ELECTRIC POWER QUALITY

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POWER QUALITY METRICS

- Frequency
- Voltage level
- Transients
- Harmonic distortion
- **Power factor**
- **Inactive/reactive power**

HISTORICAL EVOLUTION

- Power factor (a.k.a $\cos\phi$) introduced in the 1920s for sinusoidal single-phase waveforms.
- Continuing evolution of the concept of inactive power:
  - Budeanu (1927) ⇒ "reactive power" (via phasors) for multi-harmonic waveforms
  - Fryze (1931) ⇒ universal power factor (in waveform space)
  - Shepherd & Zakikhani (1972), Sharon(1973) and others ⇒ multi-component decomposition of apparent power
  - Czarnecki (1980s) ⇒ 5-component decomposition
  - Lev-Ari and Stanković (2005) ⇒ 7-component decomposition

Agreement on Reactive Power

Standard definitions [Buchholz, Schallenberger, Stanley]:

$$\|V\|^2 =< v\, v^\top >_0, \quad \|I\|^2 =< i\, i^\top >_0, \quad S = \|V\| \|I\|$$

$$P =< i^\top\, v >_0, \quad Q^2 = S^2 - P^2, \quad k_{PF} = \frac{P}{S}$$

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For single frequency and single phase/ balanced polyphase in steady state:

$$P = \|V\|\|I\| \cos\phi_1, \quad Q = \|V\|\|I\| \sin\phi_1 = Q_{B1}$$

$$k_{PF} = \cos\phi_1, \quad S_c \triangleq P + \jmath Q$$

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$$k_{PF} = \cos \phi_1, \quad S_c \triangleq P + \jmath Q$$

In multi-frequency $k_{PF} \neq \cos \phi_1$, [Steinmetz 1898]; similarly for unbalanced.

OUTLINE

➨ **SINGLE-PHASE SINUSOIDAL WAVEFORMS**

- Single-phase non-sinusoidal waveforms
- Euclidean waveform spaces
- Inactive power components
- Polyphase waveforms
- Dynamic power components

GENESIS: SINGLE-PHASE SINUSOIDAL WAVEFORMS

Waveforms (via rms phasors):

$$v(t) = \sqrt{2}\,\Re\left\{V e^{\jmath\omega t}\right\} \quad \Longrightarrow \quad V_{rms} \stackrel{\text{def}}{=} \left[\frac{1}{T}\int_0^T v^2(t)\,dt\right]^{1/2} = |V|$$

$$i(t) = \sqrt{2}\,\Re\left\{I\, e^{\jmath\omega t}\right\} \quad \Longrightarrow \quad I_{rms} = |I|$$

Instantaneous power:

$$p(t) \stackrel{\text{def}}{=} v(t)\,i(t) = \Re\{V I^*\} + \Re\{V I\, e^{2\jmath\,\omega t}\}$$

(more to come)

Average ("real") power:

$$P \stackrel{\text{def}}{=} \frac{1}{T}\int_0^T p(t)\,dt = \Re\{V I^*\}$$

SINGLE-PHASE SINUSOIDAL WAVEFORMS (2)

Power factor:

$$\text{PF} \overset{\text{def}}{=} \frac{P}{V_{rms}\, I_{rms}} = \frac{\Re\{V I^*\}}{|V|\;|I|} = \cos\phi \;\leq\; 1 \qquad \text{(direct manipulation)}$$

where $\phi \overset{\text{def}}{=} \arg\,(V/I)$

SINGLE-PHASE SINUSOIDAL WAVEFORMS (2)

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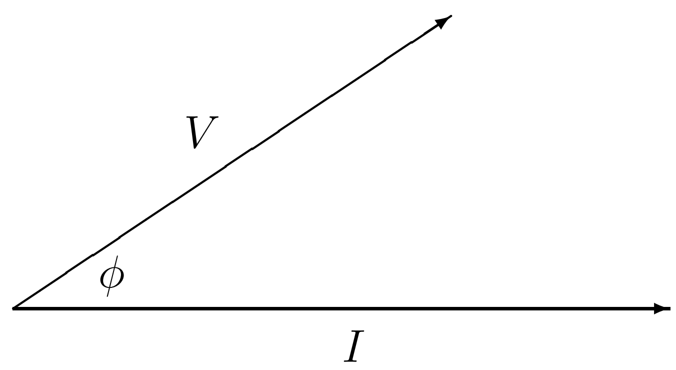
Euclidean space of phasors: $\Re\{V\, I^*\}$ is the inner product on this space

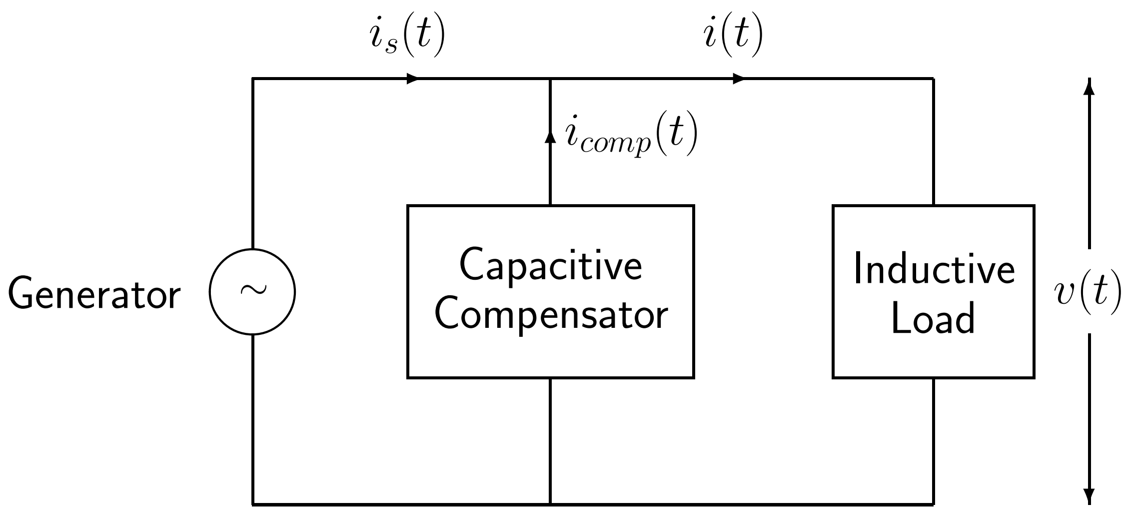


Apparent power:

- When $I = \dfrac{V}{R}$ ➡ $\phi = 0$ and $P = V_{rms} I_{rms} \stackrel{\text{def}}{=} S$
- In general $P \leq S$, and so $I_{rms} \geq \dfrac{P}{V_{rms}} \stackrel{\text{def}}{=} I_F$ ➡
  Excessive power loss in the supply line
- Utility charges extra for imperfect power factor

➡ Solution – shunt compensator

Compensation (traditional):



Exact compensation (at a single frequency) requires that

$$\Im\left\{\jmath\omega C + \frac{1}{R + \jmath\omega L}\right\} = 0 \quad \Longrightarrow \quad C = \frac{L}{|R + \jmath\omega L|^2}$$

SINGLE-PHASE SINUSOIDAL WAVEFORMS (5)

Reactive power:

$$Q \stackrel{\text{def}}{=} \Im\{V I^*\} = V_{rms} I_{rms} \sin\phi$$

Complex apparent power:

$$S_c = P + \jmath Q = V I^* = V_{rms} I_{rms} e^{\jmath\phi}$$

$$|S_c|^2 = P^2 + Q^2 \equiv S^2$$

Power quality gap: fully explained by Q (reactive power meters)

$$Q = 0 \iff \cos\phi = 1$$

OUTLINE

- Single-phase sinusoidal waveforms
- **SINGLE-PHASE NON-SINUSOIDAL WAVEFORMS**
- Euclidean waveform spaces
- Inactive power components
- Polyphase waveforms
- Dynamic power components

SINGLE-PHASE PERIODIC (NON-SINUSOIDAL) WAVEFORMS

Causes for multiple harmonics:

- Load nonlinearity ⇒ multi-harmonic current
- Non-negligible line impedance ⇒ multi-harmonic voltage
- HVDC: Transformers and converters

Metrics of harmonic distortion:

- THD
- Variation of equivalent admittances with respect to harmonic index

Challenges:

How to define V_{rms}, I_{rms}, P, S, PF, reactive power?

SINGLE-PHASE PERIODIC (NON-SINUSOIDAL) WAVEFORMS (2)

Waveforms:

$$v(t) = \sum_k \sqrt{2}\,\Re\left\{V_k\, e^{\jmath\, k\, \omega t}\right\}$$

$$i(t) = \sum_k \sqrt{2}\,\Re\left\{I_k\, e^{\jmath\, k\, \omega t}\right\}$$

$$V^2_{rms} \overset{\text{def}}{=} \frac{1}{T}\int_0^T v^2(t)\; dt = \sum_k |V_k|^2 = \sum_k V^2_{rms,k}$$

$$I^2_{rms} \overset{\text{def}}{=} \frac{1}{T}\int_0^T i^2(t)\; dt = \sum_k |I_k|^2 = \sum_k I^2_{rms,k}$$

Average power:

$$P \overset{\text{def}}{=} \frac{1}{T}\int_0^T p(t)\; dt = \frac{1}{T}\int_0^T v(t)\, i(t)\; dt = \sum_k \Re\{V_k I_k^*\} = \sum_k P_k$$

OUTLINE

- Single-phase sinusoidal waveforms
- Single-phase nonsinusoidal waveforms
- **EUCLIDEAN WAVEFORM SPACES**
- Inactive power components
- Polyphase waveforms
- Dynamic power components

EUCLIDEAN WAVEFORM SPACES

Space elements:

$v(t)$ and $i(t)$ are elements in the space of all T-periodic square-integrable waveforms.

Inner product and norm:

$$\langle\, x(\cdot)\,,\, y(\cdot)\,\rangle \overset{\text{def}}{=} \frac{1}{T}\int_T x(s)\,y(s)\,ds \quad , \qquad P = \langle\, v(\cdot)\,,\, i(\cdot)\,\rangle$$

$$V_{rms} = \|v(\cdot)\| \equiv \sqrt{\langle\, v(\cdot)\,,\, v(\cdot)\,\rangle} \quad , \qquad I_{rms} = \|i(\cdot)\| \equiv \sqrt{\langle\, i(\cdot)\,,\, i(\cdot)\,\rangle}$$

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Orthogonality of harmonics:

$$\frac{1}{T}\int_0^T \cos(k\omega t + \theta_k)\,\cos(\ell\omega t + \theta_\ell)\,dt = \begin{cases} 0 & k \neq \ell \\ \frac{1}{2}\cos\left(\theta_k - \theta_\ell\right) & k = \ell \end{cases}$$

EUCLIDEAN WAVEFORM SPACES (2)

Fourier series representation:

$$x(t) \;=\; X_0 \;+\; \sum_{k=1}^{\infty} \sqrt{2}\,\Re\{X_k\, e^{\jmath k\omega t}\} \quad , \qquad \langle\, x(\cdot)\,,\, y(\cdot)\,\rangle \;=\; \sum_{k=0}^{\infty} \Re\{X_k\, Y_k^*\}$$

$$X_k \;\stackrel{\text{def}}{=}\; \begin{cases} \frac{1}{T}\displaystyle\int_T x(t)\, dt & k = 0 \\ \frac{\sqrt{2}}{T}\displaystyle\int_T x(t)\, e^{-\jmath k\omega t}\, dt & k \geq 1 \end{cases}$$

(one-sided rms phasors)

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(one-sided rms phasors)

Voltage and current waveforms: (without DC component)

$$P \;=\; \langle\, v(\cdot)\,,\, i(\cdot)\,\rangle \;=\; \sum_{k=1}^{\infty} \Re\{V_k\, I_k^*\}$$

$$V_{rms}^2 \;\equiv\; \|v(\cdot)\|^2 \;=\; \sum_{k=1}^{\infty} |V_k|^2 \quad , \qquad I_{rms}^2 \;\equiv\; \|i(\cdot)\|^2 \;=\; \sum_{k=1}^{\infty} |I_k|^2$$

EUCLIDEAN WAVEFORM SPACES (3)

Dimension:

- Sinusoidal waveforms: dimension $= 2$

$$e_1(t) = \sqrt{2}\cos\omega t \quad , \quad e_2(t) = \sqrt{2}\sin\omega t$$

- L-harmonic waveforms: dimension $= 2L$ (no DC component)

$$\left\{ (\sqrt{2}\cos k\omega t \, , \sqrt{2}\sin k\omega t) \; ; \; 1 \le k \le L \right\}$$

- Arbitrary periodic waveforms: dimension $= \infty$ (Hilbert space)

  ➨ L-harmonic basis with $L = \infty$.

EUCLIDEAN WAVEFORM SPACES (4)

Historical footnote:

- **Stanisław Fryze (1885-1964)**, Professor at the Lwów Polytechnic (then in Poland, now in the Ukraine), originated in 1932 the function-analytic interpretation of power system waveforms.

- A contemporary of Fryze at the Lwów Polytechnic (and, apparently, a professional colleague) was ...

EUCLIDEAN WAVEFORM SPACES (4)

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A history flashback

Pioneers

Getting basics right - Buchholz, Schallenberger, Stanley
A trouble in paradise - multiple harmonics Steinmetz 1898

National schools - glowne nurty

Romania: A.D. Iliovici (1925), C. Busila, C. Budeanu, I. Antoniu (1950), F. Manea, V. Nedelcu (1960) M. Milic (1970), A. Tugulea, A. Emanuel, Poland: S. Fryze, M. Erlicki (1950), Z. Nowomiejski (1980), L. Czarnecki,
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Decussis Mirabilis: 1927-1937

Constantin Budeanu (1886-1959)

$$S^2 = P^2 + Q_B^2 + D^2$$

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$$i_a(t) = \frac{< i\, v^\top >}{< v\, v^\top >} v(t)$$

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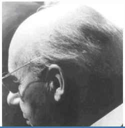

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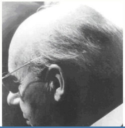

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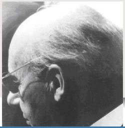

Funktionenraums

Reactive Power - Schools

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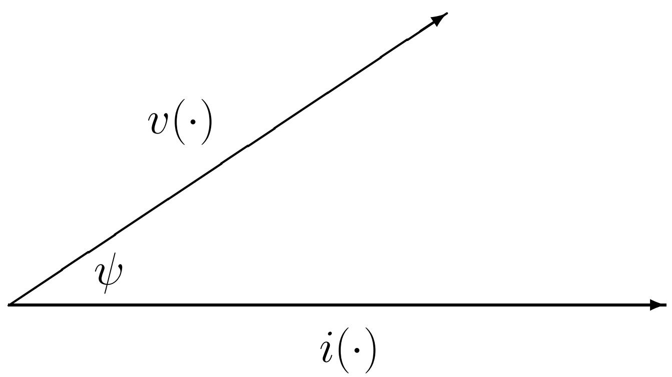
Power factor:

$$\mathsf{PF} \stackrel{\text{def}}{=} \frac{P}{V_{rms}\, I_{rms}} = \frac{\langle\, v(\cdot)\,,\, i(\cdot)\,\rangle}{\|v(\cdot)\|\; \|i(\cdot)\|} = \cos\psi \;\leq\; 1$$

where ψ is the angle between voltage and current in the **waveform space**.



$$\mathsf{PF} = 1 \quad\Longleftrightarrow\quad \psi = 0 \quad\Longleftrightarrow\quad P = S \quad\Longleftrightarrow\quad i(t) = \frac{v(t)}{R}$$

EUCLIDEAN WAVEFORM SPACES (6)

Power factor vs. phase shifts:

$$\mathsf{PF} \equiv \cos\psi = \frac{\sum_k |V_k|\,|I_k| \cos\phi_k}{\sqrt{\sum_k |V_k|^2}\sqrt{\sum_k |I_k|^2}} \quad , \qquad \text{where} \quad \phi_k \stackrel{\text{def}}{=} \arg\big(V_k/I_k\big)$$

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Warning:

$$\phi_k = 0 \quad \text{for all } k \quad \Longrightarrow \quad P < S \quad \text{(in general)}$$

because in this case

$$\text{PF} = \frac{\sum_k |V_k|\,|I_k|}{\sqrt{\sum_k |V_k|^2}\sqrt{\sum_k |I_k|^2}} < 1 \qquad \text{(why?)}$$

The guilty party: Distortion power (D)

OUTLINE

- Single-phase sinusoidal waveforms
- Single-phase nonsinusoidal waveforms
- Euclidean waveform spaces
- **INACTIVE POWER COMPONENTS**
- Polyphase waveforms
- Dynamic power components

A MENAGERIE OF INACTIVE POWER COMPONENTS

Budeanu reactive power: (Why "reactive"? Why "Budeanu"?)

$$Q_B \stackrel{\text{def}}{=} \sum_k \Im\{V_k I_k^*\} = \sum_k |V_k|\,|I_k| \sin\phi_k$$

$Q_B = 0$ when $\phi_k = 0$ for all k (but it can also vanish when $\phi_k \neq 0$)

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Distortion power: (Budeanu, 1927)

In general, (details on next page)

$$P^2 + Q_B^2 = \left| \sum_k V_k \, I_k^* \right|^2 < S^2$$

so let's define

$$D \stackrel{\text{def}}{=} \sqrt{S^2 - (P^2 + Q_B^2)}$$

A purely arithmetical (unsigned) definition, with no physical meaning.

A MENAGERIE OF INACTIVE POWER COMPONENTS (2)

Power quality gap:

$$P^2 + Q_B^2 \;=\; \left[\Re \sum_k V_k\, I_k^*\right]^2 \;+\; \left[\Im \sum_k V_k\, I_k^*\right]^2 \;=\; \left|\sum_k V_k\, I_k^*\right|^2$$

and (using Cauchy-Schwarz inequality for complex vectors)

$$\left|\sum_k V_k\, I_k^*\right|^2 \;\leq\; \left(\sum_k |V_k|^2\right)\left(\sum_k |I_k|^2\right) \;\equiv\; S^2$$

$$P^2 + Q_B^2 \;\leq\; S^2 \qquad \Longrightarrow \qquad D \;\geq\; 0$$ (terminus for Budeanu)

A MENAGERIE OF INACTIVE POWER COMPONENTS (2)

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Vanishing distortion power:

$$P^2 + Q_B^2 \;=\; S^2 \qquad \Longleftrightarrow \qquad I_k \;=\; Y\, V_k$$ for all k (frequency-independent load)

A MENAGERIE OF INACTIVE POWER COMPONENTS (3)

Equivalent load admittances:

$$Y_k \stackrel{\text{def}}{=} \frac{I_k}{V_k} = g_k - \jmath b_k$$

Var $Y_k > 0 \iff D > 0$

(Why I/V? What if $V_k = 0$?)

(How about nonlinear loads?)

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(Why I/V? What if $V_k = 0$?)

(How about nonlinear loads?)

Weighted mean expressions:

$$P = \sum_k \Re\{V_k I_k^*\} = \sum_k g_k |V_k|^2 \sim \text{weighted mean of } \{g_k\}$$

$$Q_B = \sum_k \Im\{V_k I_k^*\} = \sum_k b_k |V_k|^2 \sim \text{weighted mean of } \{b_k\}$$

A STATISTICAL (SPREAD) PERSPECTIVE

Weights: $w_k \overset{\text{def}}{=} \dfrac{|V_k|^2}{V_{rms}^2}$ $\left(\sum_k w_k = 1 \right)$ (Why?)

Weighted means:

$$\left.\begin{aligned} \mu_g &\overset{\text{def}}{=} \sum_k w_k\, g_k = \frac{P}{V_{rms}^2} \\ \mu_b &\overset{\text{def}}{=} \sum_k w_k\, b_k = \frac{Q_B}{V_{rms}^2} \end{aligned}\right\} \Longrightarrow P^2 + Q_B^2 = V_{rms}^4 \left(\mu_g^2 + \mu_b^2 \right)$$

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Weighted variances: $D = 0 \iff \sigma_g^2 = 0 = \sigma_b^2$

$$\sigma_g^2 \overset{\text{def}}{=} \sum_k w_k \left(g_k - \mu_g\right)^2$$

$$\sigma_b^2 \overset{\text{def}}{=} \sum_k w_k \left(b_k - \mu_g\right)^2$$

A STATISTICAL (SPREAD) PERSPECTIVE (2)

Mystery of distortion power solved !! ☺ $(S^2 = V_{rms}^2\, I_{rms}^2)$

$$I_{rms}^2 = \sum_k |I_k|^2 = \sum_k (g_k^2 + b_k^2)\, |V_k|^2 = V_{rms}^2 \left(\sum_k w_k\, g_k^2 + \sum_k w_k\, b_k^2 \right)$$

But

$$\mathsf{Var}\ g_k \equiv \sigma_g^2 = \sum_k w_k \left(g_k^2 - 2\mu_g\, g_k + \mu_g^2 \right) = \sum_k w_k\, g_k^2 - \mu_g^2$$

$$\mathsf{Var}\ b_k \equiv \sigma_b^2 = \sum_k w_k \left(b_k^2 - 2\mu_b\, b_k + \mu_b^2 \right) = \sum_k w_k\, b_k^2 - \mu_b^2$$

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$$\text{Var } b_k \equiv \sigma_b^2 = \sum_k w_k \left(b_k^2 - 2\mu_b \, b_k + \mu_b^2 \right) = \sum_k w_k \, b_k^2 - \mu_b^2$$

So

$$\left. \begin{aligned} \textstyle\sum_k g_k^2 \, |V_k|^2 &= V_{rms}^2 \left[\mu_g^2 + \sigma_g^2 \right] \\ \textstyle\sum_k b_k^2 \, |V_k|^2 &= V_{rms}^2 \left[\mu_b^2 + \sigma_b^2 \right] \end{aligned} \right\} \Longrightarrow I_{rms}^2 = V_{rms}^2 \left[\mu_g^2 + \sigma_g^2 + \mu_b^2 + \sigma_b^2 \right]$$

A STATISTICAL (SPREAD) PERSPECTIVE (3)

Four-component decomposition:

$$S^2 \equiv V_{rms}^2 \, I_{rms}^2 = V_{rms}^4 \left[\mu_g^2 + \mu_b^2 + \sigma_g^2 + \sigma_b^2 \right]$$

$$= P^2 + Q_B^2 + \underbrace{N_g^2 + N_b^2}_{D^2}$$

where

$$P^2 = V_{rms}^4 \, \mu_g^2 \quad , \quad Q_B^2 = V_{rms}^4 \, \mu_b^2$$

$$N_g^2 \stackrel{\text{def}}{=} V_{rms}^4 \, \sigma_g^2 \quad , \quad N_b^2 \stackrel{\text{def}}{=} V_{rms}^4 \, \sigma_b^2$$

so that

$$D^2 = V_{rms}^4 \left(\sigma_g^2 + \sigma_b^2 \right)$$

A STATISTICAL (SPREAD) PERSPECTIVE (4)

Out-of-band current: (What if $V_k = 0$?)

$$\Omega_v \overset{\text{def}}{=} \{k\,;\ \frac{|V_k|}{V_{rms}} > \varepsilon\}$$ (include only non-negligible harmonics)

$$I_{rms}^2 = \sum_{k \in \Omega_v} g_k^2 \left|V_k\right|^2 + \sum_{k \in \Omega_v} b_k^2 \left|V_k\right|^2 + \sum_{k \neq \Omega_v} \left|I_k\right|^2$$

A STATISTICAL (SPREAD) PERSPECTIVE (4)

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(include only non-negligible harmonics)

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Five-component decomposition:

$$S^2 \equiv V_{rms}^2 I_{rms}^2 = P^2 + Q_B^2 + \underbrace{N_g^2 + N_b^2 + S_\perp^2}_{D^2}$$

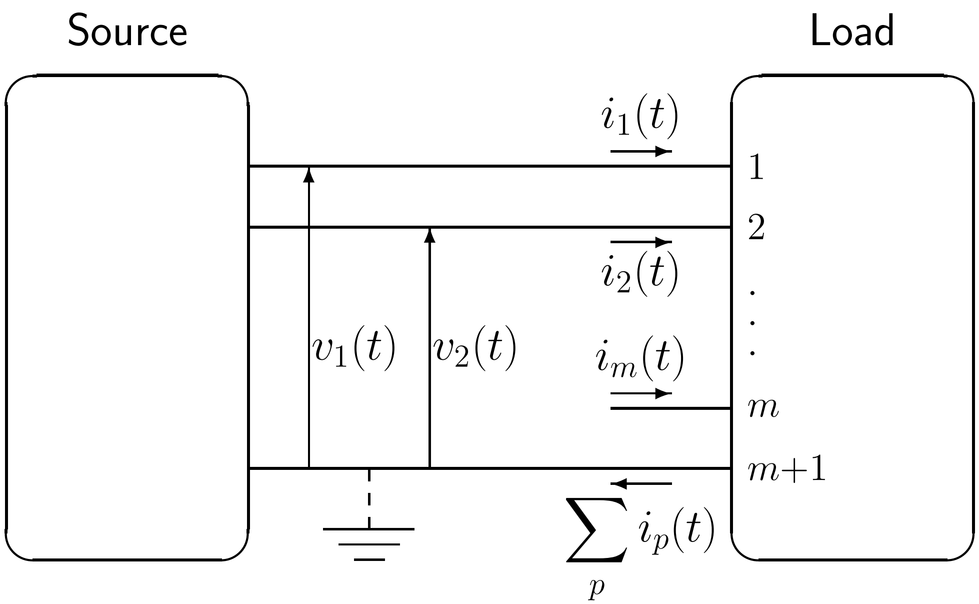
where

$$S_\perp^2 \stackrel{\text{def}}{=} V_{rms}^2 \left(\sum_{k \neq \Omega_v} |I_k|^2 \right)$$

OUTLINE

- Single-phase sinusoidal waveforms
- Single-phase nonsinusoidal waveforms
- Euclidean waveform spaces
- Inactive power components
- ➨ **POLYPHASE WAVEFORMS**
- Dynamic power components

POLYPHASE CURRENT AND VOLTAGE WAVEFORMS



$$v(t) \stackrel{\text{def}}{=} \begin{bmatrix} v_1(t) & v_2(t) & \dots & v_m(t) \end{bmatrix}$$

$$i(t) \stackrel{\text{def}}{=} \begin{bmatrix} i_1(t) & i_2(t) & \dots & i_m(t) \end{bmatrix}$$

EUCLIDEAN/HILBERT WAVEFORM SPACES

Space elements: real-valued T-periodic square-integrable polyphase waveforms

$$x(t) = \begin{bmatrix} x_1(t) & x_2(t) & \dots & x_m(t) \end{bmatrix} \qquad \text{(row vector)}$$

Inner product and norm:

$$\langle \, x \, , \, y \, \rangle \overset{\text{def}}{=} \frac{1}{T} \int_T x(t) \, y^\top(t) \, dt \qquad \| \, x \, \| \overset{\text{def}}{=} \sqrt{\langle \, x \, , \, x \, \rangle}$$

$$V_{rms} = \|v(\cdot)\| \quad , \qquad I_{rms} = \|i(\cdot)\| \quad , \qquad P = \langle \, v(\cdot) \, , \, i(\cdot) \, \rangle$$

Additivity over phases:

$$V_{rms}^2 = \sum_{p=1}^{m} V_{rms,p}^2 \quad , \qquad I_{rms}^2 = \sum_{p=1}^{m} I_{rms,p}^2 \quad , \qquad P = \sum_{p=1}^{m} P_p$$

EUCLIDEAN/HILBERT WAVEFORM SPACES (2)

Fourier series representation:

$$x(t) \;=\; X_0 \;+\; \sum_{k=1}^{\infty} \sqrt{2}\,\Re\{X_k\, e^{\jmath\, k\omega t}\}$$

(one-sided rms phasors)

$$X_k \;\stackrel{\text{def}}{=}\; \begin{cases} \frac{1}{T}\displaystyle\int_T x(t)\; dt & k = 0 \\ \frac{\sqrt{2}}{T}\displaystyle\int_T x(t)\, e^{-jk\omega t}\; dt & k \geq 1 \end{cases}$$

(row vector)

Parseval identity: ($\downarrow$ one reason for row vectors)

$$\langle\, x(\cdot)\,,\, y(\cdot)\,\rangle \;=\; \sum_{k=0}^{\infty} \Re\{X_k\, Y_k^H\} \;=\; \sum_{p=1}^{m}\sum_{k=0}^{\infty} \Re\Big\{X_k^{(p)}\,[Y_k^{(p)}]^*\Big\}$$

$X_k^{(p)} \;=\; p$-th element of X_k .

SINUSOIDAL POLYPHASE WAVEFORMS

Equivalent load admittances: (only fundamental harmonic)

$$Y_p \stackrel{\text{def}}{=} \frac{I_p}{V_p} = g_p - \jmath\, b_p \qquad \left(g_p \text{ and } b_p \text{ now represent spread over phases}\right)$$

Four-component power decomposition:

$$S^2 \equiv V_{rms}^2 I_{rms}^2 = V_{rms}^4 \left[\mu_g^2 + \sigma_g^2 + \mu_b^2 + \sigma_b^2 \right]$$

$$= P^2 + N_g^2 + Q_B^2 + N_b^2$$

where

$$\mu_g \,,\ \sigma_g^2 \quad = \quad \text{weighted mean and variance of the sequence } \left\{g_p\right\}$$

$$\mu_b \,,\ \sigma_b^2 \quad = \quad \text{weighted mean and variance of the sequence } \left\{b_p\right\}$$

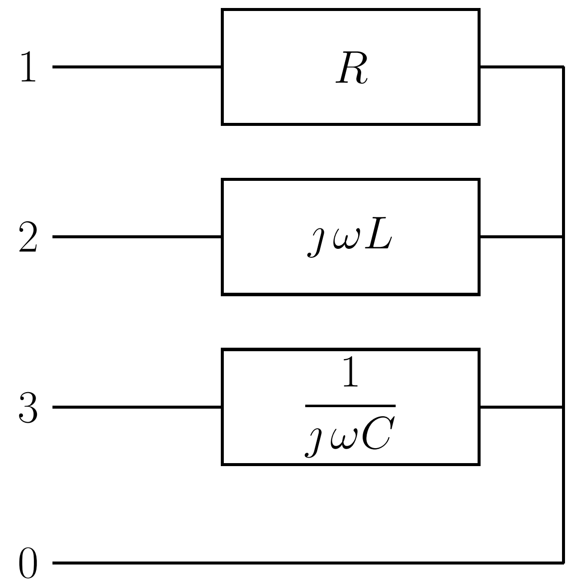
SINUSOIDAL POLYPHASE WAVEFORMS (2)

Example 1: balanced unit voltage, $R = 1 = \omega L = 1/\omega C$

$g_1 = 1$, $b_1 = 0$ ⇨ $\phi_1 = 0°$

$g_2 = 0$, $b_2 = 1$ ⇨ $\phi_2 = 90°$

$g_3 = 0$, $b_3 = -1$ ⇨ $\phi_3 = -90°$



Power components:

$S = 3$, $P = 1$, $\mathbf{Q_B = 0}$, $N_g = \sqrt{2}$, $N_b = \sqrt{6}$ ⇨ $\mathsf{PF} = \frac{1}{3}$

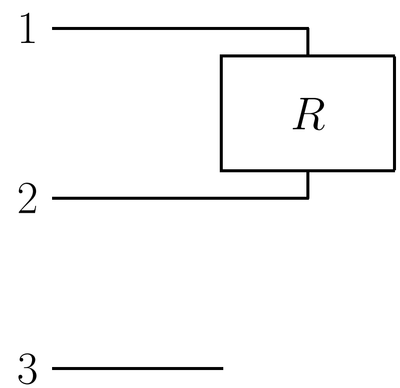
SINUSOIDAL POLYPHASE WAVEFORMS (3)

Example 2: balanced unit voltage, $R = 1$

$g_1 = \frac{3}{2}$, $b_1 = \frac{\sqrt{3}}{2}$ ⇨ $\phi_1 = 30^\circ$

$g_2 = \frac{3}{2}$, $b_2 = -\frac{\sqrt{3}}{2}$ ⇨ $\phi_2 = -30^\circ$

$g_3 = 0$, $b_3 = 0$ ⇨ $\phi_3 = \text{n/a}$



Power components:

$S = \sqrt{18}$, $P = 3$, $\mathbf{Q_B = 0}$, $N_g = \frac{3\sqrt{2}}{2}$, $N_b = \frac{3\sqrt{2}}{2}$ ⇨ $\text{PF} = \frac{\sqrt{2}}{2}$

SINUSOIDAL POLYPHASE WAVEFORMS (4)

Balanced voltage and current: ("standard" conditions, only positive sequence)

$$Y_1 = Y_2 = Y_3 \quad \Longrightarrow \quad g_p = g \text{ and } b_p = b \quad \text{for all } p$$

Also

$$\phi_1 = \phi_2 = \phi_3 \stackrel{\text{def}}{=} \phi$$

SINUSOIDAL POLYPHASE WAVEFORMS (4)

Balanced voltage and current: (“standard” conditions, only positive sequence)

$$Y_1 = Y_2 = Y_3 \implies g_p = g \text{ and } b_p = b \quad \text{for all } p$$

Also

$$\phi_1 = \phi_2 = \phi_3 \stackrel{\text{def}}{=} \phi$$

Power components:

$$N_g = 0 = N_b \implies S^2 = P^2 + Q_B^2$$

and

$$\text{PF} \equiv \cos\psi = \cos\phi$$

Indistinguishable from single-phase sinusoidal case.

NON-SINUSOIDAL POLYPHASE WAVEFORMS

Equivalent load admittances:

$$Y_k^{(p)} \stackrel{\text{def}}{=} \frac{I_k^{(p)}}{V_k^{(p)}} = g_k^{(p)} - \jmath\, b_k^{(p)} \qquad \big(k \text{ is harmonic index, } p \text{ is phase index}\big)$$

Seven-component power decomposition: (ANOVA: groups = harmonics)

$$S^2 \equiv V_{rms}^2 I_{rms}^2 = V_{rms}^4 \Big[\mu_g^2 + \underbrace{\sigma_{gs}^2 + \sigma_{gu}^2}_{\sigma_g^2} + \mu_b^2 + \underbrace{\sigma_{bs}^2 + \sigma_{bu}^2}_{\sigma_b^2} \Big] + S_\perp^2$$

$$= P^2 + N_{gs}^2 + N_{gu}^2 + Q_B^2 + N_{bs}^2 + N_{bu}^2 + S_\perp^2$$

where ("u" = within, "s" = between)

$\mu_g \,,\ \sigma_g^2 \;=\;$ weighted mean and variance of the "2D" sequence $\left\{ g_k^{(p)} \right\}$

$\mu_b \,,\ \sigma_b^2 \;=\;$ weighted mean and variance of the "2D" sequence $\left\{ b_k^{(p)} \right\}$

Variance components:

$$\mu_g(k) \stackrel{\text{def}}{=} \sum_p g_k^{(p)} \frac{|V_k^{(p)}|^2}{\sum_i |V_k^{(i)}|^2} \quad , \quad \mu_b(k) \stackrel{\text{def}}{=} \sum_p b_k^{(p)} \frac{|V_k^{(p)}|^2}{\sum_i |V_k^{(i)}|^2}$$

$$\sigma_{gu}^2 \stackrel{\text{def}}{=} \sum_k \left(\sum_p \left[g_k^{(p)} - \mu_g(k) \right]^2 \frac{|V_k^{(p)}|^2}{V_{rms}^2} \right) \quad \text{(within)}$$

$$\sigma_{gs}^2 \stackrel{\text{def}}{=} \sum_k \left(\sum_p \left[\mu_g(k) - \mu_g \right]^2 \frac{|V_k^{(p)}|^2}{V_{rms}^2} \right) \quad \text{(between)}$$

$$\sigma_{bu}^2 \stackrel{\text{def}}{=} \sum_k \left(\sum_p \left[b_k^{(p)} - \mu_b(k) \right]^2 \frac{|V_k^{(p)}|^2}{V_{rms}^2} \right) \quad \text{(within)}$$

$$\sigma_{bs}^2 \stackrel{\text{def}}{=} \sum_k \left(\sum_p \left[\mu_b(k) - \mu_b \right]^2 \frac{|V_k^{(p)}|^2}{V_{rms}^2} \right) \quad \text{(between)}$$

OUTLINE

- Single-phase sinusoidal waveforms
- Single-phase nonsinusoidal waveforms
- Euclidean waveform spaces
- Inactive power components
- Polyphase waveforms

➨ **DYNAMIC POWER COMPONENTS**

DYNAMIC POWER COMPONENTS

Time-variant inner product and norm:

$$\langle\, x(\cdot)\,,\, y(\cdot)\,\rangle(t) \;\stackrel{\text{def}}{=}\; \frac{1}{T}\int_{t-T}^{t} x(t)\, y^{\top}(t)\, dt \quad , \qquad \|x(\cdot)\|(t) \;\stackrel{\text{def}}{=}\; \sqrt{\langle\, x(\cdot)\,,\, x(\cdot)\,\rangle(t)}$$

$$V_{rms}(t) \;=\; \|v(\cdot)\|(t) \quad , \qquad I_{rms}(t) \;=\; \|i(\cdot)\|(t) \quad , \qquad P(t) \;=\; \langle\, v(\cdot)\,,\, i(\cdot)\,\rangle(t)$$

DYNAMIC POWER COMPONENTS

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Dynamic 7-component approach: (transient conditions)

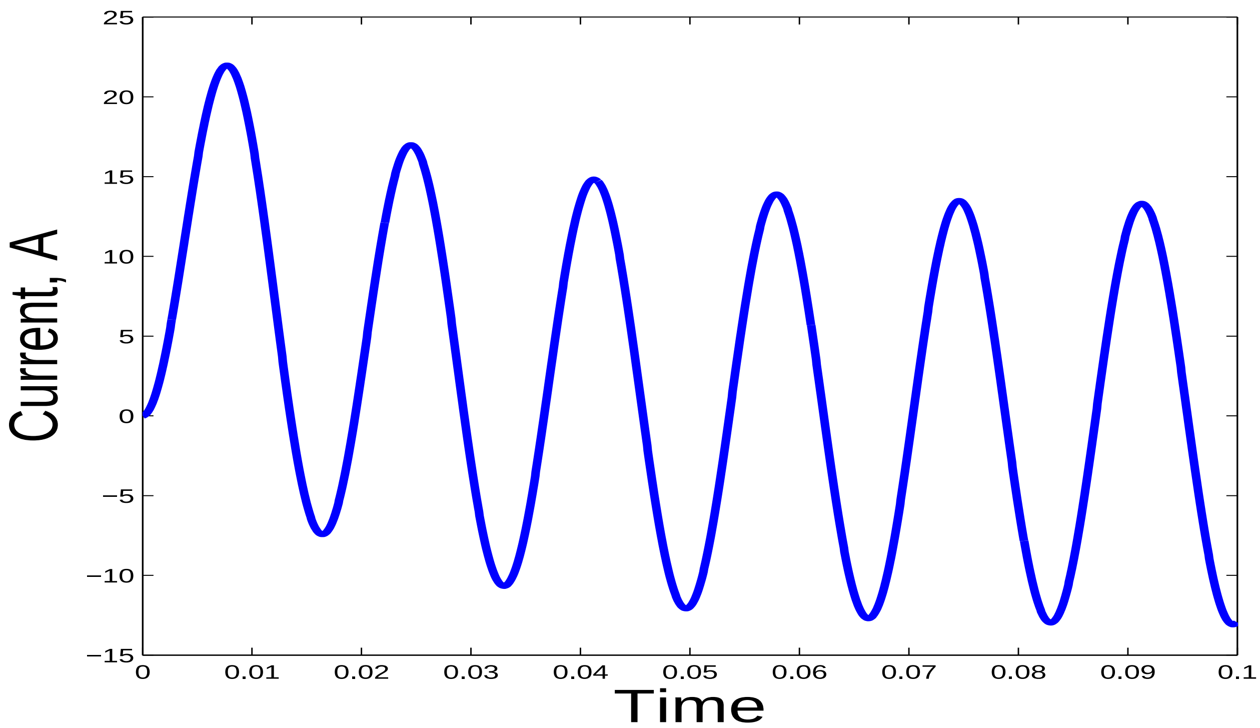
$$S^2(t) \;\equiv\; V^2_{rms}(t)\, I^2_{rms}(t)$$

$$= P^2(t) \;+\; N^2_{gs}(t) \;+\; N^2_{gu}(t) \;+\; Q^2_B(t) \;+\; N^2_{bs}(t) \;+\; N^2_{bu}(t) \;+\; S^2_{\perp}(t)$$

- Time-invariant for periodic waveforms

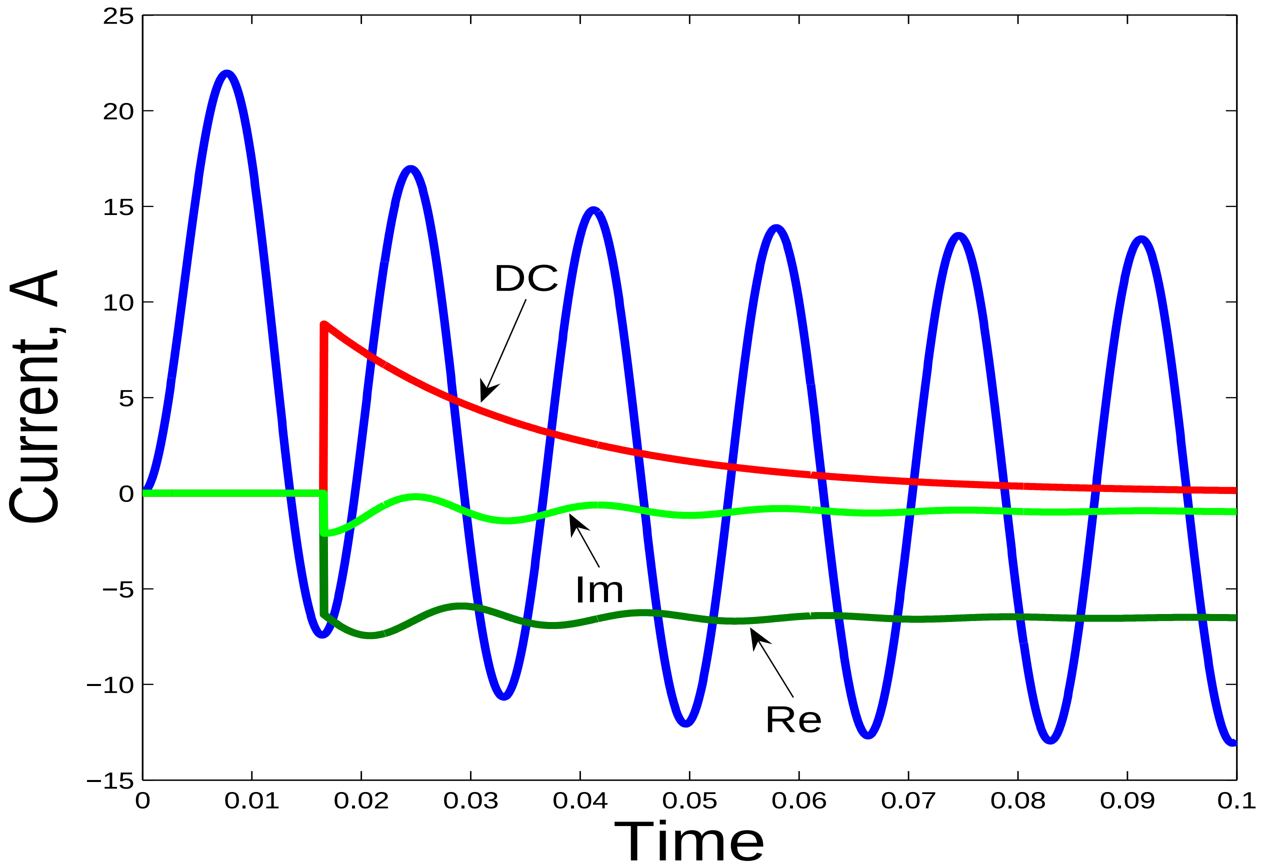
A Simple Example

Consider a simple RL circuit (R=0.1, L=0.002) with a $\sin$ excitation (V=10), at 60Hz



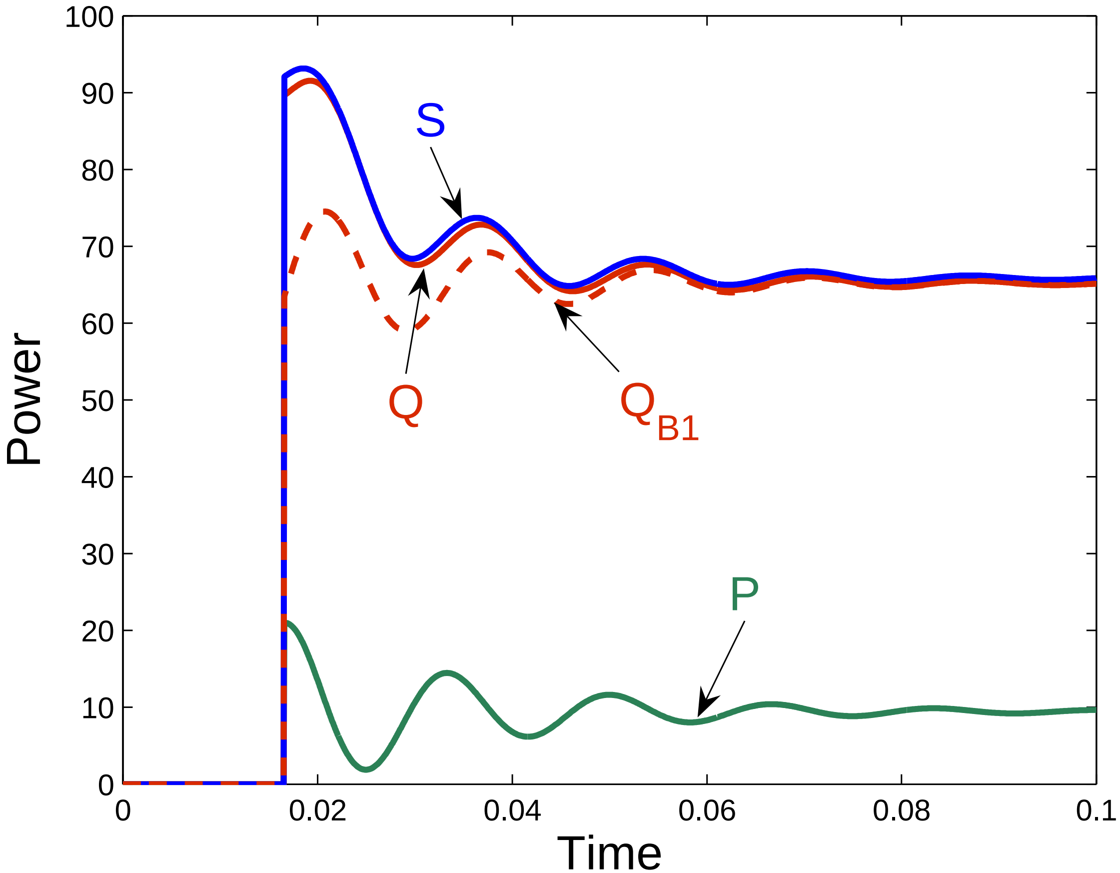
A Simple Example (2)

The dynamic phasors (according to our definition)



Reactive Power in the Example

Our inductor example



DYNAMIC POWER COMPONENTS (2)

Industrial example:

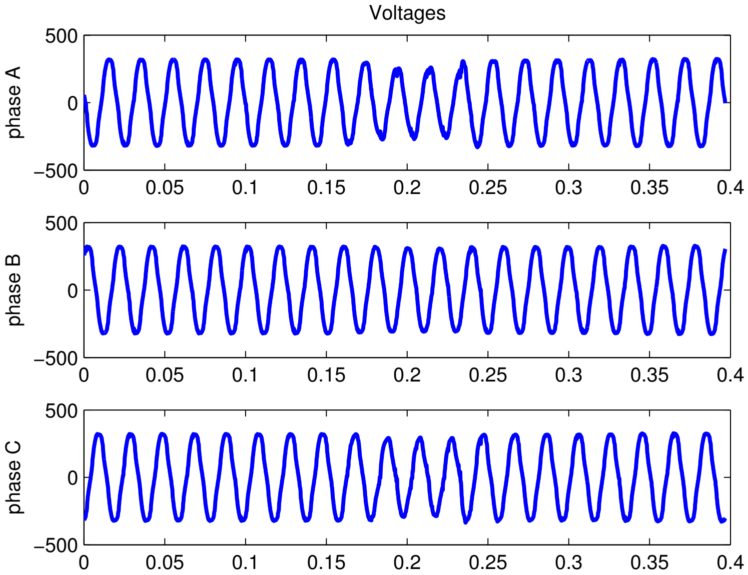
- Power flow during a fault (sag): data collected from a large paper mill.
- Sampling rate is 140 samples/cycle: sufficient to cover multiple harmonics.
- Figures show ten cycles before the fault, and ten cycles after the fault.

DYNAMIC POWER COMPONENTS (2)

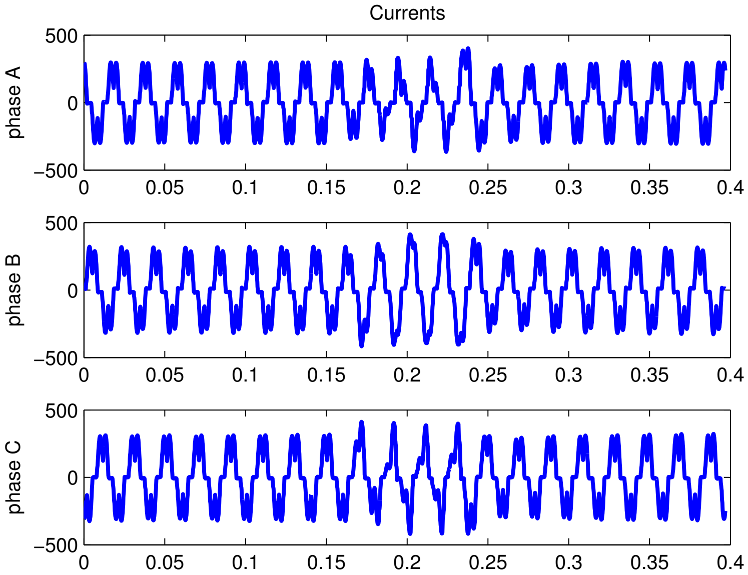
Industrial example:

- Power flow during a fault (sag): data collected from a large paper mill.
- Sampling rate is 140 samples/cycle: sufficient to cover multiple harmonics.
- Figures show ten cycles before the fault, and ten cycles after the fault.
- Noticeable current distortion even before the fault: significant values of N_{gs} and N_{bs}
- Fault causes significant increase in load imbalance: N_{gu} and N_{bu} increase by (approximately) a factor of 4 during the transient.

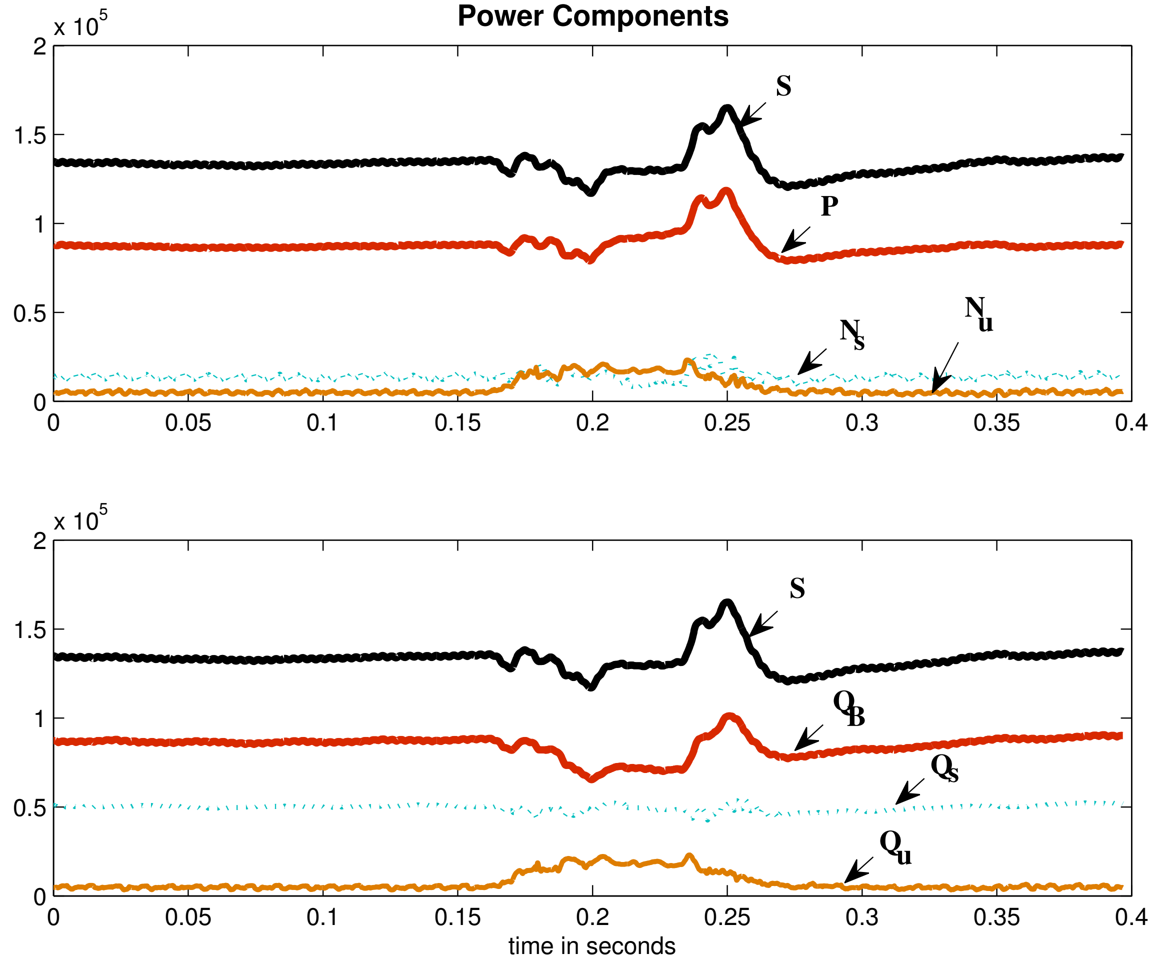
Paper Mill - 1


Voltages
phase A
phase B
phase C
500
0
−500
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4

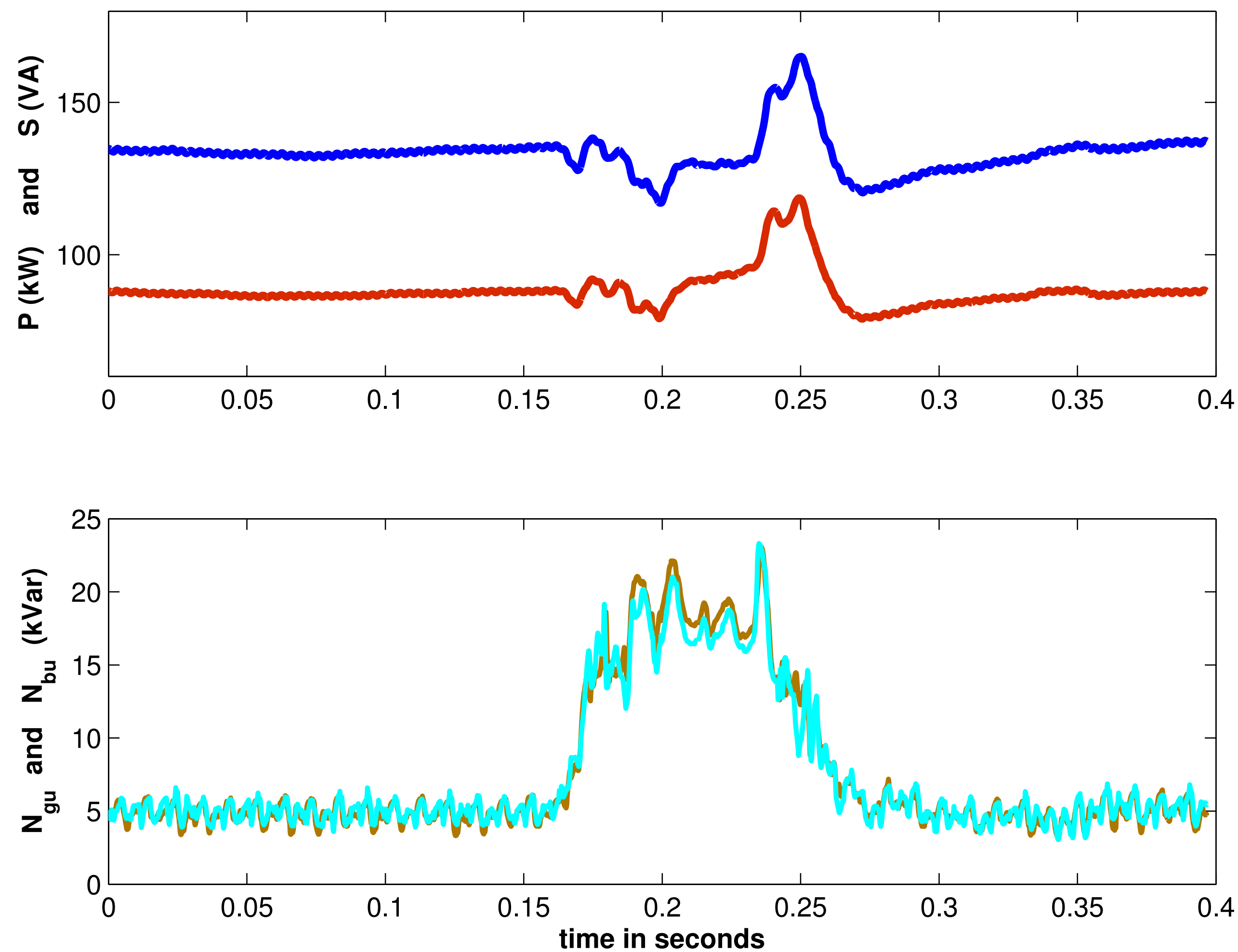
Paper Mill - 2


Currents
phase A
phase B
phase C
500
0
–500
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4

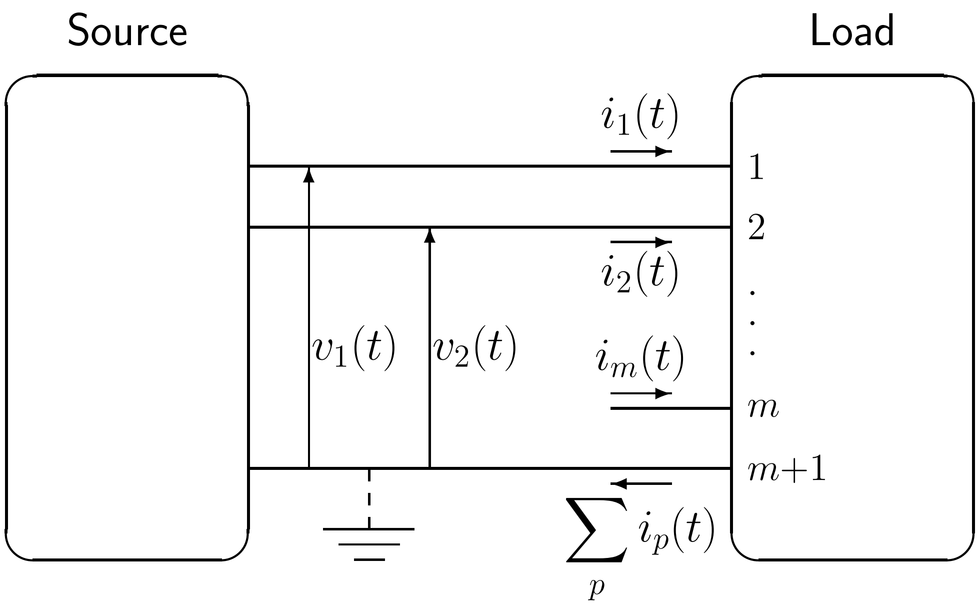
DYNAMIC POWER COMPONENTS (3)



DYNAMIC POWER COMPONENTS (4)



INSTANTANEOUS DYNAMIC POWER COMPONENTS



$$v(t) \stackrel{\text{def}}{=} \begin{bmatrix} v_1(t) & v_2(t) & \ldots & v_m(t) \end{bmatrix} \in \mathbb{R}^m$$

$$i(t) \stackrel{\text{def}}{=} \begin{bmatrix} i_1(t) & i_2(t) & \ldots & i_m(t) \end{bmatrix} \in \mathbb{R}^m$$

INSTANTANEOUS DYNAMIC POWER COMPONENTS (2)

Instantaneous real and apparent power: (Akagi & Nabae approach)

$$p(t) \;=\; v(t)\, i^{\top}(t) \qquad , \qquad s(t) \;\stackrel{\text{def}}{=}\; \sqrt{\left[v(t)v^{\top}(t)\right]\;\left[i(t)i^{\top}(t)\right]}$$

Notice: $s^2(t) - p^2(t) \;=\; 0$ for single-phase systems.

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Notice: $s^2(t) - p^2(t) = 0$ for single-phase systems.

Lagrange identity:

$$s^2(t) - p^2(t) = \sum_{k<\ell} q_{k\ell}^2(t) = \tfrac{1}{2} \left\| Q(t) \right\|_F^2$$

where (skew-symmetric matrix)

$$Q(t) \stackrel{\text{def}}{=} i^{\top}(t)\, v(t) - v^{\top}(t)\, i(t) = \Big[q_{k\ell}(t)\Big]_{k,\ell=1:m}$$

THREE PHASE SYSTEMS

$$\left.\begin{aligned} v(t) &= \begin{bmatrix} v_a(t) & v_b(t) & v_c(t) \end{bmatrix} \\ i(t) &= \begin{bmatrix} i_a(t) & i_b(t) & i_c(t) \end{bmatrix} \end{aligned}\right\} \Rightarrow \quad q_{ab}(t), q_{bc}(t), q_{ac}(t)$$

Coordinate transform: any orthogonal matrix M

$$\left.\begin{aligned} \begin{bmatrix} v_\alpha(t) & v_\beta(t) & v_c0) \end{bmatrix} &= \begin{bmatrix} v_a(t) & v_b(t) & v_c(t) \end{bmatrix} M \\ \begin{bmatrix} i_\alpha(t) & i_\beta(t) & i_c0) \end{bmatrix} &= \begin{bmatrix} i_a(t) & i_b(t) & i_c(t) \end{bmatrix} M \end{aligned}\right\} \Rightarrow \quad q_{\alpha\beta}(t), q_{\alpha o}(t), q_{\beta o}(t)$$

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$$\left.\begin{aligned} v(t) &= \begin{bmatrix} v_a(t) & v_b(t) & v_c(t) \end{bmatrix} \\ i(t) &= \begin{bmatrix} i_a(t) & i_b(t) & i_c(t) \end{bmatrix} \end{aligned}\right\} \Longrightarrow \quad q_{ab}(t), q_{bc}(t), q_{ac}(t)$$

Park transform:

$$\left.\begin{aligned} \begin{bmatrix} v_\alpha(t) & v_\beta(t) & v_c0) \end{bmatrix} &= \begin{bmatrix} v_a(t) & v_b(t) & v_c(t) \end{bmatrix} M \\ \begin{bmatrix} i_\alpha(t) & i_\beta(t) & i_c0) \end{bmatrix} &= \begin{bmatrix} i_a(t) & i_b(t) & i_c(t) \end{bmatrix} M \end{aligned}\right\} \Longrightarrow \quad q_{\alpha\beta}(t), q_{\alpha o}(t), q_{\beta o}(t)$$

where

$$M = \frac{1}{\sqrt{3}} \begin{pmatrix} \sqrt{2} & 0 & 1 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{6}}{2} & 1 \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{6}}{2} & 1 \end{pmatrix}$$

(instantaneous "equivalent" of symmetric sequence components)

Special case: vanishing zero-sequence components

$$v_o(t) = 0 = i_o(t) \quad \Longrightarrow \quad q_{\alpha o}(t) = 0 = q_{\beta o}(t) \quad , \quad q_{\alpha\beta}(t) \stackrel{\text{def}}{=} q_{AN}(t)$$

THREE PHASE SYSTEMS (2)

Interpretations:

- **Vanishing zero-sequence** (Akagi-Nabae) – when $v_0(t) = 0 = i_0(t)$ there is only one non-zero (signed) reactive power quantity $q_{AN}(t)$, which can be conveniently expressed in terms of the Park transform.

- **Vector Calculus Approach** (Dai, Liu and Gretsch, 2004) – these three reactive power quantities can be viewed as the elements of a *cross product* between the current and voltage vectors.

THREE PHASE SYSTEMS (2)

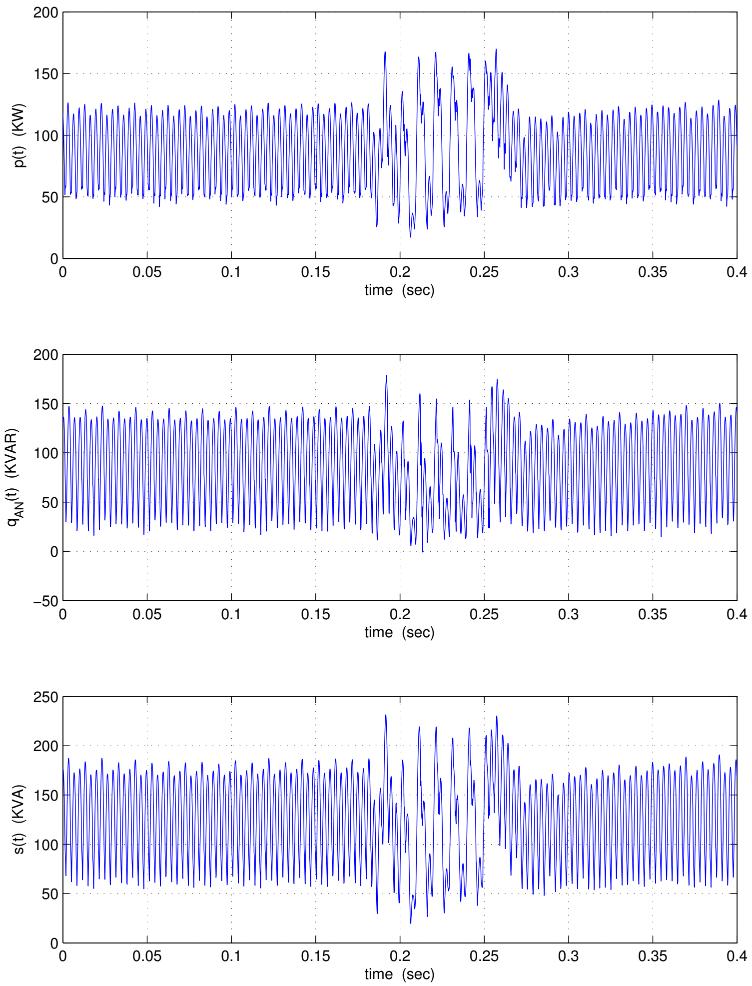
Interpretations:

- **Vanishing zero-sequence** (Akagi-Nabae) – when $v_0(t) = 0 = i_0(t)$ there is only one non-zero (signed) reactive power quantity $q_{AN}(t)$, which can be conveniently expressed in terms of the Park transform.
- **Vector Calculus Approach** (Dai, Liu and Gretsch, 2004) – these three reactive power quantities can be viewed as the elements of a *cross product* between the current and voltage vectors.

Warnings:

- Both interpretations fail when $m > 3$. Vectors, such a $v(t)$ and $i(t)$ consist of m elements, but there are $\frac{m(m-1)}{2}$ distinct instantaneous reactive power quantities.
- Instantaneous powers are "noisy" – hence poor indicators of power events.

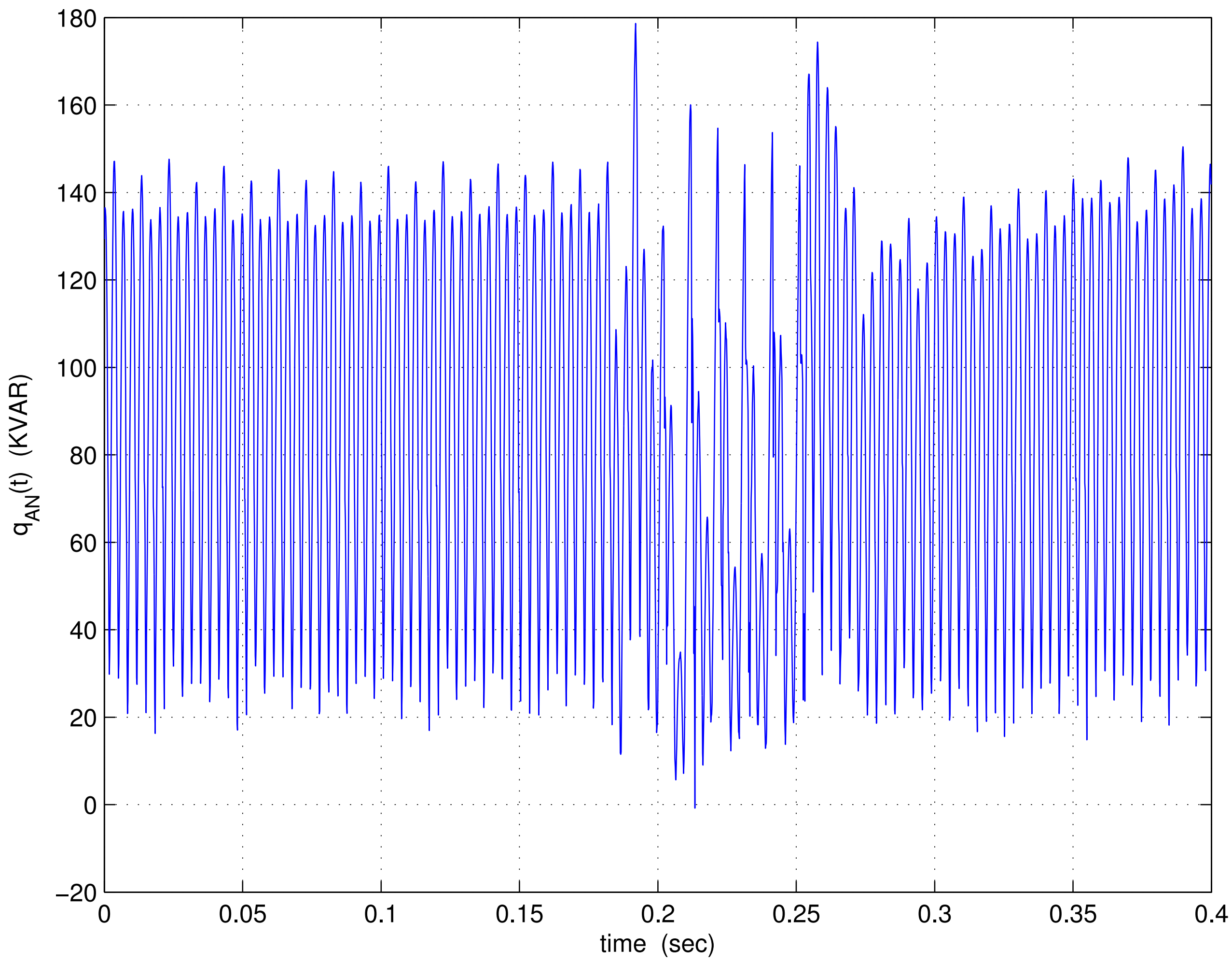
THREE PHASE SYSTEMS (3)



Observations:

- The transient is noticeable in all three waveforms: $s(t)$, $p(t)$ and $q_{AN}(t)$.
- Duration of transient is not easily discernible from either one.
- We get no information about the nature of the fault.

THREE PHASE SYSTEMS (5)



Two samples

Phasor estimate

$$\widehat{X}_1(t) \stackrel{\text{def}}{=} \frac{1}{\sqrt{2}}\, e^{-j\omega t} \left[x(t) \,+\, jx(t-\tfrac{T}{4}) \right]$$

Simple domain transition

$$x(t) \;=\; \sqrt{2}\, \Re\left\{ \widehat{X}_1(t)\, e^{j\omega t} \right\}$$

as well as

$$\begin{aligned} \left\| \widehat{X}_1(t) \right\|^2 &\stackrel{\text{def}}{=} \widehat{X}_1(t)\, \widehat{X}_1^H(t) \\ &= \frac{1}{2} \left[x(t)\, x^T(t) + x(t-\tfrac{T}{4})\, x^T(t-\tfrac{T}{4}) \right] \\ &\stackrel{\text{def}}{=} X_{rms}^2(t) \end{aligned}$$

Two samples

Phasor estimate

$$\widehat{X}_1(t) \stackrel{\text{def}}{=} \frac{1}{\sqrt{2}}\, e^{-j\omega t} \left[x(t) \;+\; jx(t - \tfrac{T}{4}) \right]$$

Simple domain transition

$$x(t) \;=\; \sqrt{2}\, \Re\left\{ \widehat{X}_1(t)\, e^{j\omega t} \right\}$$

as well as

$$\left\| \widehat{X}_1(t) \right\|^2 \stackrel{\text{def}}{=} \widehat{X}_1(t)\, \widehat{X}_1^H(t)$$

$$= \frac{1}{2} \left[x(t)\, x^T(t) + x(t - \tfrac{T}{4})\, x^T(t - \tfrac{T}{4}) \right]$$

$$\stackrel{\text{def}}{=} X_{rms}^2(t)$$

Two samples (2)

Power Definitions

$$P(t) \stackrel{\text{def}}{=} \Re\left\{\widehat{V}_1(t)\,\widehat{I}_1^H(t)\right\}$$

$$Q(t) \stackrel{\text{def}}{=} \Im\left\{\widehat{V}_1(t)\,\widehat{I}_1^H(t)\right\}$$

$$S(t) \stackrel{\text{def}}{=} V_{rms}(t)\,I_{rms}(t) \equiv \left\|\widehat{V}_1(t)\right\|\,\left\|\widehat{I}_1(t)\right\|$$

Admittances and weights

$$G(t) - jB(t) \stackrel{\text{def}}{=} \frac{\widehat{I}_1(t)}{\widehat{V}_1(t)}$$

$$w_k(t) \stackrel{\text{def}}{=} \frac{1}{V_{rms}^2(t)}\left|\widehat{V}_1^{(k)}(t)\right|^2$$

$$\mathcal{M}_w X \stackrel{\text{def}}{=} \sum_k w_k\,x_k$$

Two samples (2)

Power Definitions

$$P(t) \stackrel{\text{def}}{=} \Re\left\{\widehat{V}_1(t)\,\widehat{I}_1^H(t)\right\}$$

$$Q(t) \stackrel{\text{def}}{=} \Im\left\{\widehat{V}_1(t)\,\widehat{I}_1^H(t)\right\}$$

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Two samples (3)

Means and Variances

$$\mu_g(t) \stackrel{\text{def}}{=} \mathcal{M}_w\, G(t) = \frac{P(t)}{V_{rms}^2(t)}$$

$$\mu_b(t) \stackrel{\text{def}}{=} \mathcal{M}_w\, B(t) = \frac{Q(t)}{V_{rms}^2(t)}$$

$$\mathcal{M}_w\, X^2 = \mathcal{M}_w \left(X - \mu_x \mathbf{1}\right)^2 + \mu_x^2$$

$$\mathcal{M}_w\, G^2(t) = \mu_g^2(t) + \sigma_g^2(t)$$

$$\mathcal{M}_w\, B^2(t) = \mu_b^2(t) + \sigma_b^2(t)$$

Power decomposition

$$S^2(t) = V_{rms}^4 \left[\mu_g^2(t) + \sigma_g^2(t) + \mu_b^2(t) + \sigma_b^2(t) \right]$$

$$= P^2(t) + N_g^2(t) + Q^2(t) + N_b^2(t)$$

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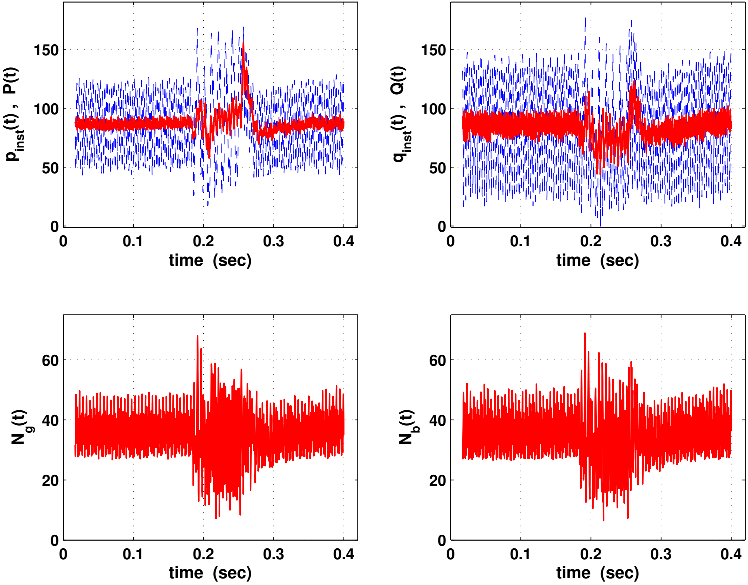
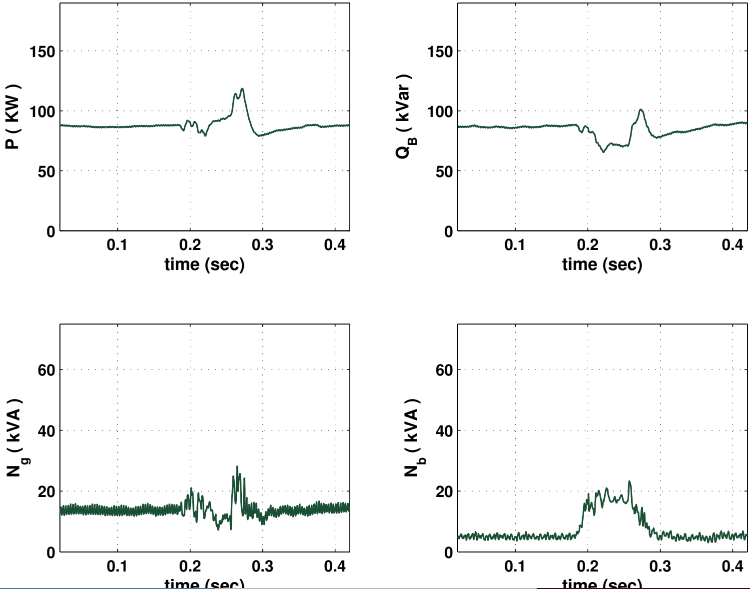
$$\begin{aligned} S^2(t) &= V_{rms}^4 \left[\mu_g^2(t) + \sigma_g^2(t) + \mu_b^2(t) + \sigma_b^2(t) \right] \\ &= P^2(t) + N_g^2(t) + Q^2(t) + N_b^2(t) \end{aligned}$$

Back to the paper mill example



Comparing with Full-cycle quantities



Sequence Components

Phasor Transformations

$$\left[\, \widehat{V}_0(t) \quad \widehat{V}_+(t) \quad \widehat{V}_-(t) \,\right] \stackrel{\text{def}}{=} \widehat{V}_1(t)\, \mathcal{W}$$

$$\left[\, \widehat{I}_0(t) \quad \widehat{I}_+(t) \quad \widehat{I}_-(t) \,\right] \stackrel{\text{def}}{=} \widehat{I}_1(t)\, \mathcal{W}$$

$$\mathcal{W} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha^* & \alpha \\ 1 & \alpha & \alpha^* \end{pmatrix}, \quad \alpha \stackrel{\text{def}}{=} e^{j2\pi/3}$$

Additive Decompositions

$$P(t) = P_0(t) + P_+(t) + P_-(t)$$

$$Q(t) = Q_0(t) + Q_+(t) + Q_-(t)$$

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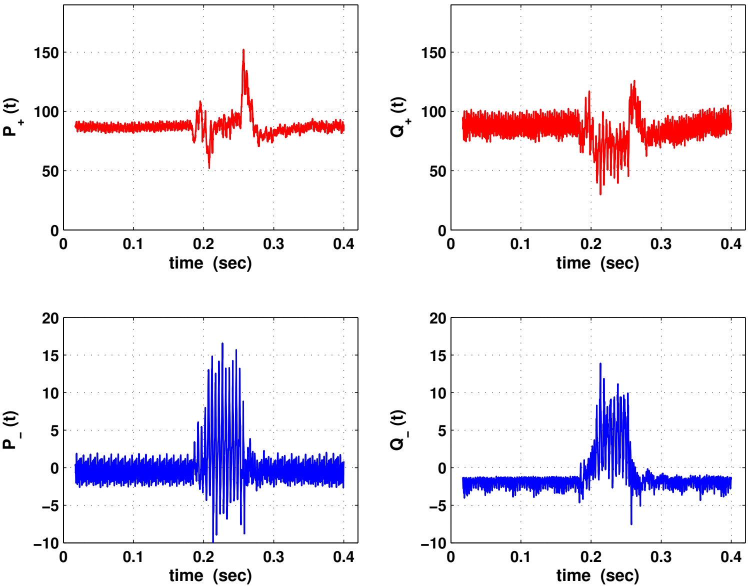
Fast Detection



Conclusions

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- The electric energy engineering - Ars Longa, Vita Brevis.

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