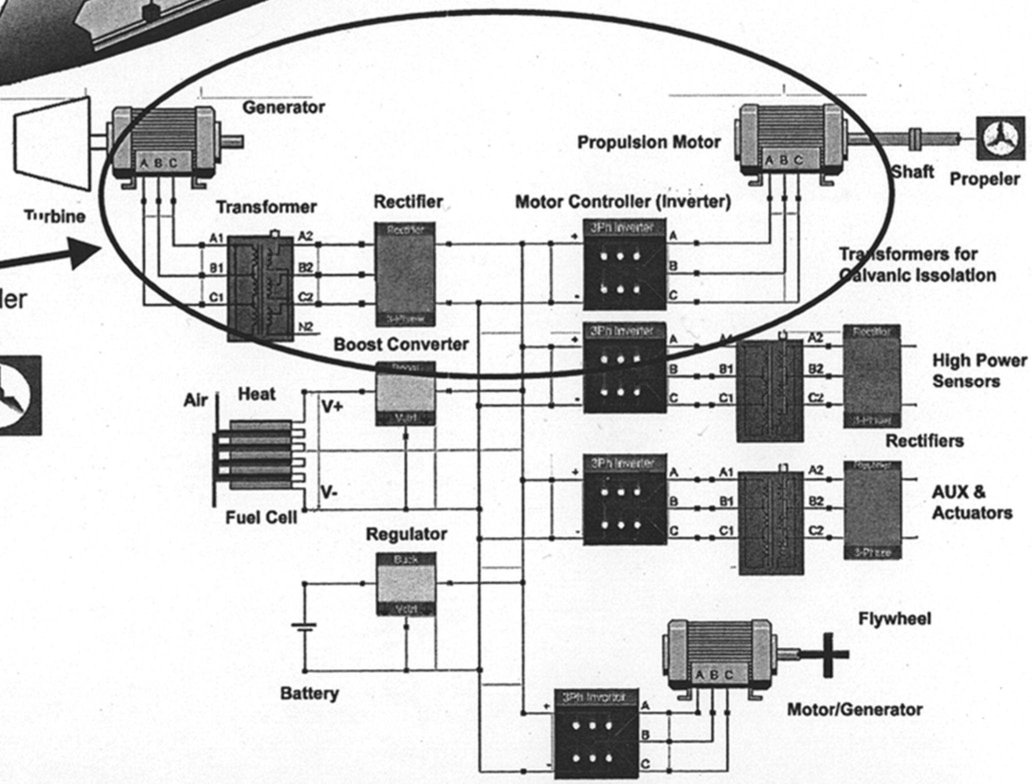
Advanced Electrical Power Systems

TE - 8

Electrical power systems are made of many complex energy conversion machines

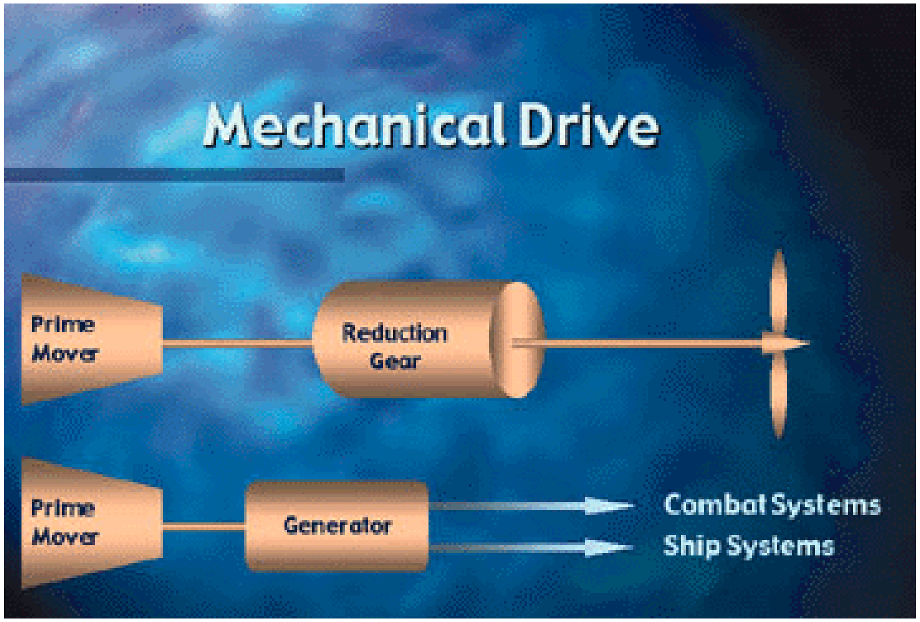


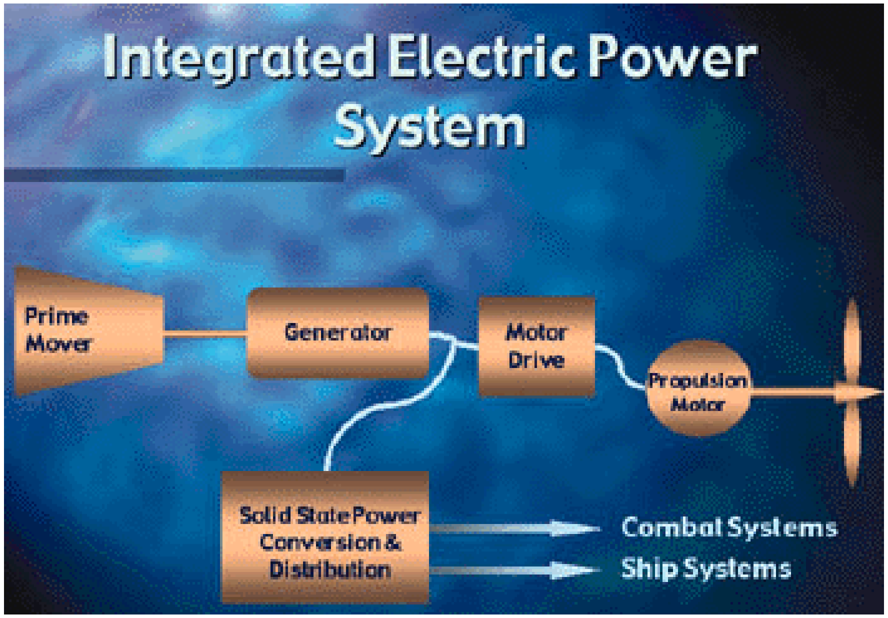




[redacted]

Electric Ship Power System

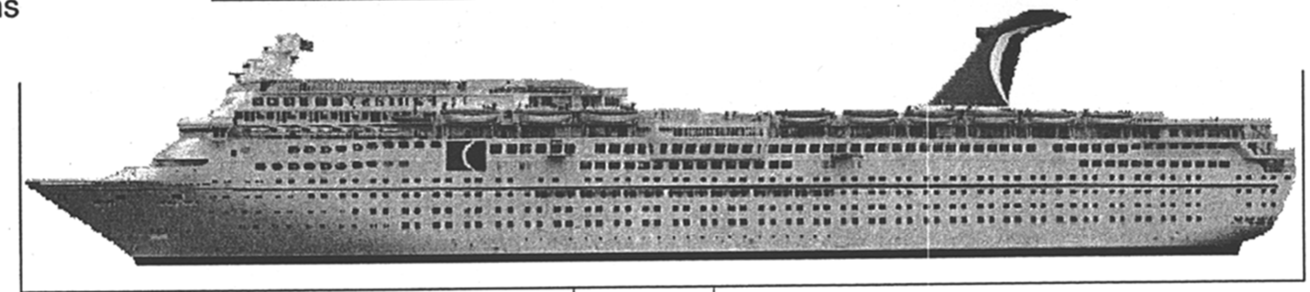




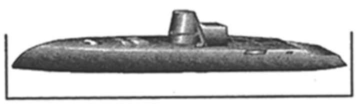
http://www.navy.mil/navydata/cno/n87/usw/issue_9/power_system.html

Advanced Electrical Power Systems

Future Electrical Power System Requirements



856 ft.



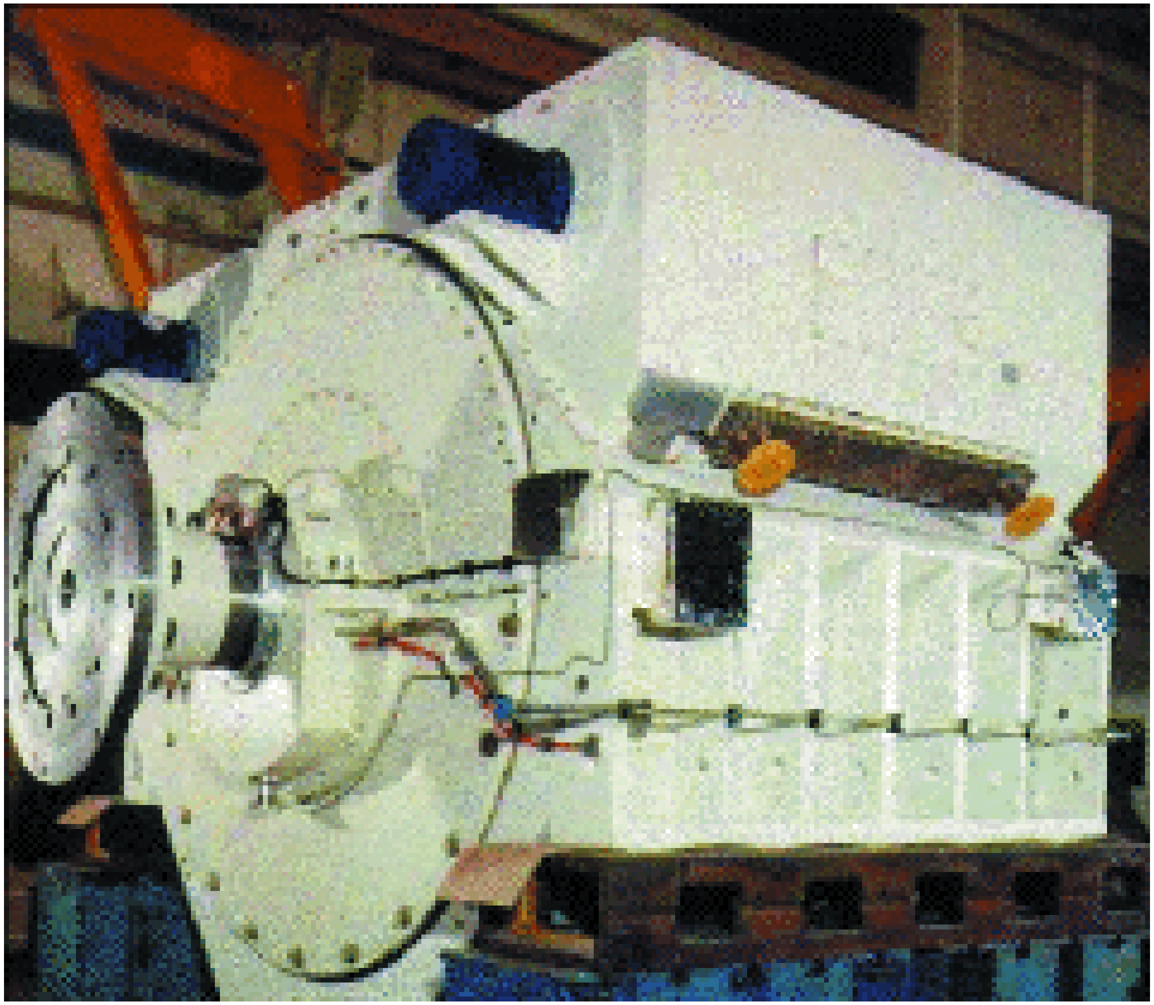
221 ft.

An Electric Navy Ship = 10x Cruise Ship Power Density

With pulse power, An Electric Warship = 20x Cruise Ship Power Density

Metrics	Fantasy Cruise Liner	Navy Littoral Combatant
Length, max: ft. Breadth, DWL: ft . Draught, DWL: ft.	856 103 26	221 22 7.2
Displacement: LT	~40,000	~2,000
Total Propulsion & Service Power Output: kW	42,240	~15,000→ 20,000
Specific Power Output: kW/LT	1.1	~7.5→10

Propulsion Motor 19MW



http://navy-matters.beedall.com/images/20mw.gif

Electric Mining Trucks

One of the haul trucks fitted with a pantograph for the trolley-assist system, which uses electric power from the main grid to move the trucks up the steep pit ramps.

Electric Shovel and Truck

Electric shovels load the broken material into trucks for transport from the pit. Initial pit equipment included shovels with 11 cubic yard buckets and 85 ton production trucks. For the 1990s, the mine is converting its fleet to 170 and 190 ton trucks.

Unmanned Aerial Vehicle

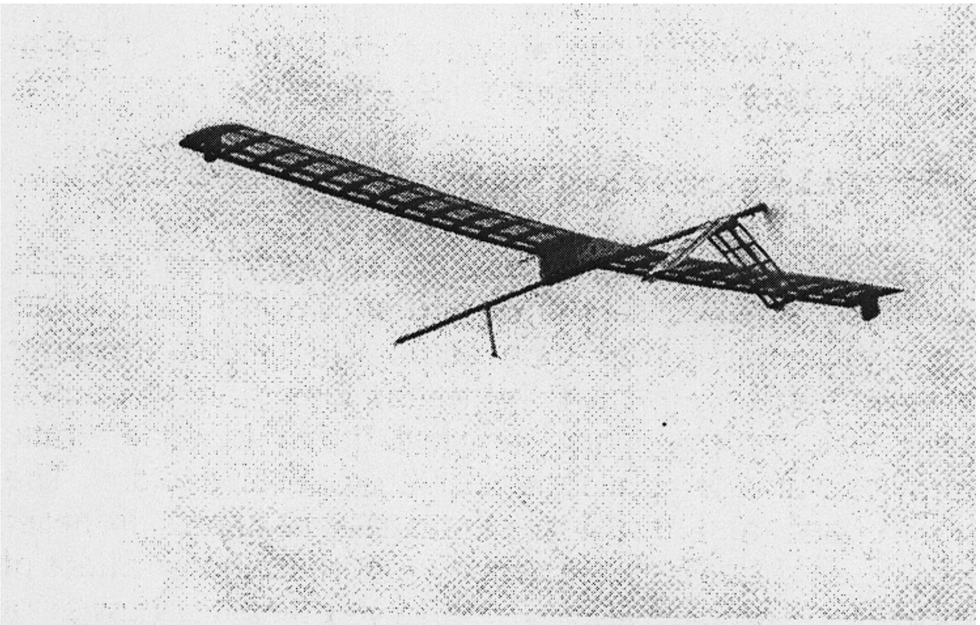


Figure 1: Boeing's Solar Powered UAV in Flight over El Mirage Dry Lake

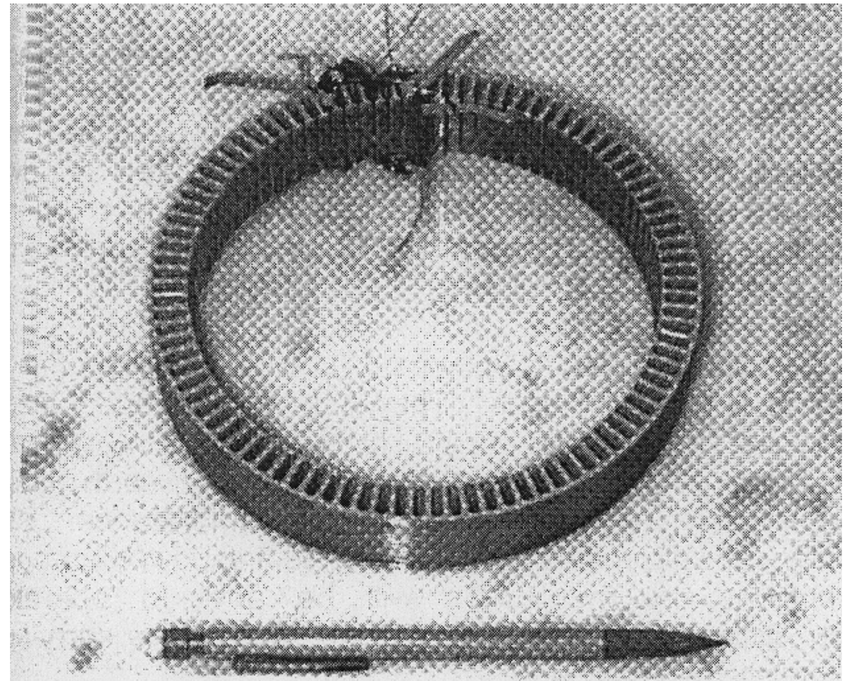


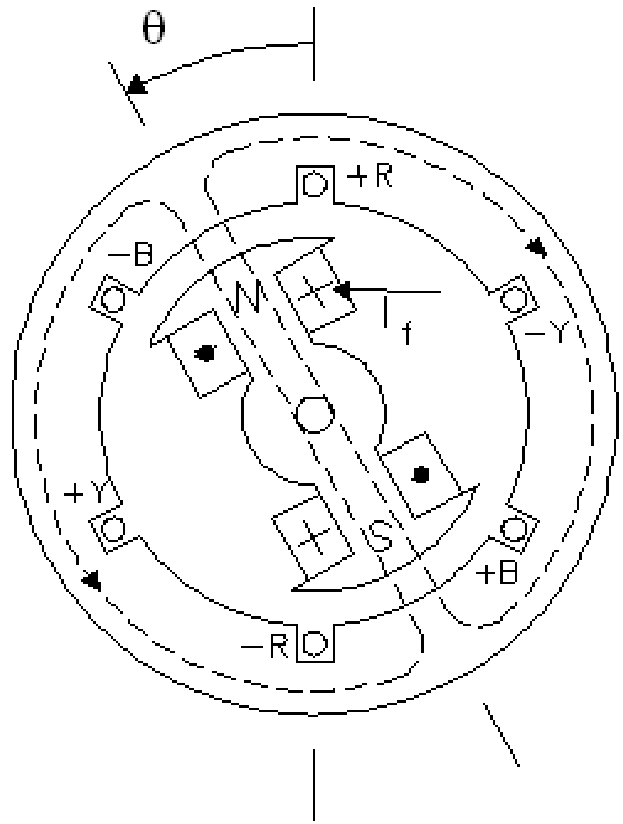
Figure 10: Picture of AVEOX 6.0-Inch Diameter Stator

Manned Aerial Vehicle

Sonex Aircraft
E-Flight Initiative

brushless dc motor
efficiency: 96%
motor weight: 50 lbs.
flight time: 1 hr.
motor power: 100 hp.
voltage: 270 Vdc

Reference Frame Theory



Introduced by R.H. Park in 1929 to model synchronous machines

Three-Phase Transformation to the Arbitrary Reference Frame

$$f_{qd0s} = K_s f_{abcs}$$

$$f_{qd0s} = \begin{bmatrix} f_{qs} \\ f_{ds} \\ f_{0s} \end{bmatrix} \qquad f_{abcs} = \begin{bmatrix} f_{as} \\ f_{bs} \\ f_{cs} \end{bmatrix}$$

f = voltage, current, or flux linkage

q = *q*-axis (quadrature axis)
d = *d*-axis (direct axis)
0 = zero sequence

a = *a*-phase
b = *b*-phase
c = *c*-phase

The Reference Frame Transformation

$$K_s = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin(\theta) & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

ω = reference frame speed (rad/sec)
θ = reference frame position (rad)

The Inverse Transformation

$$f_{abcs} = K_s^{-1} f_{qd0s}$$

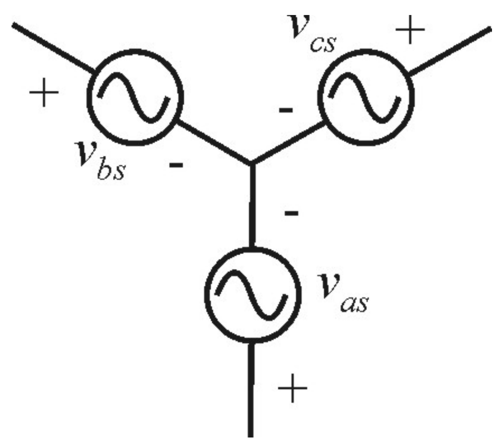
$$K_s^{-1} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 1 \\ \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) & 1 \end{bmatrix}$$

Example: Three-Phase Set of Voltages

$$v_{as} = \sqrt{2} V_s \cos\left(\theta_e + \phi_v\right)$$

$$v_{bs} = \sqrt{2} V_s \cos\left(\theta_e + \phi_v - \frac{2\pi}{3}\right)$$

$$v_{cs} = \sqrt{2} V_s \cos\left(\theta_e + \phi_v + \frac{2\pi}{3}\right)$$



$$\theta_e = \omega_e t$$

$$\omega_e = 2\pi f$$

f - electric frequency (Hz)

ω_e - electric radian frequency (rad/sec)

θ_e - electrical position (rad)

V_s - rms Voltage (V)

ϕ_v - phase shift (rad)

Transform to the Arbitrary Reference Frame

$$v_{qd0s} = K_s v_{abcs}$$

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{0s} \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin(\theta) & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \sqrt{2} V_s \cos(\theta_e + \phi_v) \\ \sqrt{2} V_s \cos\left(\theta_e + \phi_v - \frac{2\pi}{3}\right) \\ \sqrt{2} V_s \cos\left(\theta_e + \phi_v + \frac{2\pi}{3}\right) \end{bmatrix}$$

Voltages in Arbitrary Reference Frame

_q_-axis voltage

$$v_{qs} = \frac{2}{3}\sqrt{2}V_s\left[\cos\left(\theta\right)\cos\left(\theta_e+\phi_v\right)+\cos\left(\theta-\tfrac{2\pi}{3}\right)\cos\left(\theta_e+\phi_v-\tfrac{2\pi}{3}\right)+\cos\left(\theta+\tfrac{2\pi}{3}\right)\cos\left(\theta_e+\phi_v+\tfrac{2\pi}{3}\right)\right]$$

using the identity,

$$\cos\left(x\right)\cos\left(y\right)+\cos\left(x-\tfrac{2\pi}{3}\right)\cos\left(y-\tfrac{2\pi}{3}\right)+\cos\left(x+\tfrac{2\pi}{3}\right)\cos\left(y+\tfrac{2\pi}{3}\right)=\frac{3}{2}\cos\left(x-y\right)$$

$$\boxed{v_{qs} = \sqrt{2}V_s\cos\left(\theta-\theta_e-\phi_v\right)}$$

_d_-axis voltage

$$v_{ds} = \frac{2}{3}\sqrt{2}V_s\left[\sin\left(\theta\right)\cos\left(\theta_e+\phi_v\right)+\sin\left(\theta-\tfrac{2\pi}{3}\right)\cos\left(\theta_e+\phi_v-\tfrac{2\pi}{3}\right)+\sin\left(\theta+\tfrac{2\pi}{3}\right)\cos\left(\theta_e+\phi_v+\tfrac{2\pi}{3}\right)\right]$$

using the identity,

$$\sin\left(x\right)\cos\left(y\right)+\sin\left(x-\tfrac{2\pi}{3}\right)\cos\left(y-\tfrac{2\pi}{3}\right)+\sin\left(x+\tfrac{2\pi}{3}\right)\cos\left(y+\tfrac{2\pi}{3}\right)=\frac{3}{2}\sin\left(x-y\right)$$

$$\boxed{v_{ds} = \sqrt{2}V_s\sin\left(\theta-\theta_e-\phi_v\right)}$$

zero sequence voltage

$$\boxed{v_{0s} = \frac{1}{3}\sqrt{2}V_s\left[\cos\left(\theta_e+\phi_v\right)+\cos\left(\theta_e+\phi_v-\tfrac{2\pi}{3}\right)+\cos\left(\theta_e+\phi_v+\tfrac{2\pi}{3}\right)\right]=0}$$

Numerical Example

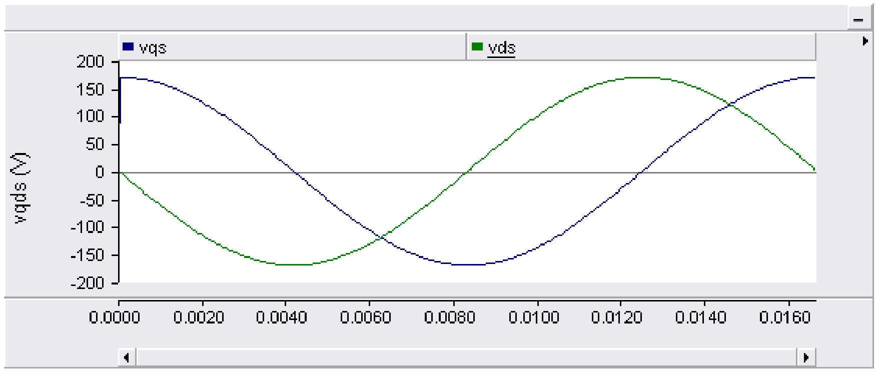
208 V, 3-phase, *f* = 60 Hz (208 V line-to-line rms)

$$V_s = \frac{208\text{ V}}{\sqrt{3}} = 120\text{ V} \qquad \phi_v = 0$$

1. Stationary reference frame $\theta = 0$

$$v_{qs}^{s} = \sqrt{2}V_s \cos\left(\theta_e\right)$$

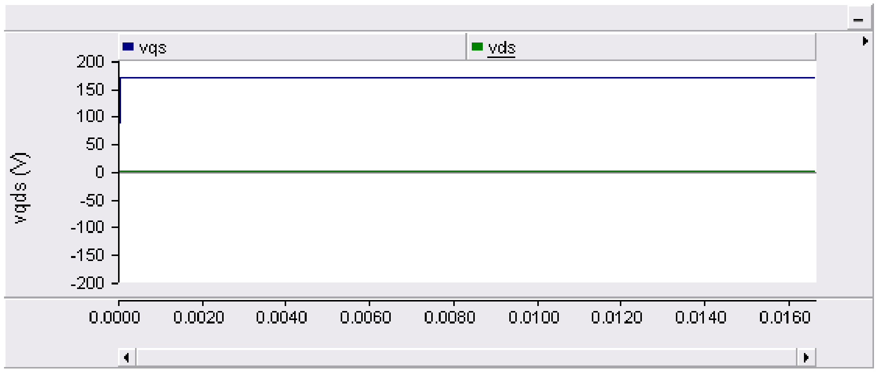
$$v_{ds}^{s} = -\sqrt{2}V_s \sin\left(\theta_e\right)$$



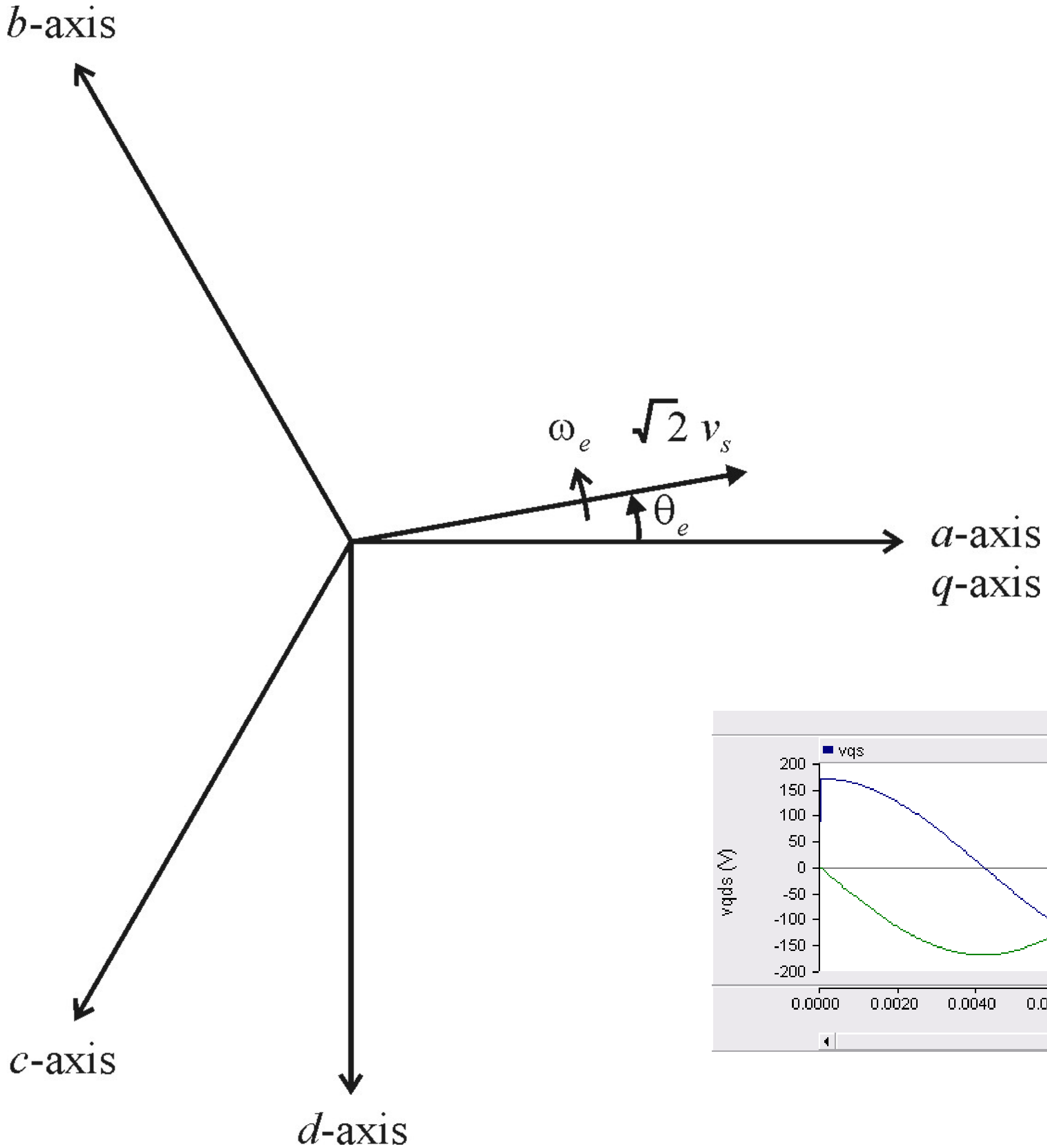
2. Synchronous reference frame $\theta = \theta_e$

$$v_{qs}^{e} = \sqrt{2}V_s \cos\left(\phi_v\right) = \sqrt{2}V_s$$

$$v_{ds}^{e} = -\sqrt{2}V_s \sin\left(\phi_v\right) = 0$$

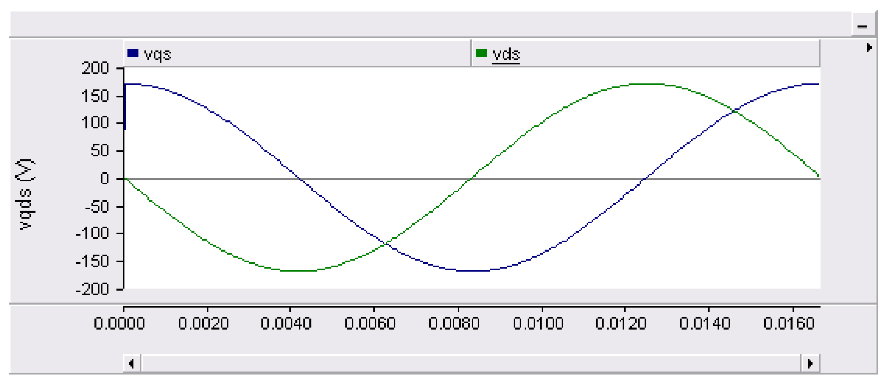


Axis Sketch with θ=0

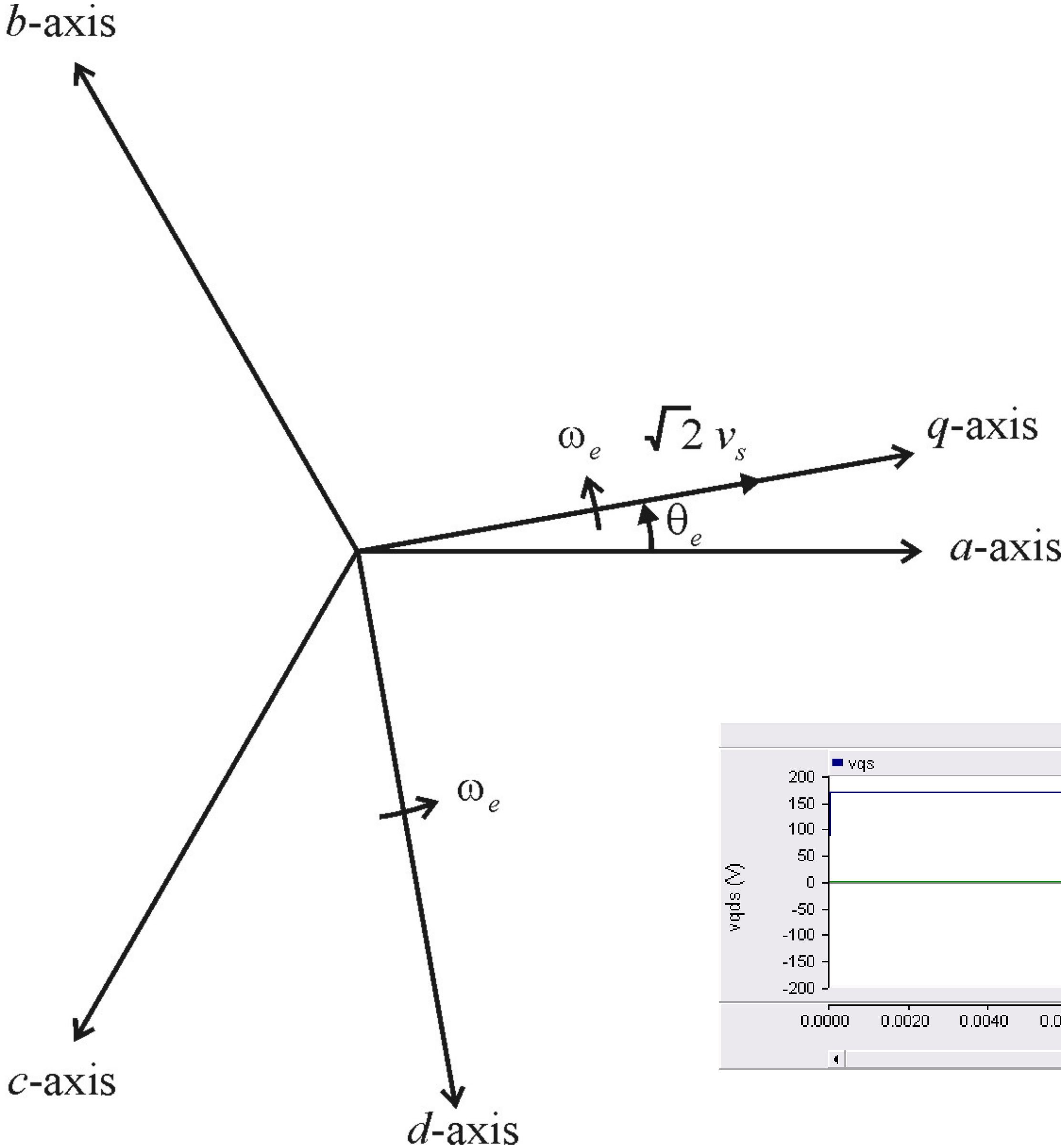


$$v_{qs}^{s} = \sqrt{2}V_s \cos(\theta_e)$$

$$v_{ds}^{s} = -\sqrt{2}V_s \sin(\theta_e)$$

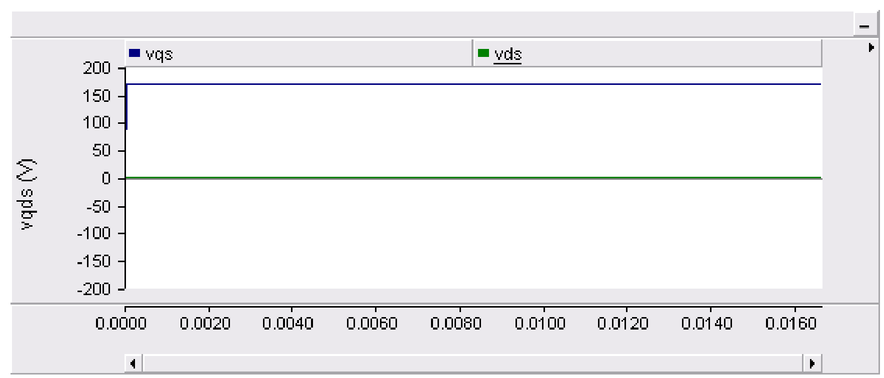


Axis Sketch with $\theta=\theta_e$



$$v_{qs}^{e} = \sqrt{2}V_s \cos\left(\phi_v\right) = 170\,\mathrm{V}$$

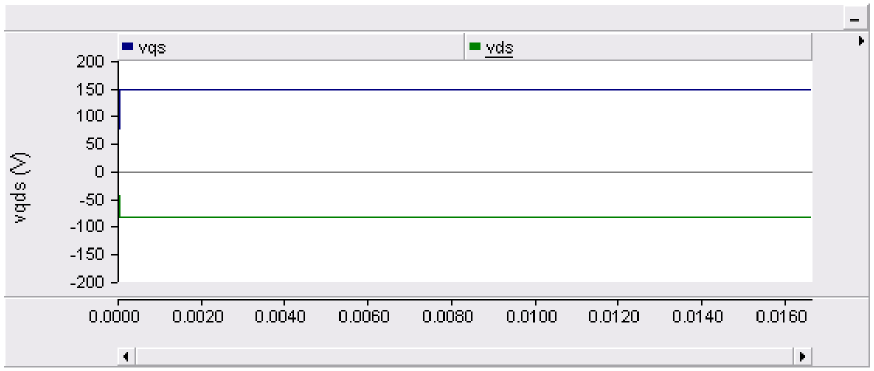
$$v_{ds}^{e} = -\sqrt{2}V_s \sin\left(\phi_v\right) = 0$$



$\theta=\theta_e$ and $\phi_v=30^o$

$$v_{qs}^{e} = \sqrt{2}\,V_s \cos\left(\phi_v\right) = 147\,\mathrm{V}$$

$$v_{ds}^{e} = -\sqrt{2}\,V_s \sin\left(\phi_v\right) = -85\,\mathrm{V}$$



Commonly Used Reference Frames

Arbitrary	$\omega = \omega$	f_{qs}, f_{ds}, f_{0s}	K_s
Stationary	$\omega = 0$	$f_{qs}^{s}, f_{ds}^{s}, f_{0s}$	K_s^s
Synchronous	$\omega = \omega_e$	$f_{qs}^{e}, f_{ds}^{e}, f_{0s}$	K_s^e
Rotor	$\omega = \omega_r$	$f_{qs}^{r}, f_{ds}^{r}, f_{0s}$	K_s^r

compact notation

$$v_{qd0s} = K_s v_{abcs}$$

$$v_{qd0s}^{e} = K_s^{e} v_{abcs}$$

Notes: In all reference frames $f_{0s} = \frac{1}{3}\left(f_{as} + f_{bs} + f_{cs}\right)$

Real Power in the *q*-*d* Reference Frame

$$P_{in} = v_{as}\, i_{as} + v_{bs}\, i_{bs} + v_{cs}\, i_{cs} = \begin{bmatrix} v_{as} & v_{bs} & v_{cs} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} = v_{abcs}^{T}\, i_{abcs}$$

$$P_{in} = \left(K_s^{-1}\, v_{qd0s}\right)^{T} \left(K_s^{-1} i_{qd0s}\right) = v_{qd0s}^{T} \left(K_s^{-1}\right)^{T} K_s^{-1}\, i_{qd0s}$$

$$\left(K_s^{-1}\right)^{T} K_s^{-1} = \begin{bmatrix} \frac{3}{2} & 0 & 0 \\ 0 & \frac{3}{2} & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$P_{in} = \frac{3}{2}\left(v_{qs} i_{qs} + v_{ds} i_{ds}\right) + 3 v_{0s} i_{0s}$$

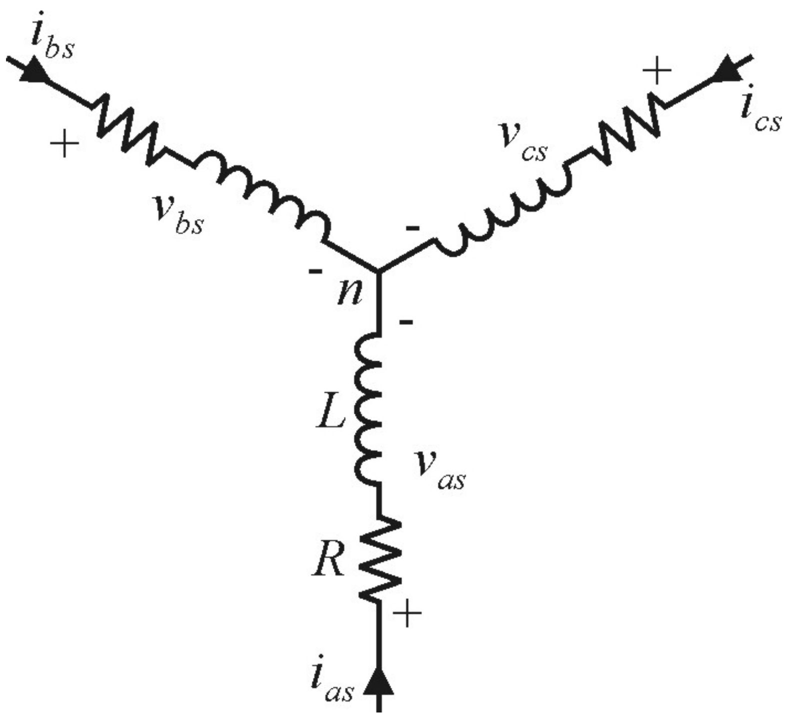
Reference Frame Transformation

Developed by R.H. Park in 1929 for analysis of synchronous machines.

Allows treatment of balanced three-phase ac systems as two-phase dc systems.

- This leads to application of classical control theory
- Also simplifies control equations of some systems

Transforming Circuit Elements: R-L Example



voltage equations

$$v_{as} = Ri_{as} + p\lambda_{as}$$

$$v_{bs} = Ri_{bs} + p\lambda_{bs}$$

$$v_{cs} = Ri_{cs} + p\lambda_{cs}$$

note: $p = \dfrac{d}{dt}$

flux linkage equations

$$\lambda_{as} = Li_{as}$$

$$\lambda_{bs} = Li_{bs}$$

$$\lambda_{cs} = Li_{cs}$$

Transform Flux Linkage Equations

1. Compress equations

$$\lambda_{abcs} = Li_{abcs}$$

2. Transform equations

$$K_s\lambda_{abcs} = K_sLi_{abcs} = LK_si_{abcs}$$

$$\downarrow$$

$$\lambda_{qd0s} = Li_{qd0s}$$

3. Expand equations

$$\lambda_{qs} = Li_{qs}$$

$$\lambda_{ds} = Li_{ds}$$

$$\lambda_{0s} = Li_{0s}$$

Transform Voltage Equations

1. Compress equations

$$v_{abcs} = Ri_{abcs} + p\lambda_{abcs}$$

2. Transform equations

$$K_s v_{abcs} = K_s R i_{abcs} + K_s p\lambda_{abcs}$$

$$\downarrow$$

$$v_{qd0s} = Ri_{qd0s} + K_s p\{K_s^{-1}\lambda_{qd0s}\}$$

$$= Ri_{qd0s} + K_s K_s^{-1} p\lambda_{qd0s} + K_s p\{K_s^{-1}\}\lambda_{qd0s}$$

$$K_s K_s^{-1} = I$$

$$p\{K_s^{-1}\} = \omega \begin{bmatrix} -\sin(\theta) & \cos(\theta) & 0 \\ -\sin\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) & 0 \\ -\sin\left(\theta + \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) & 0 \end{bmatrix}$$

$$K_s p\{K_s^{-1}\} = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$v_{qd0s} = Ri_{qd0s} + p\lambda_{qd0s} + \omega\lambda_{dqs} \qquad \lambda_{dqs} = \begin{bmatrix} \lambda_{ds} \\ -\lambda_{qs} \\ 0 \end{bmatrix}$$

3. Expand equations

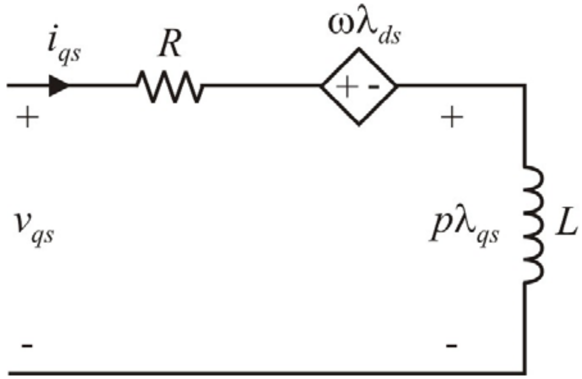
$$v_{qs} = Ri_{qs} + p\lambda_{qs} + \omega\lambda_{ds}$$

$$v_{ds} = Ri_{ds} + p\lambda_{ds} - \omega\lambda_{qs}$$

$$v_{0s} = Ri_{0s} + p\lambda_{0s}$$

Equivalent Circuit

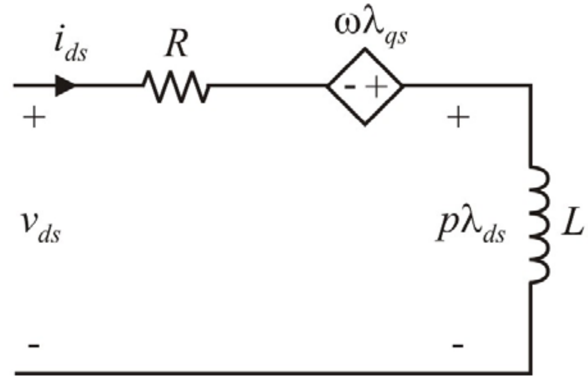
q-axis



$$v_{qs} = Ri_{qs} + p\lambda_{qs} + \omega\lambda_{ds}$$

$$\lambda_{qs} = Li_{qs}$$

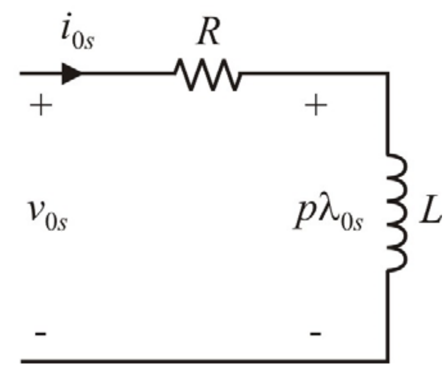
d-axis



$$v_{ds} = Ri_{ds} + p\lambda_{ds} - \omega\lambda_{qs}$$

$$\lambda_{ds} = Li_{ds}$$

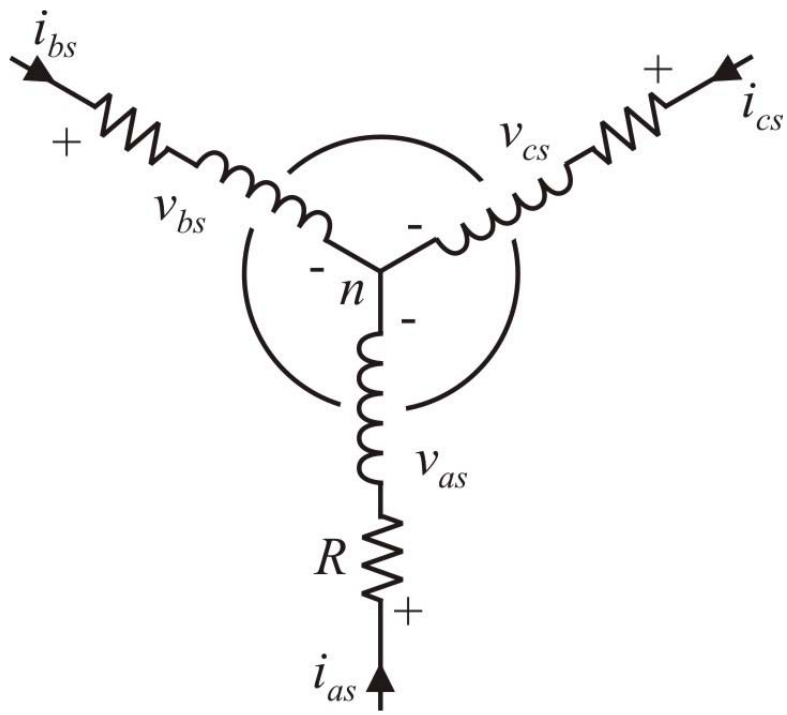
zero sequence



$$v_{0s} = Ri_{0s} + p\lambda_{0s}$$

$$\lambda_{0s} = Li_{0s}$$

Coupled Inductors



voltage equations

$$v_{abcs} = Ri_{abcs} + p\lambda_{abcs}$$

$$v_{qd0s} = Ri_{qd0s} + p\lambda_{qd0s} + \omega\lambda_{dqs} \qquad \lambda_{dqs} = \begin{bmatrix} \lambda_{ds} \\ -\lambda_{qs} \\ 0 \end{bmatrix}$$

Coupled Inductors

flux linkage equations

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = \begin{bmatrix} L_{ls} + L_{ms} & -\frac{1}{2} L_{ms} & -\frac{1}{2} L_{ms} \\ -\frac{1}{2} L_{ms} & L_{ls} + L_{ms} & -\frac{1}{2} L_{ms} \\ -\frac{1}{2} L_{ms} & -\frac{1}{2} L_{ms} & L_{ls} + L_{ms} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

$$\lambda_{abcs} = \mathbf{L}_s \, i_{abcs}$$

$$K_s \lambda_{abcs} = K_s \mathbf{L}_s \, i_{abcs}$$

$$\downarrow$$

$$\lambda_{qd0s} = K_s \mathbf{L}_s \, K_s^{-1} \, i_{qd0s}$$

$$K_s \mathbf{L}_s K_s^{-1} = \begin{bmatrix} L_{ls} + \frac{3}{2} L_{ms} & 0 & 0 \\ 0 & L_{ls} + \frac{3}{2} L_{ms} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix}$$

expanded form

$$\lambda_{qs} = \left(L_{ls} + \frac{3}{2} L_{ms} \right) i_{qs}$$

$$\lambda_{ds} = \left(L_{ls} + \frac{3}{2} L_{ms} \right) i_{ds}$$

$$\lambda_{0s} = L_{ls} i_{0s}$$

Transformation of Circuit Elements

Balanced three-phase ac circuits transformed to two-phase dc circuits (neglecting the zero sequence and assuming analysis in the synchronous reference frame)

Flux linkage equations for inductive circuits were used for generality to other circuits; including electric machines

Coupling terms between the *q*- and *d*-axes result from the transformation. These will later be viewed as back-emf terms when observing electric machinery in the synchronous reference frame.

Balanced Steady-State Voltages

$$v_{as} = \sqrt{2}V_s \cos\left(\omega_e t + \phi_v\right)$$

$$v_{bs} = \sqrt{2}V_s \cos\left(\omega_e t + \phi_v - \frac{2\pi}{3}\right)$$

$$v_{cs} = \sqrt{2}V_s \cos\left(\omega_e t + \phi_v + \frac{2\pi}{3}\right)$$

$$\tilde{V}_{as} = V_s e^{j\phi_v} \longrightarrow \tilde{V}_{as} = V_s \angle \phi_v$$

$$\tilde{V}_{bs} = \tilde{V}_s e^{-j\frac{2\pi}{3} + j\phi_v}$$

$$\tilde{V}_{cs} = \tilde{V}_s e^{j\frac{2\pi}{3} + j\phi_v}$$

Synchronous Reference Frame *q*-*d* Voltages

Synchronous: $\omega = \omega_e$

$$V_{qs}^{e} = \sqrt{2} V_s \cos\left(\phi_v\right)$$

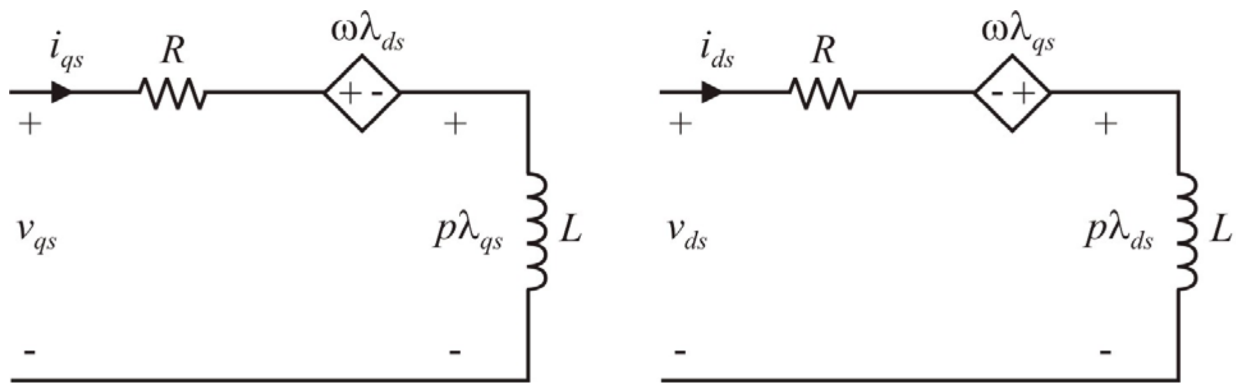
$$V_{ds}^{e} = -\sqrt{2} V_s \sin\left(\phi_v\right)$$

note:

$$\sqrt{2}\, V_s = \sqrt{\left(V_{qs}^{e}\right)^2 + \left(V_{ds}^{e}\right)^2}$$

$$\phi_v = \tan^{-1}\left(\frac{-V_{ds}^{e}}{V_{qs}^{e}}\right)$$

Steady-State Calculation ($\omega = \omega_e$)



steady-state equations

$$V_{qs}^{e} = RI_{qs}^{e} + \omega_e LI_{ds}^{e}$$

$$V_{ds}^{e} = RI_{ds}^{e} - \omega_e LI_{qs}^{e}$$

steady-state equations in matrix form

$$\begin{bmatrix} V_{qs}^e \\ V_{ds}^e \end{bmatrix} = \begin{bmatrix} R & \omega_e L \\ -\omega_e L & R \end{bmatrix} \begin{bmatrix} I_{qs}^e \\ I_{ds}^e \end{bmatrix}$$

solve for currents

$$\begin{bmatrix} I_{qs}^e \\ I_{ds}^e \end{bmatrix} = \frac{1}{R^2 + \omega_e{}^2 L^2} \begin{bmatrix} R & -\omega_e L \\ \omega_e L & R \end{bmatrix} \begin{bmatrix} V_{qs}^e \\ V_{ds}^e \end{bmatrix}$$

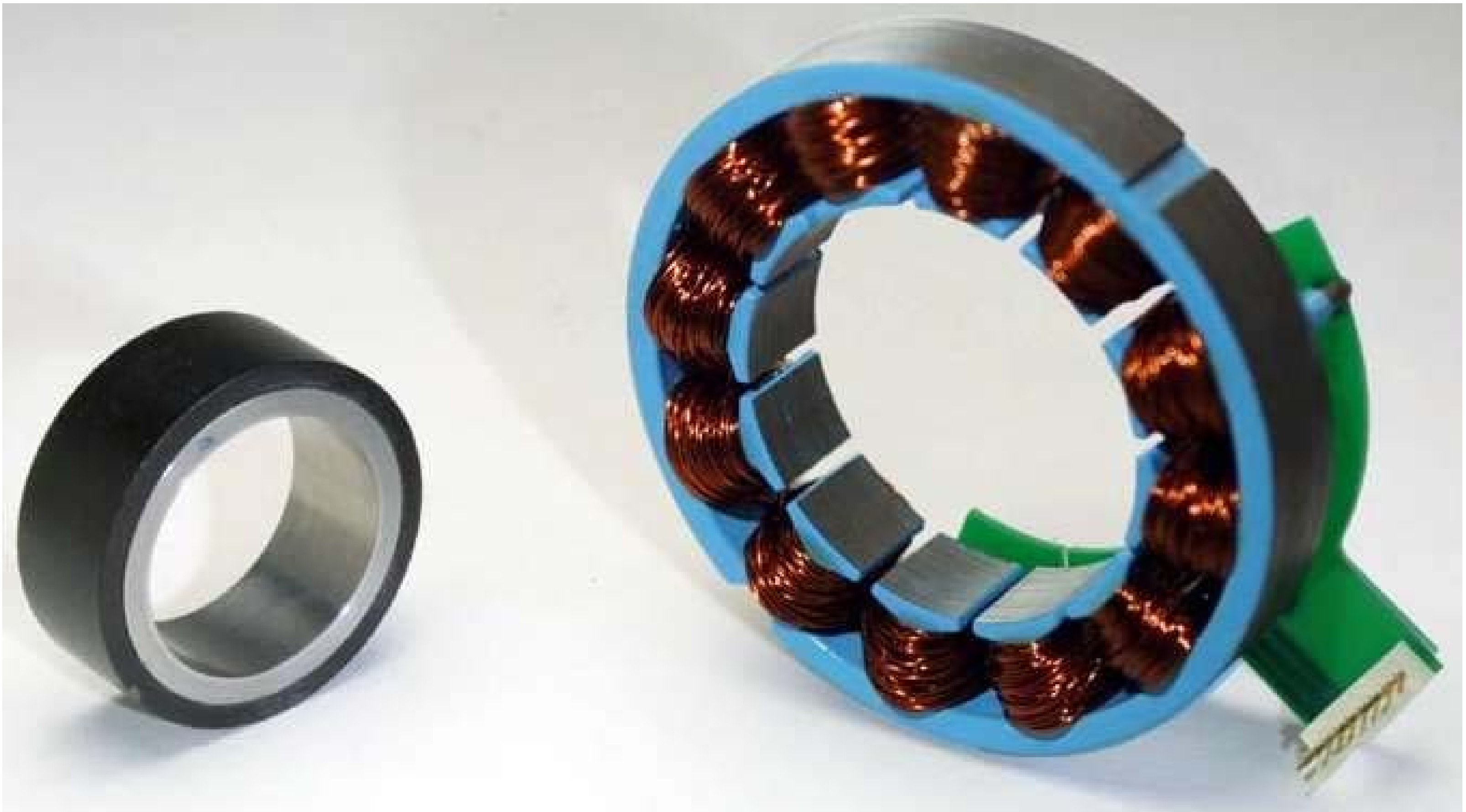
Steady-State *q-d* Calculations

In the synchronous reference frame, the *q-d* circuits are supplied from dc and the corresponding dc solution is steady-state (inductors treated as short-circuit, capacitors treated as open-circuit)

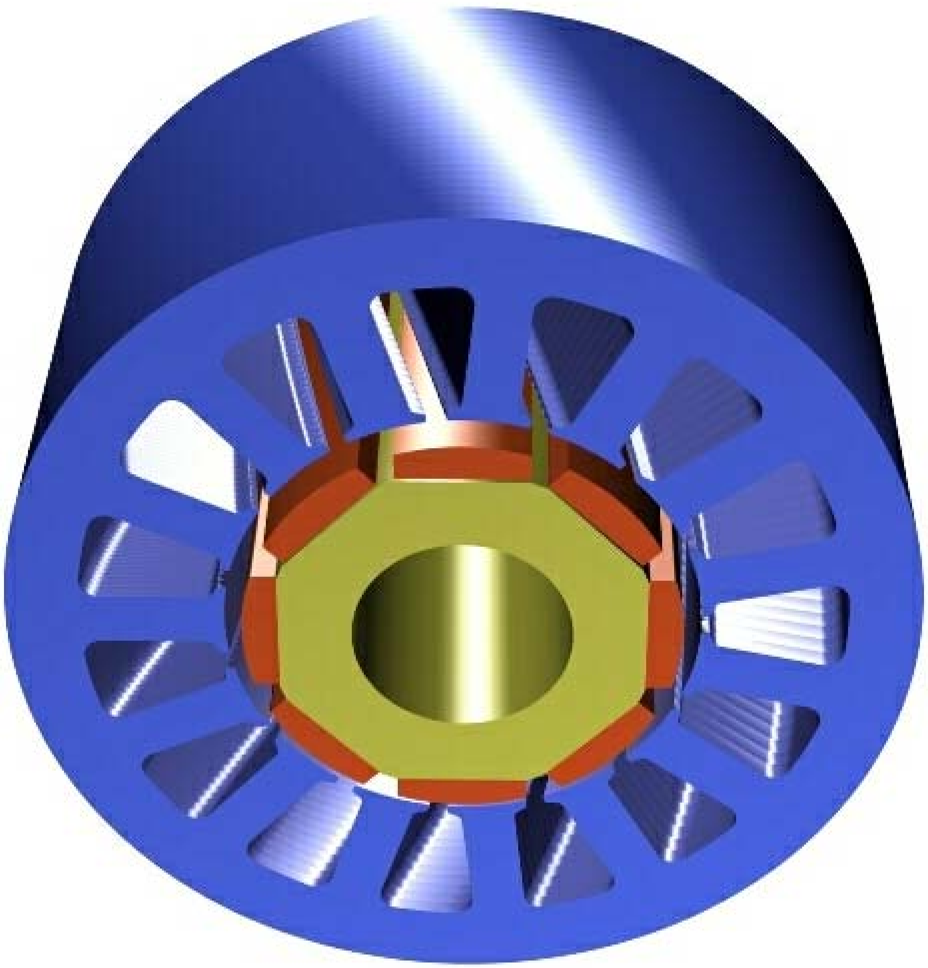
Can be used to analyze steady-state operation of electric machines

Equations can be linearized about the dc operating point for application of control theory

PMSM Rotor and Stator

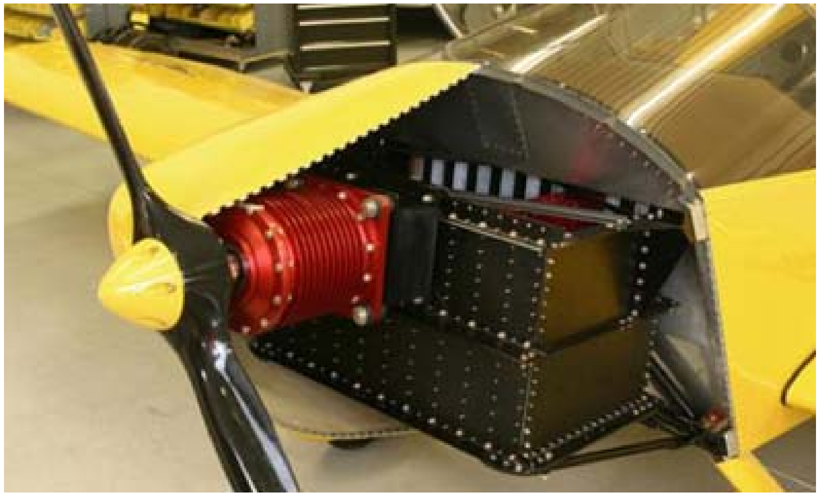


Rotor and Stator Designs



Some PMSM Applications

Electric bike



Electric airplane

Remote operated vehicle

All-terrain vehicle

Permanent Magnet Motor Drives

Permanent magnet AC motors (PMAC) :

- quasi-rectangular fed BLDC Motor
- Sinusoidally fed PMSM

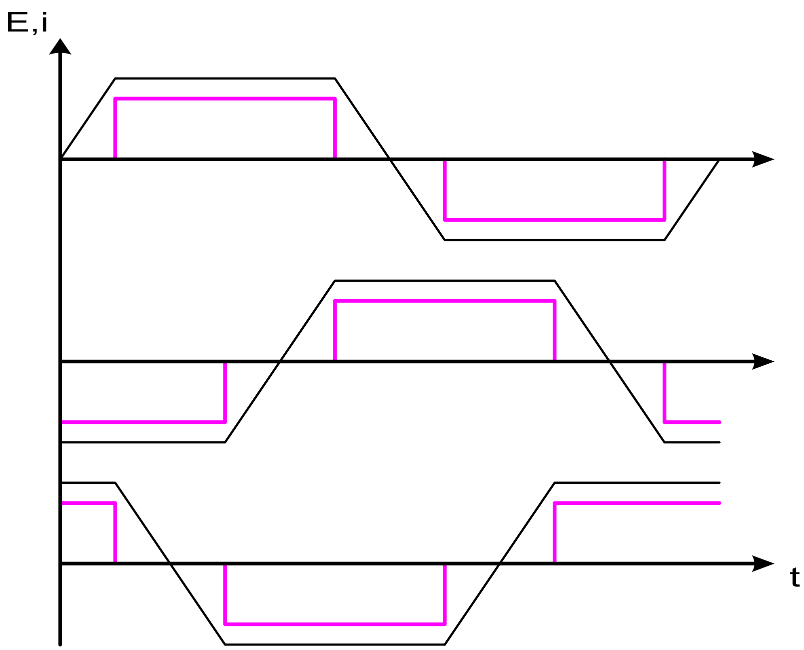
BLDC motors:

- Two phases are conducting at each instant of time
- A low resolution position sensor (Hall Sensor) is enough for providing commutation instants
- Higher torque density compared to PMSM at low and medium speeds
- Commutation torque pulsation
- The performance deteriorates at high speeds

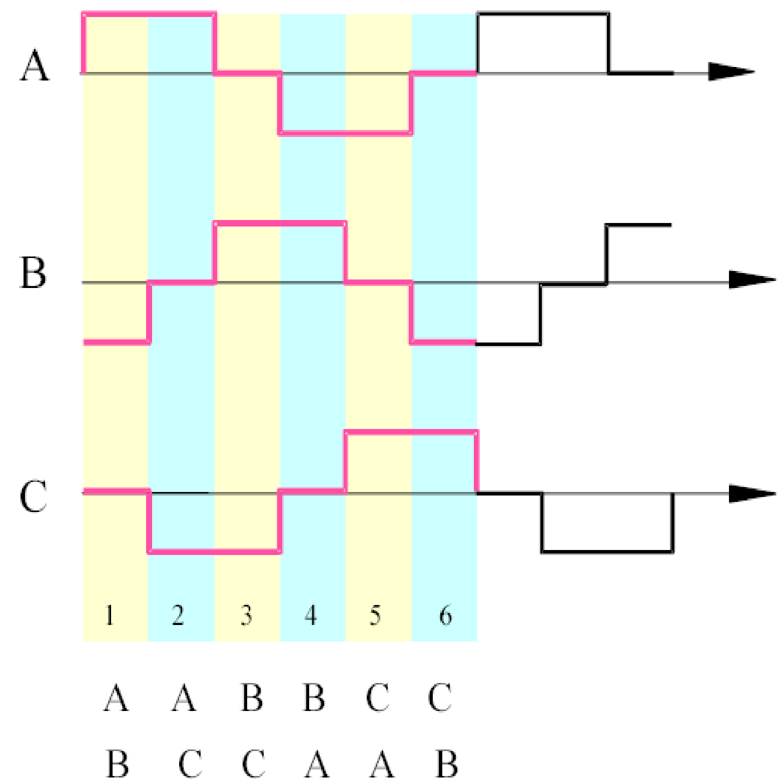
PMSM :

- Position information is needed at each instant of time
- More advanced control techniques with faster transient response is applicable to this kind of drive over the whole speed range.

BLDC Motors



Back-EMF and
phase current



BLDC Current Waveform

BLDC Motors

Equations governing three phase BLDC motor:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L-M & 0 & 0 \\ 0 & L-M & 0 \\ 0 & 0 & L-M \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix}$$

where *ea* , *eb* , and *ec* are trapezoidal back-EMFs.

The electromagnetic torque is expressed as:

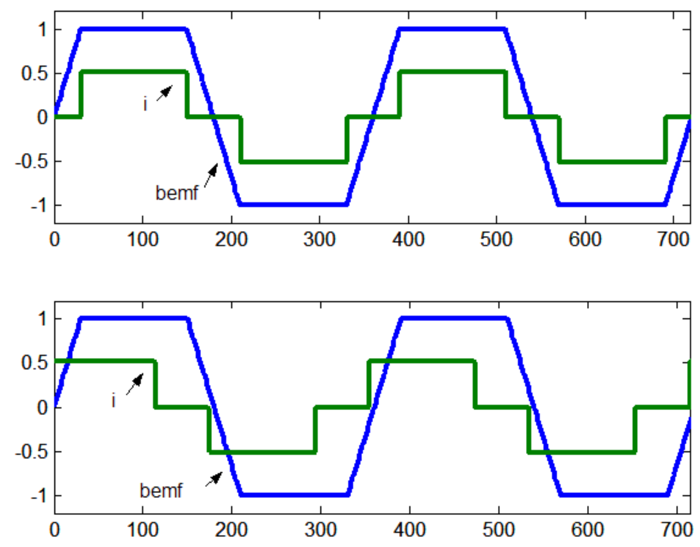
$$T_e = \frac{1}{\omega_r}(e_a i_a + e_b i_b + e_c i_c)$$

And, the interaction of *Te* with the load torque determines how the motor speed builds up:

$$T_e = T_L + J\frac{d\omega_r}{dt} + B\omega_r$$

High Speed Operation of BLDC Motors

- The BLDC operation above rated speed is being performed by advance angle technique
- At a given torque or a given speed, it is difficult to find the exact advance-angle to be applied.
- At high speeds the stator winding inductance's can cause the phase current to deviate significantly from the ideal rectangular waveform which will reduce the torque production at high speeds



PMSM

Transformation from stationary frame of reference to rotating frame of reference

Equations governing three phase PMSM motor:

$$v_{ds} = r_s i_{ds} - \omega L_{qs} i_{qs} + L_{ds}\frac{di_{ds}}{dt}$$

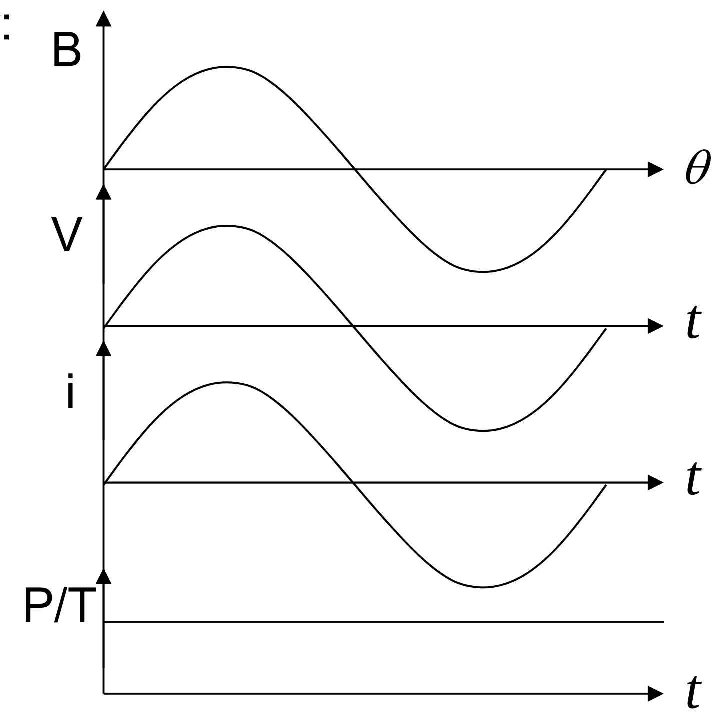
$$v_{qs} = r_s i_{qs} + \omega (L_{ds} i_{ds} + \psi_m) + L_{qs}\frac{di_{qs}}{dt}$$

Torque equation:

$$T_e = P_n \left[\psi_m i_{qs} + \left(L_{qs} - L_{ds}\right) i_{ds} i_{qs}\right]$$

Electromechanical motion equation

$$T_e = T_L + J\frac{d\omega_r}{dt} + B\omega_r$$



PMSM

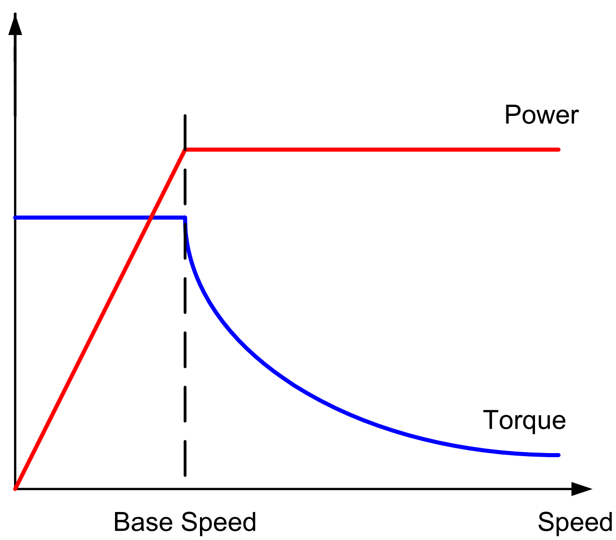
High Speed Operation of PMSM

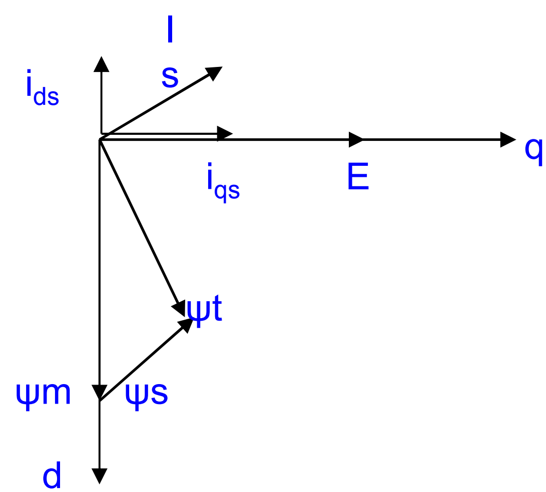
The PMSM operation above rated speed is being performed by injecting negative id (field weakening) and considering current and voltage limitation:

$$(i_{ds})^2 + (i_{qs})^2 = I_{rated}^2$$

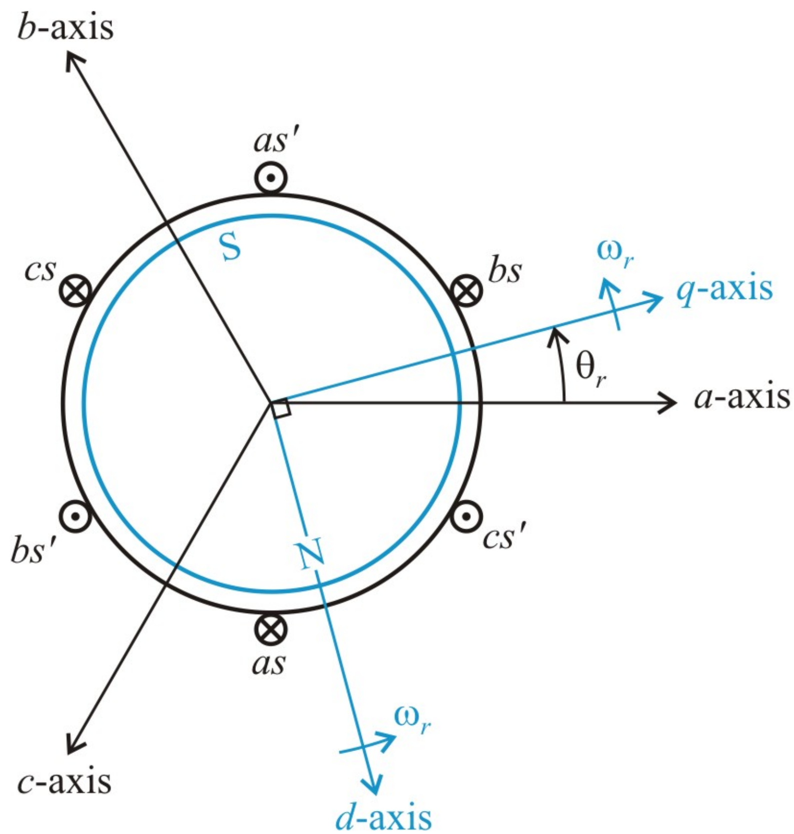
$$(\psi_m + L_{ds} i_{ds}^r)^2 + (L_{qs} i_{qs})^2 = (\frac{V_{rated}}{\omega_r})^2$$

Based on the above two limits the amount of d axis current to be injected is known at any given speed

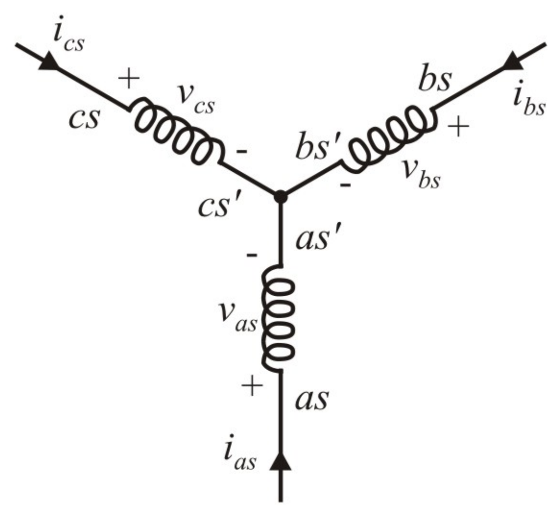




Permanent Magnet Synchronous Machines (PMSMs)



motor construction

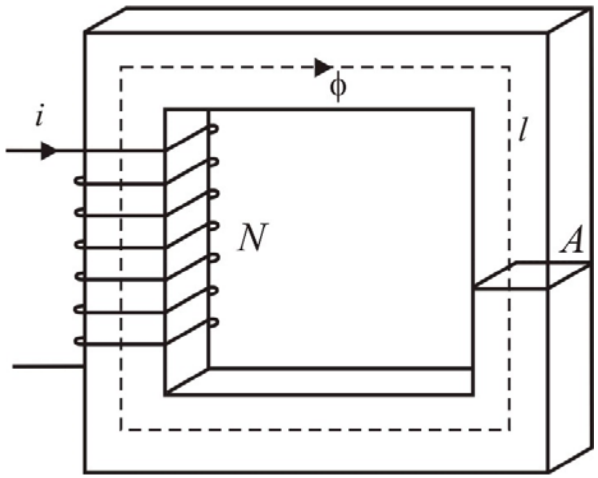


winding connection

assume: 1. Round rotor
2. Sinusoidal flux linkages

$$L_{mq} = L_{md} = \frac{3}{2} L_{ms}$$

Basic Magnetic Equations



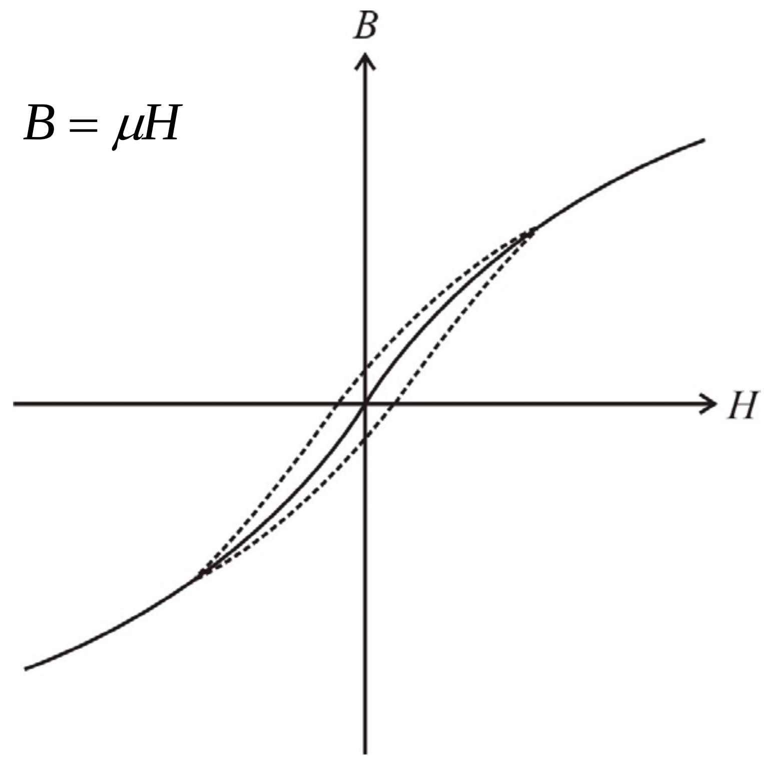
$$\Im = Ni$$

$$\Re = \frac{l}{\mu A}$$

$$\phi = \frac{\Im}{\Re} = \frac{Ni}{\Re}$$

$$\mu = \mu_r \mu_o$$

$$B = \mu H$$



$$\lambda = N\phi = \frac{N^2 i}{\Re}$$

define inductance $$L = \frac{N^2}{\Re}$$

$$\lambda = Li$$

flux linkage and current are proportional to *B* and *H*

$$H = \frac{Ni}{l} \qquad B = \frac{\phi}{A} = \frac{\lambda}{N \cdot A}$$

Basic Magnetic Definitions

$\mathfrak{I}$ = MMF (A·t)

$\mathfrak{R}$ = reluctance (1/H)

l = flux path length (m)

A = cross sectional area (m^2)

$\mu_o = 4\pi*10^{-7}$ H/m permeability of free space

μ_r = relative permeability $\mu_r = 4000$ iron, $\mu_r = 2000$ steel

Φ = magnetic flux (Wb=V·sec)

λ = Flux linkage (V·sec)

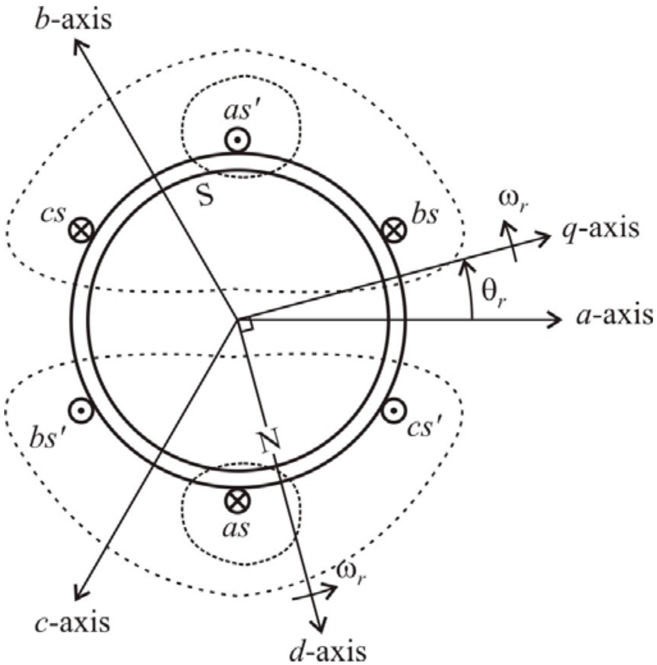
L = inductance (H=Ω·sec)

B = flux density (V·sec/m^2)

H = magnetic field intensity (A·t/m)

Inductance Terms

consider a current in the *a*-phase winding



self inductance L_{asas}

$$L_{asas} = L_{ls} + L_{ms}$$

L_{ls} - stator leakage inductance (H)

L_{ms} - stator magnetizing inductance (H)

mutual inductance L_{asbs} (determines the component of λ_{as} due to i_{bs})

$$L_{asbs} = \cos\left(120^{\circ}\right) L_{ms} = -\frac{1}{2} L_{ms}$$

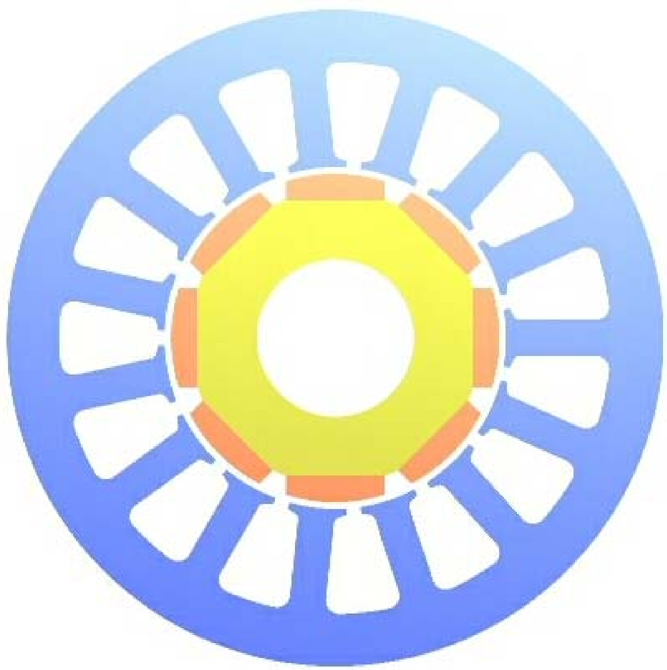
remaining inductance terms

$$L_{asas} = L_{bsbs} = L_{cscs} = L_{ls} + L_{ms}$$

$$L_{asbs} = L_{ascs} = L_{bsas} = L_{bscs} = L_{csas} = L_{csbs} = -\frac{1}{2} L_{ms}$$

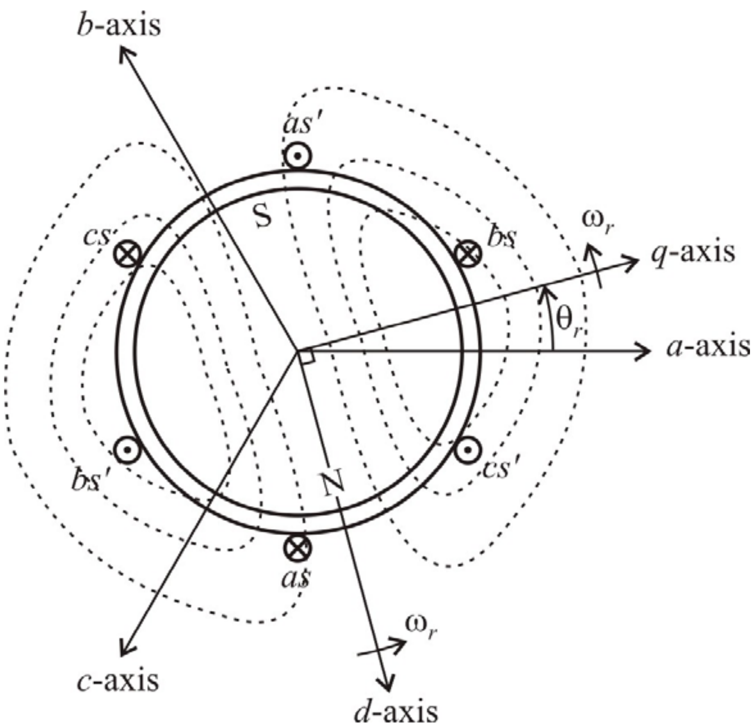
Inductance Related to Machine Dimensions

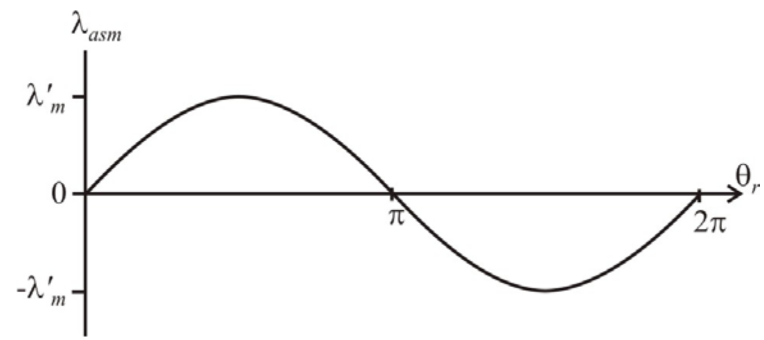
$$L_{ms} = \left(\frac{N_s}{2} \right)^2 \frac{\pi \mu_0 r l}{g}$$



Magnet Flux

magnet flux linking the *a*-phase





$$\lambda_{asm} = \lambda_m' \sin(\theta_r)$$

magnet flux linking the *b*- and *c*-phase

$$\lambda_{bsm} = \lambda_m' \sin\left(\theta_r - \frac{2\pi}{3}\right)$$

$$\lambda_{csm} = \lambda_m' \sin\left(\theta_r + \frac{2\pi}{3}\right)$$

PMSM Equations

machine coil variables

$$v_{as} = r_s i_{as} + p\lambda_{as}$$

$$v_{bs} = r_s i_{bs} + p\lambda_{bs}$$

$$v_{cs} = r_s i_{cs} + p\lambda_{cs}$$

flux linkages

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = \begin{bmatrix} L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L_{ls} + L_{ms} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \lambda_m' \begin{bmatrix} \sin(\theta_r) \\ \sin\left(\theta_r - \frac{2\pi}{3}\right) \\ \sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix}$$

Reduced Flux Linkage Equations

$$\lambda_{as} = (L_{ls} + L_{ms}) i_{as} - \frac{1}{2} L_{ms} i_{bs} - \frac{1}{2} L_{ms} i_{cs} + \lambda_m' \sin(\theta_r)$$

$$= \left(L_{ls} + \frac{3}{2} L_{ms} \right) i_{as} - \frac{1}{2} L_{ms} (i_{as} + i_{bs} + i_{cs}) + \lambda_m' \sin(\theta_r)$$

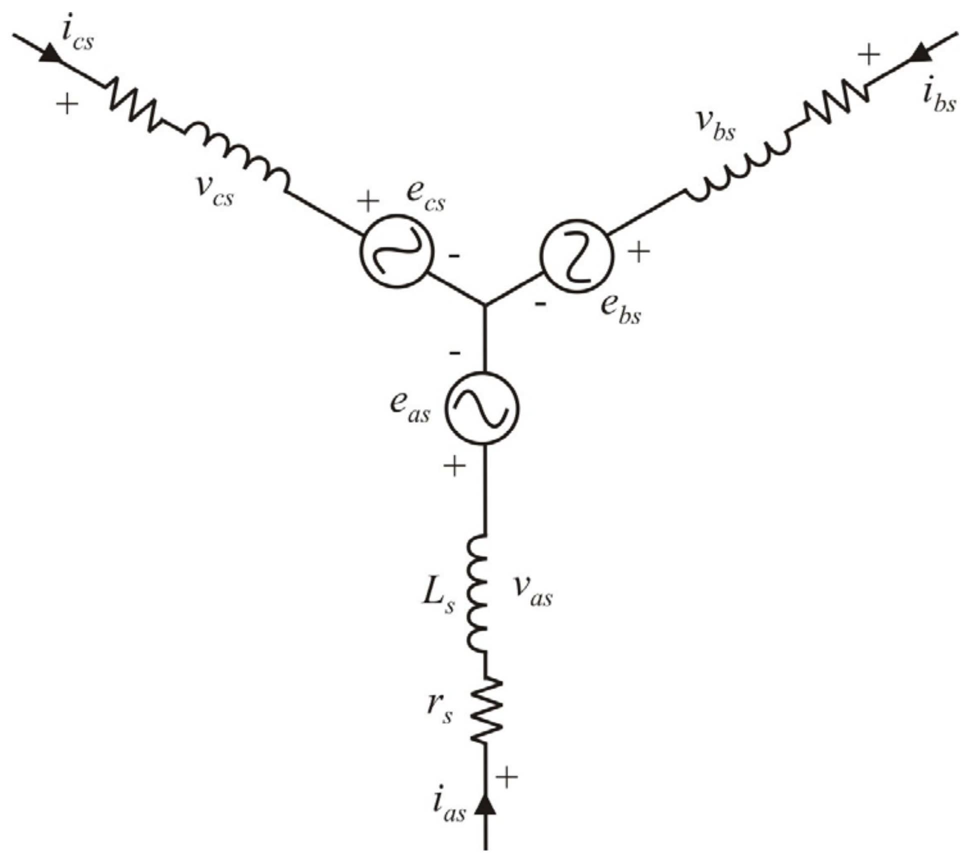
for a wye connection,
$i_{as} + i_{bs} + i_{cs} = 0$

$$\lambda_{as} = L_s i_{as} + \lambda_m' \sin(\theta_r)$$

where $L_s = L_{ls} + (3/2)L_{ms}$

$$v_{as} = r_s i_{as} + L_s p i_{as} + \omega_r \lambda_m' \cos(\theta_r)$$
$$= r_s i_{as} + L_s p i_{as} + e_{as}$$

e_{as} - a-phase back-emf



Coil Voltage Equations

machine variables

$$v_{as} = r_s i_{as} + p\lambda_{as}$$

$$v_{bs} = r_s i_{bs} + p\lambda_{bs}$$

$$v_{cs} = r_s i_{cs} + p\lambda_{cs}$$

rotor reference frame, *q*-*d* variables

$$v_{qs}^r = r_s i_{qs}^r + \omega_r \lambda_{ds}^r + p\lambda_{qs}^r$$

$$v_{ds}^r = r_s i_{ds}^r - \omega_r \lambda_{qs}^r + p\lambda_{ds}^r$$

$$v_{0s} = r_s i_{0s} + p\lambda_{0s}$$

Flux Linkage Expressions

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = \begin{bmatrix} L_{ls} + L_{ms} & -\frac{1}{2} L_{ms} & -\frac{1}{2} L_{ms} \\ -\frac{1}{2} L_{ms} & L_{ls} + L_{ms} & -\frac{1}{2} L_{ms} \\ -\frac{1}{2} L_{ms} & -\frac{1}{2} L_{ms} & L_{ls} + L_{ms} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \lambda'_m \begin{bmatrix} \sin(\theta_r) \\ \sin\left(\theta_r - \frac{2\pi}{3}\right) \\ \sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix}$$

compress equations

$$\lambda_{abcs} = \mathbf{L}_s \, i_{abcs} + \lambda_{abcm}$$

Transform Flux Linkages

$$K_s^r \lambda_{abcs} = K_s^r \mathbf{L}_s \, i_{abcs} + K_s^r \lambda_{abcm}$$

$$\downarrow$$

$$\lambda_{qd0s}^r = K_s^r \mathbf{L}_s \left(K_s^r \right)^{-1} i_{qd0s}^r + \lambda_{qd0m}^r$$

inductance

$$K_s^r \mathbf{L}_s \left(K_s^r \right)^{-1} = \begin{bmatrix} L_{ls} + \frac{3}{2} L_{ms} & 0 & 0 \\ 0 & L_{ls} + \frac{3}{2} L_{ms} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix}$$

define

$$L_s = L_{ls} + \frac{3}{2} L_{ms}$$

magnet flux linkage

$$\lambda_{qd0m}^r = K_s^r \lambda_{abcm} = \begin{bmatrix} 0 \\ \lambda_m' \\ 0 \end{bmatrix}$$

Flux Linkage Equations in the Rotor Reference Frame

$$\lambda_{qs}^{r} = L_s i_{qs}^{r}$$

$$\lambda_{ds}^{r} = L_s i_{ds}^{r} + \lambda_m'$$

$$\lambda_{0s} = L_{ls} i_{0s}$$

compared to *a*-*b*-*c* variables:
no coupled terms and no rotor position dependence

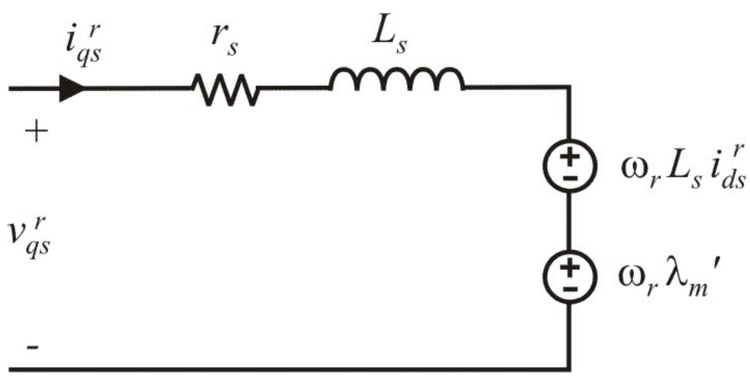
$$\begin{bmatrix} \lambda_{qs}^{r} \\ \lambda_{ds}^{r} \\ \lambda_{0s} \end{bmatrix} = \begin{bmatrix} L_s & 0 & 0 \\ 0 & L_s & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} \begin{bmatrix} i_{qs}^{r} \\ i_{ds}^{r} \\ i_{0s} \end{bmatrix} + \begin{bmatrix} 0 \\ \lambda_m' \\ 0 \end{bmatrix}$$

Equivalent Circuit Model

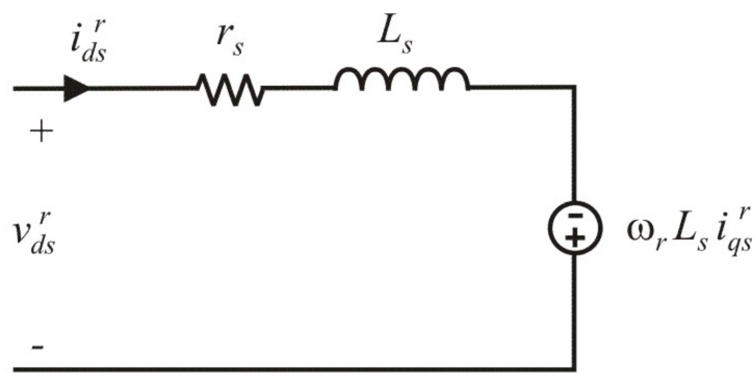
substitute flux linkage equations into voltage equations

$$v_{qs}^r = r_s i_{qs}^r + L_s p i_{qs}^r + \omega_r L_s i_{ds}^r + \omega_r \lambda_m'$$

$$v_{ds}^r = r_s i_{ds}^r + L_s p i_{ds}^r - \omega_r L_s i_{qs}^r$$



q-axis circuit



d-axis circuit

General Machine with *P* Poles

$$\theta_r = \frac{P}{2}\theta_{rm}$$

$$\omega_r = \frac{P}{2}\omega_{rm}$$

θ_r - electrical rotor position (rad)

θ_{rm} - mechanical rotor position (rad)

ω_r - electrical rotor speed (rad/sec)

ω_{rm} - mechanical rotor speed (rad/sec)

Torque Equation from Power

$$P_{out} = \frac{3}{2}\left[\omega_r L_s i_{ds}^r i_{qs}^r + \omega_r \lambda_m' i_{qs}^r - \omega_r L_s i_{qs}^r i_{ds}^r\right] = \frac{3}{2}\omega_r \lambda_m' i_{qs}^r$$

$$P_{out} = T_e \omega_{rm} = \frac{2}{P} T_e \omega_r \longrightarrow T_e = \frac{P}{2}\frac{P_{out}}{\omega_r}$$

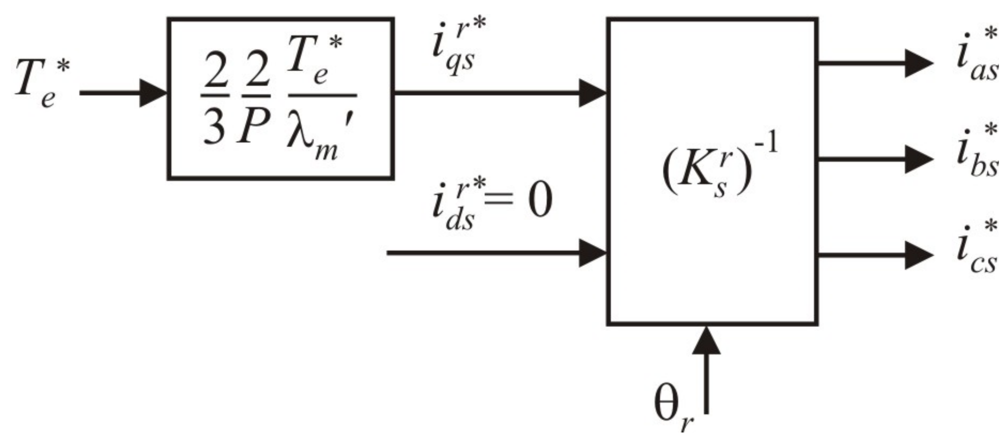
$$T_e = \frac{3}{2}\frac{P}{2}\lambda_m' i_{qs}^r$$

Torque Control

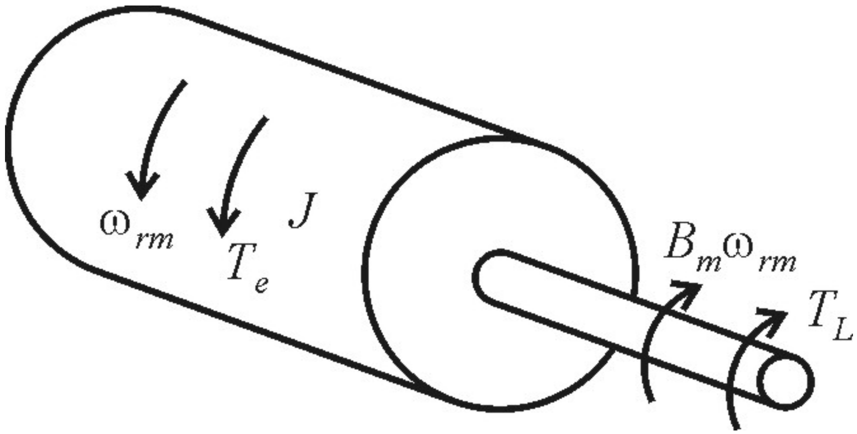
in *q*-*d*-0 variables

$$T_e = \frac{3}{2}\frac{P}{2}\lambda_m' i_{qs}^r$$

torque control



Mechanical Equations



$$\sum T = J\frac{d\omega_{rm}}{dt}$$

$$T_e - B_m\omega_{rm} - T_L = Jp\omega_{rm}$$

also $\omega_{rm} = p\theta_{rm}$

T_e - electrical torque (N·m)
B_m - friction constant (Kg·m^2/sec)
J - total inertia (Kg·m^2)
ω_{rm} - mechanical speed (rad/sec)
T_L - load torque (N·m)
θ_{rm} - mechanical rotor position (rad)

PMSM Model

The PMSM can be modeled with a fairly straightforward resistance, inductance, and back-emf term in each phase.

If the zero sequence current can be neglected, the *q*-*d* model represents the machine as a two-phase circuit with dc quantities in the steady-state. The two circuits are coupled by back-emf terms.

The torque equation in the *q*-*d* model is simplified compared to the machine-variable (*a*-*b*-*c*) model. This property will be used for developing a torque control.

The *q*-*d* model also leads to simple control in other systems such as induction machines, active rectifiers, and active filters.

PMSM Steady-State Equations

round rotor machine $L_q = L_d = L_s$

constant $v_{qs}^r \; v_{ds}^r \; i_{qs}^r \; i_{ds}^r$

use notation $V_{qs}^r \; V_{ds}^r \; I_{qs}^r \; I_{ds}^r$

with $p = \dfrac{d}{dt} = 0$

$$V_{qs}^r = r_s I_{qs}^r + \omega_r L_s I_{ds}^r + \omega_r \lambda_m'$$

$$V_{ds}^r = r_s I_{ds}^r - \omega_r L_s I_{qs}^r$$

$$T_e = \frac{3}{2}\frac{P}{2}\lambda_m' I_{qs}^r = B_m \omega_{rm} + T_L$$

Steady-State Solutions

solving for voltages

$$\begin{bmatrix} V_{qs}^r \\ V_{ds}^r \end{bmatrix} = \begin{bmatrix} r_s & \omega_r L_s \\ -\omega_r L_s & r_s \end{bmatrix} \begin{bmatrix} I_{qs}^r \\ I_{ds}^r \end{bmatrix} + \begin{bmatrix} \omega_r \lambda_m' \\ 0 \end{bmatrix}$$

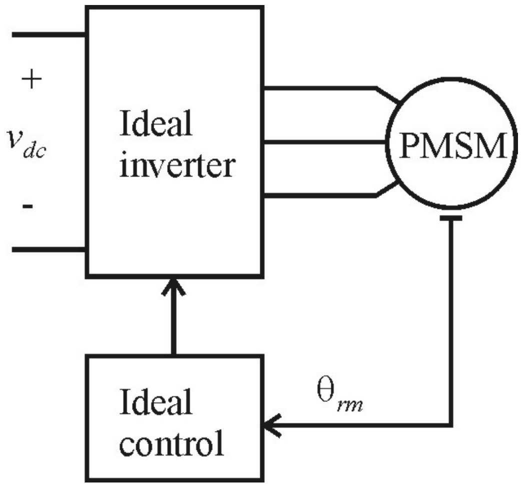
solving for currents

$$\begin{bmatrix} I_{qs}^r \\ I_{ds}^r \end{bmatrix} = \frac{\begin{bmatrix} r_s & -\omega_r L_s \\ \omega_r L_s & r_s \end{bmatrix} \begin{bmatrix} V_{qs}^r - \omega_r \lambda_m' \\ V_{ds}^r \end{bmatrix}}{r_s^2 + \omega_r^2 L_s^2}$$

$$I_{qs}^r = \frac{r_s \left(V_{qs}^r - \omega_r \lambda_m'\right) - \omega_r L_s V_{ds}^r}{r_s^2 + \omega_r^2 L_s^2}$$

$$I_{ds}^r = \frac{\omega_r L_s \left(V_{qs}^r - \omega_r \lambda_m'\right) + r_s V_{ds}^r}{r_s^2 + \omega_r^2 L_s^2}$$

PMSM with Drive (Steady-State)



inverter voltages

$$v_{as} = \sqrt{2} V_s \cos(\theta_r + \phi_v)$$

$$v_{bs} = \sqrt{2} V_s \cos\left(\theta_r + \phi_v - \frac{2\pi}{3}\right)$$

$$v_{cs} = \sqrt{2} V_s \cos\left(\theta_r + \phi_v + \frac{2\pi}{3}\right)$$

$$V_{qs}^r = \sqrt{2} V_s \cos(\phi_v)$$

$$V_{ds}^r = -\sqrt{2} V_s \sin(\phi_v)$$

$$V_s = \frac{1}{\sqrt{2}} \sqrt{\left(V_{qs}^r\right)^2 + \left(V_{ds}^r\right)^2}$$

$$\phi_v = \tan^{-1}\left(\frac{-V_{ds}^r}{V_{qs}^r}\right)$$

resulting currents

$$i_{as} = \sqrt{2} I_s \cos(\theta_r + \phi_i)$$

$$i_{bs} = \sqrt{2} I_s \cos\left(\theta_r + \phi_i - \frac{2\pi}{3}\right)$$

$$i_{cs} = \sqrt{2} I_s \cos\left(\theta_r + \phi_i + \frac{2\pi}{3}\right)$$

$$I_{qs}^r = \sqrt{2} I_s \cos(\phi_i)$$

$$I_{ds}^r = -\sqrt{2} I_s \sin(\phi_i)$$

$$I_s = \frac{1}{\sqrt{2}} \sqrt{\left(I_{qs}^r\right)^2 + \left(I_{ds}^r\right)^2}$$

$$\phi_i = \tan^{-1}\left(\frac{-I_{ds}^r}{I_{qs}^r}\right)$$

Ideal Drive Calculations with $\phi_v = 0$

motor parameters

$P := 2 \qquad r_s := 2.9 \cdot \Omega \qquad L_s := 11.4 \cdot mH \qquad \lambda'_m := 0.156\, V \cdot s$

operating conditions

$V_s := 90 \cdot V \qquad \phi_v := 0 \cdot rad$

$\omega_{rm} := 6000 \cdot RPM \qquad \omega_{rm} = 628 \frac{rad}{s}$

$\omega_r := \frac{P}{2} \cdot \omega_{rm} \qquad \omega_r = 628 \frac{rad}{s}$

***q*- and *d*-axis voltages and currents**

$V_{qsr} := \sqrt{2} \cdot V_s \cdot \cos(\phi_v) \qquad V_{qsr} = 127V$

$V_{dsr} := -\sqrt{2} \cdot V_s \cdot \sin(\phi_v) \qquad V_{dsr} = 0V$

$$\begin{pmatrix} I_{qsr} \\ I_{dsr} \end{pmatrix} := \begin{pmatrix} r_s & \omega_r \cdot L_s \\ -\omega_r \cdot L_s & r_s \end{pmatrix}^{-1} \cdot \begin{pmatrix} V_{qsr} - \omega_r \cdot \lambda'_m \\ V_{dsr} \end{pmatrix}$$

$I_{qsr} = 1.42A$

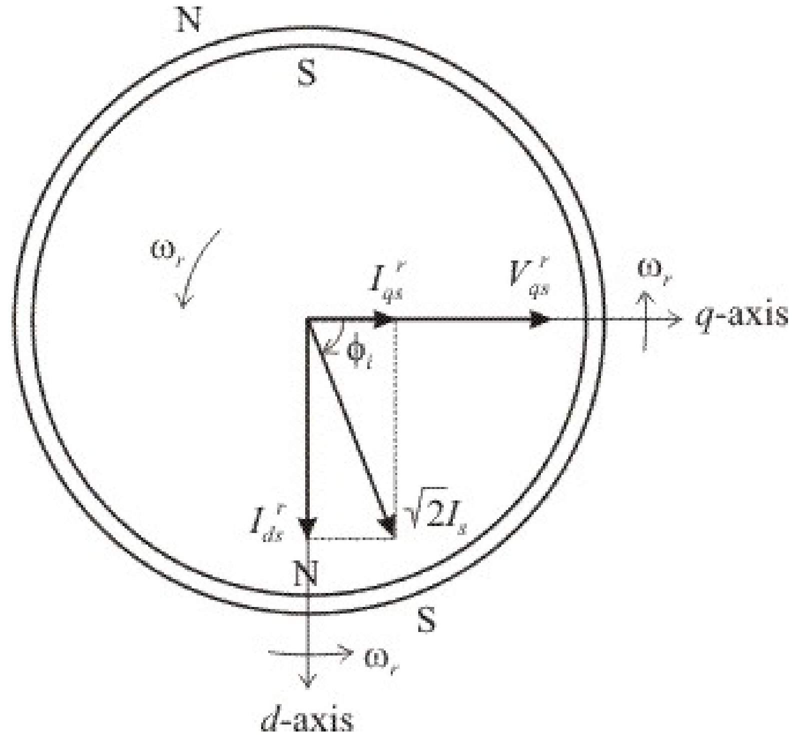
$I_{dsr} = 3.51A$

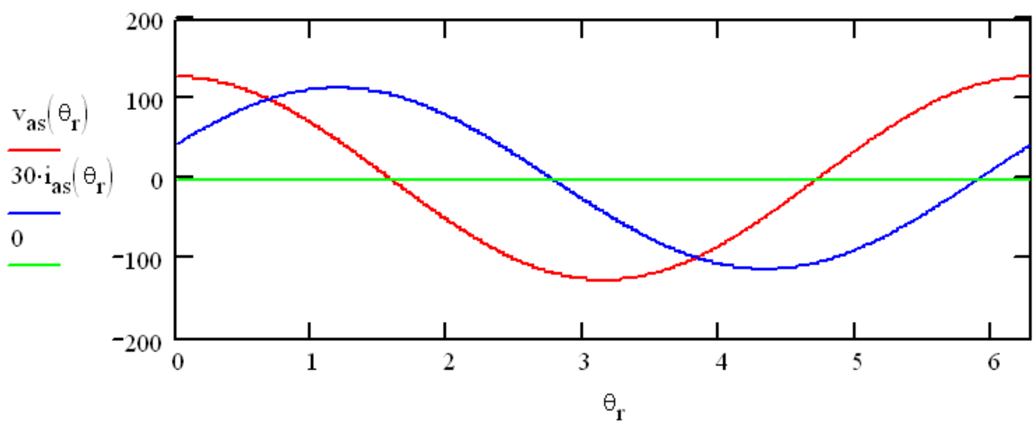
$I_s := \frac{1}{\sqrt{2}} \cdot \sqrt{I_{qsr}^{\ 2} + I_{dsr}^{\ 2}} \qquad I_s = 2.7A \qquad \sqrt{2} \cdot I_s = 3.79A$

$\phi_i := \operatorname{atan}\left(\frac{-I_{dsr}}{I_{qsr}}\right) \qquad \phi_i = -68\, deg$

$T_e := \frac{3}{2} \cdot \frac{P}{2} \cdot \lambda'_m \cdot I_{qsr} \qquad T_e = 0.33\, N \cdot m$

PMSM Steady-Sate Example ϕ_v=0





Ideal Drive Calculations with $\phi_i = 0$

put all current in the *q*-axis

$\phi_i := 0 \cdot deg$

$I_{qsr} := \sqrt{2} \cdot I_s \cdot \cos(\phi_i)$ $\qquad I_{qsr} = 3.79A$

$I_{dsr} := -\sqrt{2} \cdot I_s \cdot \sin(\phi_i)$ $\qquad I_{dsr} = 0\,A$

$$\begin{pmatrix} V_{qsr} \\ V_{dsr} \end{pmatrix} := \begin{pmatrix} r_s & \omega_r \cdot L_s \\ -\omega_r \cdot L_s & r_s \end{pmatrix} \cdot \begin{pmatrix} I_{qsr} \\ I_{dsr} \end{pmatrix} + \begin{pmatrix} \omega_r \cdot \lambda'_m \\ 0 \end{pmatrix}$$

$V_{qsr} = 109V$

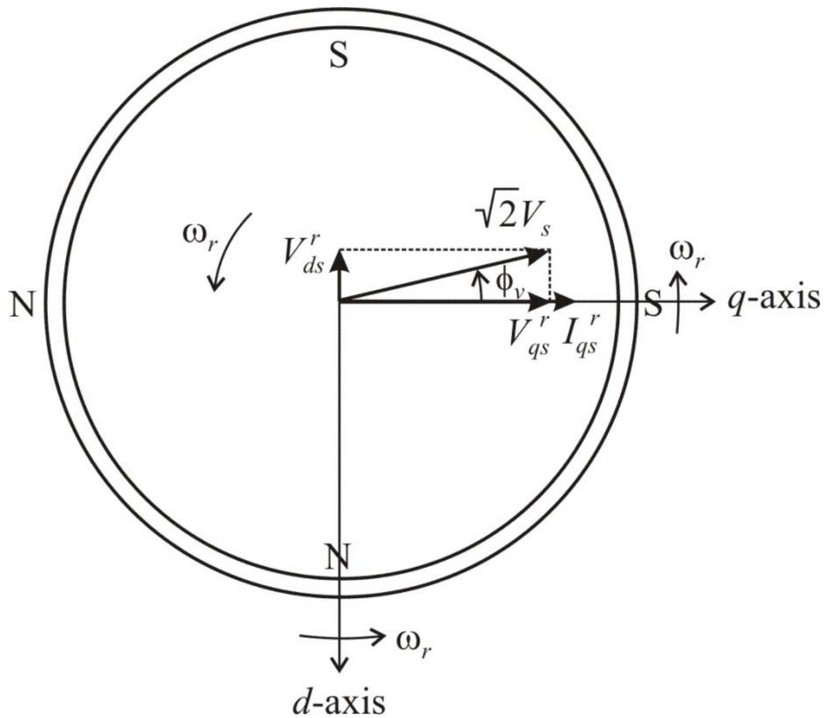
$V_{dsr} = -27.1V$

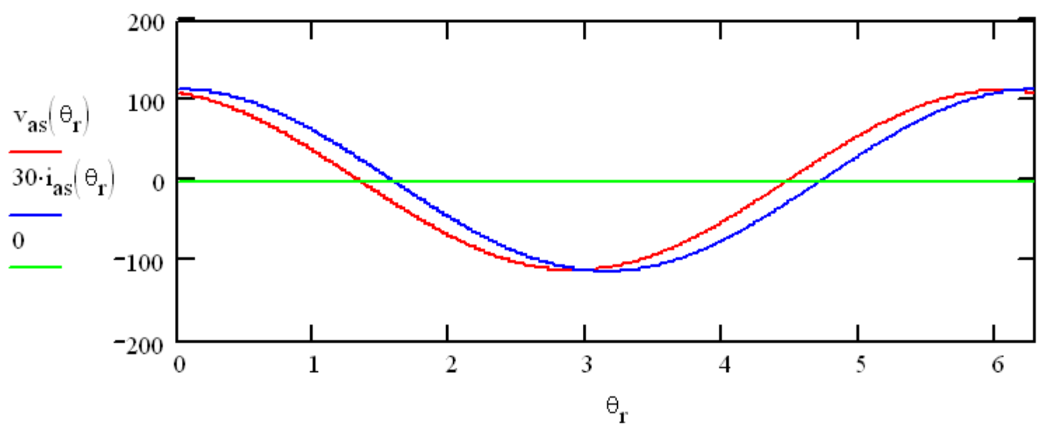
$V_s := \frac{1}{\sqrt{2}} \cdot \sqrt{V_{qsr}^{\;2} + V_{dsr}^{\;2}}$ $\qquad V_s = 79.4V \qquad \sqrt{2} \cdot V_s = 112V$

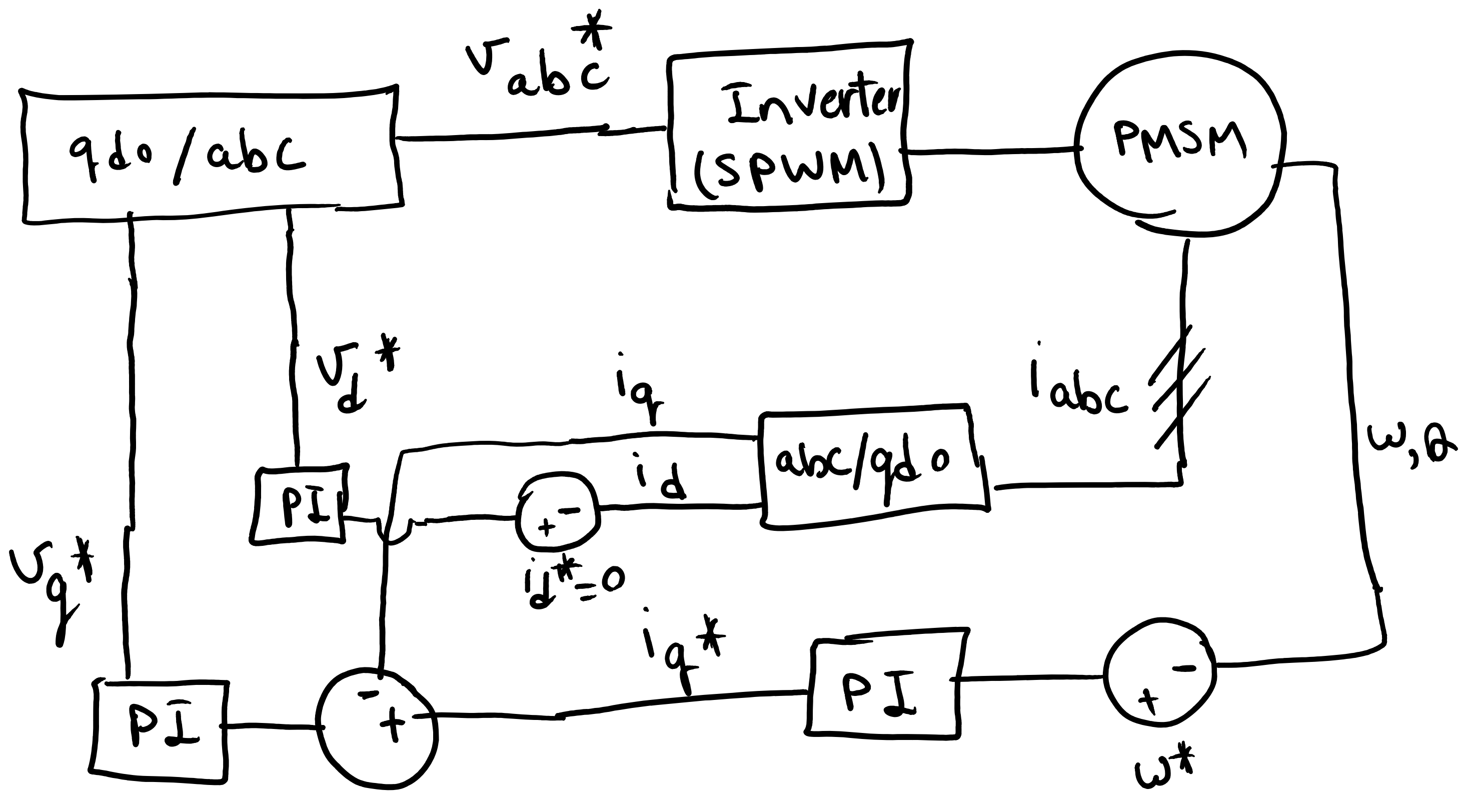
$\phi_v := \text{atan}\left(\frac{-V_{dsr}}{V_{qsr}}\right)$ $\qquad \phi_v = 14\,deg$

$T_e := \frac{3}{2} \cdot \frac{P}{2} \cdot \lambda'_m \cdot I_{qsr}$ $\qquad T_e = 0.89\,N \cdot m$

PMSM Steady-Sate Example with $\phi_i = 0$



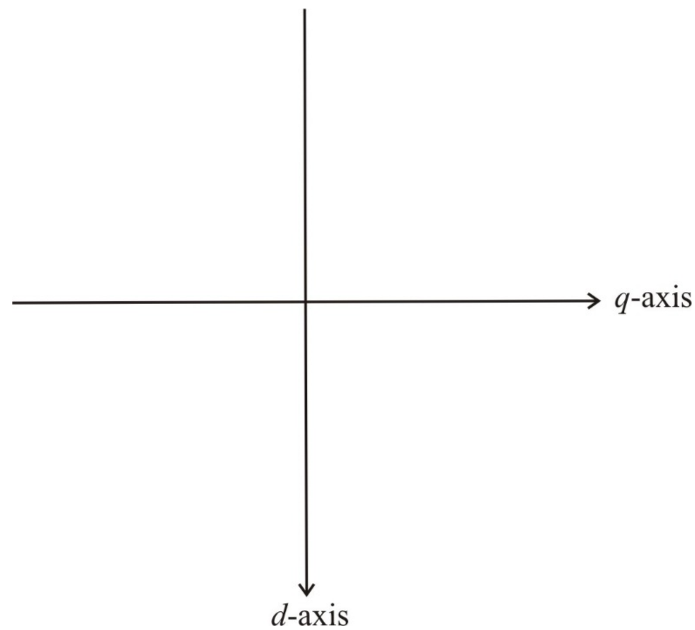




qdo / abc
v abc *
Inverter (SPWM)
PMSM
v d *
i q
abc/qdo
i abc
i d
ω, θ
PI
+ -
i d * = 0
v q *
i q *
PI
- +
PI
+ -
ω*

Reference Frame Theory

A three-phase inductive load is connected to a three-phase voltage source. The inductance is $L = 1\,\text{mH}$ and the source has parameters of $V_s = 120\,\text{V}$, $\phi_v = 45^\circ$, and $f = 60\,\text{Hz}$ (where $\theta_e = \omega_e t = 2\pi f t$). Calculate the steady-state *q*- and *d*-axis voltages and currents in the synchronous reference frame. Sketch these quantities and the rotating voltage and current vectors on the graph below.



Sketch the *q*- and *d*-axis voltages in the synchronous reference frame for two cycles of θ_e.

Permanent-Magnet Ac Drives

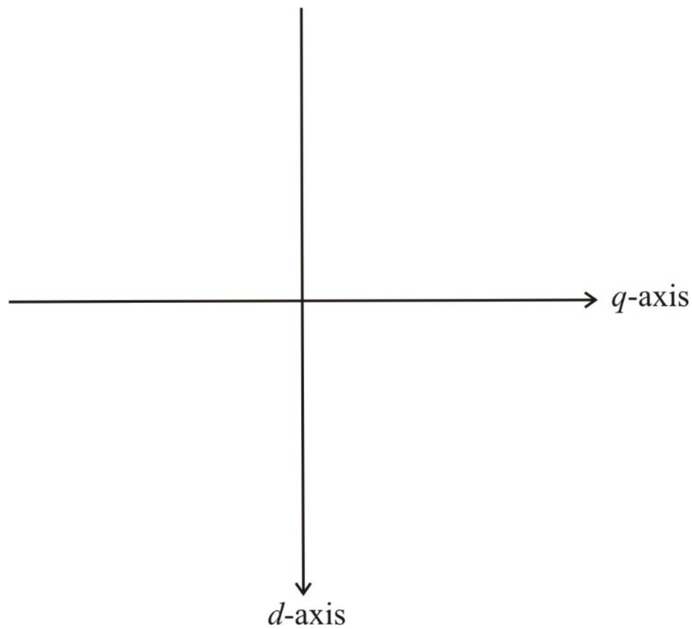
A PMAC machine with the parameters

$$P = 4 \qquad r_s = 1\Omega \qquad L_s = 5\,\text{mH} \qquad \lambda_m'^r = 0.95\,\text{V}\cdot\text{s}$$

is operating with negligible friction and without load. Given that the applied voltage and phase angle is

$$V_s = 270\,\text{V} \qquad \phi_v = 20^\circ$$

Determine the steady-state no-load mechanical rotor speed. At no-load speed, compute the electrical input power, mechanical output power, and power losses. Compute the *q*- and *d*-axis voltages and currents. Sketch these quantities and the rotating voltage and current vectors on the graph below.



Is the machine operating with a leading or lagging power factor?