ECE 692
Advanced Topics on Power System Stability

4 – Transient Stability

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Instructor: Kai Sun
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• Transient stability analysis on an SMIB system
• Direct methods
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Transient Stability

- The ability of the power system to maintain synchronism when subjected to a severe disturbance such as a fault on transmission facilities, loss of generation or loss of a large load.
  - The system response to such disturbances involves large excursions of generator rotor angles, power flows, bus voltages, and other system variables.
  - Stability is influenced by the nonlinear characteristics of the system.
  - If the resulting angular separation between the machines in the system remains within certain bounds, the system maintains synchronism.
  - If loss of synchronism due to transient instability occurs, it will usually be evident within 2-3 seconds of the initial disturbances.
Methods for Transient Stability Analysis

Analyzing a dynamic system’s transient stability following a given disturbance

- **Time-domain simulation** solves the nonlinear differential-algebraic equations (DAEs) with known initial values $x=x_0$ and $t=t_0$ by using step-by-step numerical integration techniques (either explicit or implicit).

\[
\dot{x} = f(x,u) \quad \text{DE} \\
0 = g(x,u) \quad \text{AE}
\]

- **Direct methods** are based on Lyapunov’s second method and they determine stability without explicitly solving the DAEs:
  1. Define a Transient Energy Function (TEF) as a possible Lyapunov function
  2. Compare the TEF to a critical energy, denoted by $V_{cr}$, to judge stability
Direct Methods for Transient Stability Analysis

Lyapunov’s second method:

\[ \dot{x} = f(x, u) \]  \hspace{1cm} (1)

Its equilibrium \( x_s \), which satisfies \( f(x_s, u)=0 \), is \textit{stable} if there exists a positive definite function \( V(x) \), called a \textit{Lyapunov function}, such that its total derivative \( dV(x)/dt \) with respect to (1) is not positive, i.e.

- \( V(x) \geq 0 \), with equality if and only if \( x = x_s \)
- \( dV(x)/dt \leq 0 \)

Its equilibrium \( x_s \) is \textit{asymptotically stable} if \( dV(x)/dt \) is negative definite, i.e.

- \( dV(x)/dt \leq 0 \), with equality if and only if \( x = x_s \)
Transient Energy Function (TEF) Approach

- The TEF based methods are a special case of the more general Lyapunov’s 2nd method. The TEF is one possible Lyapunov function.
- Consider a rolling ball analogy: two quantities are required to determine whether the “ball” (state) will go outside the “bowl” (stability region)
  - The initial kinetic energy of the ball
  - The height of the rim at the crossing point (depending on the direction of the initial motion)

![Diagram of a ball rolling on the inner surface of a bowl]

**Rim: Potential energy boundary surface (PEBS)**

**SEP (stable equilibrium point)**

**Figure 13.59** A ball rolling on the inner surface of a bowl
Application to Power Systems

• Initially the system is operating at an SEP $x_s$.

• If a fault occurs, the system gains kinetic energy $V_{KE}$ and potential energy $V_{PE}$ during the fault-on period and hence moves away from $x_s$.

• After fault clearing, $V_{KE}$ is converted into $V_{PE}$.

• To avoid instability, the system must be capable of absorbing $V_{KE}$ at a time when the system moves toward the old SEP $x_s$ or a new SEP.

• For a given post-disturbance network configuration, there is a maximum or critical amount of transient energy $V_{cr}$ that the system can absorb without loss of synchronism.

• Therefore, assessment of transient stability for the state of the system at fault clearing ($x_{cl}$) requires:
  1. Estimate TEF $V(x_{cl})$ that adequately describes the transient energy responsible for separating one or more generators from the rest of the system.
  2. Estimate the critical energy $V_{cr}$.
  3. Calculate the transient energy margin $V_{cr} - V(x_{cl})$. 
4.2 Transient Stability Analysis on an SMIB System
SMIB System

\[
\frac{2H}{\omega_0} \frac{d^2 \delta}{dt^2} = P_m - P_e = P_a
\]

\( P_a \) is called Accelerating Power

\[
P_e = P_{\text{max}} \sin \delta = \frac{E'_E E_B}{X_T} \sin \delta
\]

**Figure 13.3** Power-angle relationship
Response to a step change in $P_m$

Consider a sudden increase in $P_m$: $P_{m0} \rightarrow P_{m1}$.

New SEP $b$ satisfying $P_e(\delta_1) = P_{m1}$

- **$a \rightarrow b$**: Due to the rotor’s inertia, $\delta$ cannot jump from $\delta_0$ to $\delta_1$, so $P_a = P_{m1} - P_e(\delta_0) > 0$ and $\omega_r$ increases from $\omega_0$. **When $b$ is reached, $P_a = 0$ but $\omega_r > \omega_0$, so $\delta$ continues to increase.**

- **$b \rightarrow c$**: $\delta > \delta_1$ and $P_a < 0$, so $\omega_r$ decreases until $c$. **At $c$, $\omega_r = \omega_0$ and $\delta$ reaches the peak value $\delta_{max}$.**

- **$c \rightarrow$** At $c$, the rotor starts to decelerate (since $P_a < 0$) with $\omega_r < \omega_0$ and $\delta$ decreases.

- With all resistances (damping) neglected, $\delta$ and $\omega_r$ oscillate around new SEP $b$ with a constant amplitude.
Equal-Area Criterion (EAC)

- Neglect all losses:

\[
\frac{d^2 \delta}{dt^2} = \frac{\omega_0}{2H}(P_m - P_e)
\]

\[
\frac{d}{dt} \left[ \frac{d\delta}{dt} \right]^2 = \frac{\omega_0}{H}(P_m - P_e) \frac{d\delta}{dt}
\]

\[
\left[ \frac{d\delta}{dt} \right]^2 = \int \frac{\omega_0}{H}(P_m - P_e) d\delta
\]

\[
\Delta \omega_r^2 = \frac{\omega_0}{H} \int (P_m - P_e)d\delta
\]

\[
\frac{1}{2} \left( 2H \omega_0 \right) \cdot \left( \frac{\Delta \omega_r}{\omega_0} \right)^2 = \int (P_m - P_e)d\delta
\]

Moment of inertia in p.u.

\[
\Delta \omega_r = \frac{d\delta}{dt} = 0 \text{ at } \delta_{\text{max}} \text{ if it exists}
\]

\[
0 = \int_{\delta_0}^{\delta_{\text{max}}} (P_m - P_e)d\delta
\]

\[
= \int_{\delta_0}^{\delta_1} (P_m - P_e)d\delta + \int_{\delta_1}^{\delta_{\text{max}}} (P_m - P_e)d\delta
\]

\[
= |A_1| - |A_2| = 0
\]
• **Equal-Area Criterion (EAC):** The stability is maintained only if a decelerating area $|A_2| \geq$ the accelerating area $|A_1|$ can be located above $P_m$ (from b to d, i.e. the Unstable Equilibrium Point or UEP).

If $|A_2|<|A_1|$, $\delta$ will continue increasing at UEP (since $\omega_r>\omega_0$) and lose stability.

For the case with a step change in $P_m$, the new $P_m$ does matter for transient stability.

![Diagram](image-url)  
**FIGURE 11.12**  
Equal-area criterion—maximum power limit.
Transient stability limit for a step change of $P_m$

Following a step change $P_{m0} \rightarrow P_m$, solve the transient stability limit of $P_m$:

- Assume $|A_1| = |A_2|$ in order to solve the limit of $P_m$:

$$P_m(\delta_1 - \delta_0) - \int_{\delta_0}^{\delta_1} P_{\text{max}} \sin \delta d \delta = \int_{\delta_1}^{\delta_{\text{max}}} P_{\text{max}} \sin \delta d \delta - P_m(\delta_{\text{max}} - \delta_1)$$

$$-P_m \delta_0 + P_{\text{max}} (\cos \delta_1 - \cos \delta_0) = -P_{\text{max}} (\cos \delta_{\text{max}} - \cos \delta_1) - P_m \delta_{\text{max}}$$

$$(\delta_{\text{max}} - \delta_0)P_m = P_{\text{max}} (\cos \delta_0 - \cos \delta_{\text{max}})$$

- At the UEP, $P_m = P_{\text{max}} \sin \delta_{\text{max}}$

$$(\delta_{\text{max}} - \delta_0) \sin \delta_{\text{max}} + \cos \delta_{\text{max}} = \cos \delta_0$$

- Solve $\delta_{\text{max}}$ to calculate the transient stability limit for a step change of $P_m$:

$$P_m = P_{\text{max}} \sin \delta_{\text{max}} \quad \delta_1 = \pi - \delta_{\text{max}}$$

Questions: Can we increase $P_m$ beyond that transient limit? If yes, how much further can we increase $P_m$?

![Graph showing transient stability limit](image-url)

**FIGURE 11.12**

Equal-area criterion—maximum power limit.
Solve $\delta_{max}$ by the Newton-Raphson method

$$(\delta_{max} - \delta_0) \sin \delta_{max} + \cos \delta_{max} = \cos \delta_0$$

• The nonlinear function form:
  $$f(\delta_{max}) = \cos \delta_0 = c$$

• Select an initial estimate:
  $$\pi / 2 < \delta_{max}^{(k)} < \pi$$

• Calculate iterative solutions by the N-R algorithm:

  $$\delta_{max}^{(k+1)} = \delta_{max}^{(k)} + \Delta \delta_{max}^{(k)}$$

  where

  $$\Delta \delta_{max}^{(k)} = \frac{c - f(\delta_{max}^{(k)})}{\frac{df}{d\delta_{max}^{(k)}}} = \frac{c - f(\delta_{max}^{(k)})}{(\delta_{max}^{(k)} - \delta_0) \cos \delta_{max}^{(k)}}$$

• Give a solution when a specific accuracy $\varepsilon$ is reached, i.e.

  $$\left| \delta_{max}^{(k+1)} - \delta_{max}^{(k)} \right| \leq \varepsilon$$
Response to a three-phase fault

\[ P_e = P_{\text{max}} \sin \delta = \left( \frac{E'}{E_B} \right) \sin \delta \]

- \( P_{e,\text{during\,fault}} \ll P_{e,\,\text{post-fault}} \)
- \( P_{e,\text{post-fault}} < P_{e,\text{pre-fault}} \) for a permanent fault (cleared by tripping the fault circuit)
- \( P_{e,\text{post-fault}} = P_{e,\text{pre-fault}} \) for a temporary fault
Stable

Unstable

\[ A_1 = A_2 \]

\[ A_1 > A_2 \]

(c) Response to a fault cleared in \( t_{c1} \) seconds - stable case

(d) Response to a fault cleared in \( t_{c2} \) seconds - unstable case
Critical Clearing Angle (CCA)

- Consider a simple case
  - A three-phase fault at the sending end
  - $P_e$, during fault = 0 if all resistances are neglected
  - Critical Clearing Angle $\delta_c$

$$\int_{\delta_0}^{\delta_c} P_m d\delta = \int_{\delta_c}^{\delta_{\text{max}}} (P_{\text{max}} \sin \delta - P_m) d\delta$$

$$|A_1| \quad \quad |A_2|$$

Integrating both sides:

$$P_m (\delta_c - \delta_0) = P_{\text{max}} (\cos \delta_c - \cos \delta_{\text{max}}) - P_m (\delta_{\text{max}} - \delta_c)$$

$$\cos \delta_c = \frac{P_m}{P_{\text{max}}} (\delta_{\text{max}} - \delta_0) + \cos \delta_{\text{max}}$$

$$= \frac{P_m}{P_{\text{max}}} (\pi - 2\delta_0) - \cos \delta_0$$

$$= \pi - \delta_0$$
Critical Clearing Time (CCT)

- Solve the CCT from the CCA:

Since $P_e$, during fault = 0 for this case, during the fault:

\[
\frac{2H}{\omega_0} \frac{d^2 \delta}{dt^2} = P_m
\]

\[
\frac{d\delta}{dt} = \frac{\omega_0}{2H} P_m \int_0^t dt = \frac{\omega_0}{2H} P_m t
\]

\[
\delta = \frac{\omega_0}{4H} P_m t^2 + \delta_0
\]

\[
t_c = \sqrt{\frac{4H(\delta_c - \delta_0)}{\omega_0 P_m}}
\]
• For a more general case: $P_e$ (during fault) > 0

\[ P_m (\delta_c - \delta_0) - \int_{\delta_0}^{\delta_c} P_{2\text{max}} \sin \delta d\delta = \int_{\delta_c}^{\delta_{\max}} P_{3\text{max}} \sin \delta d\delta - P_m (\delta_{\max} - \delta_c) \]

\[ \cos \delta_c = \frac{P_m (\delta_{\max} - \delta_0) + P_{3\text{max}} \cos \delta_{\max} - P_{2\text{max}} \cos \delta_0}{P_{3\text{max}} - P_{2\text{max}}} \]

• $|A_1| = V_{KE}(\delta_c)$, i.e. the kinetic energy at $\delta_c$

• $|A_1| + |A_3| = V(\delta_c) = V_{KE}(\delta_c) + V_{PE}(\delta_c)$, total energy at $\delta_c$

• $|A_2| + |A_3| = V_{PE}(\delta_u) = V_{cr}$, i.e. the largest potential energy

• If and only if $V(\delta_c) \leq V_{cr}$ (i.e. $|A_1| \leq |A_2|$), the generator is stable
Factors influencing transient stability

- How heavily the generator is loaded.
- The generator output during the fault. This depends on the fault location and type
- The fault-clearing time
- The post-fault transmission system reactance
- The generator reactance. A lower reactance increases peak power and reduces initial rotor angle.
- The generator inertia. The higher the inertia, the slower the rate of change in angle. This reduces the kinetic energy gained during fault; i.e. the accelerating area $A_1$ is reduced.
- The generator internal voltage magnitude ($E'$). This depends on the field excitation
- The infinite bus voltage magnitude $E_B$
EAC for a Two-Machine System

• Two interconnected machines respectively with $H_1$ and $H_2$
  – The system can be reduced to an equivalent SMIB system

\[
\frac{d^2 \delta_1}{dt^2} = \frac{\omega_0}{2H_1} (P_{m1} - P_{e1}) = \frac{\omega_0}{2H_1} P_{a1}
\]
\[
\frac{d^2 \delta_2}{dt^2} = \frac{\omega_0}{2H_2} (P_{m2} - P_{e2}) = \frac{\omega_0}{2H_2} P_{a2}
\]

\[
\frac{2}{\omega_0} \frac{H_1 H_2}{H_1 + H_2} \frac{d^2 \delta_{12}}{dt^2} = \frac{H_2 P_{a1} - H_1 P_{a2}}{H_1 + H_2} = \frac{H_2 P_{m1} - H_1 P_{m2}}{H_1 + H_2} - \frac{H_2 P_{e1} - H_1 P_{e2}}{H_1 + H_2}
\]

\[
\frac{2H_{12}}{\omega_0} \frac{d^2 \delta_{12}}{dt^2} = P_{m,12} - P_{e,12}
\]

\[
\int_{\delta_{12,0}}^{\delta_{12,max}} \frac{\omega_0}{H_{12}} (P_{m,12} - P_{e,12}) d\delta = 0
\]
An Example with Multiple Faults

\[ \delta_{d_1} \quad \delta_{d_2} \quad \delta_{c_1} \quad \delta_{c_2} \quad \delta_{d_3} \quad \delta_{c_3} \quad \delta_{\text{max}} \]

\[ Pe_{,0} \quad Pe_{,1} \quad Pe_{,3}=Pe_{,2} \]

Diagram showing the relationship between control variables \( P_e \) and \( P_m \) with multiple fault points marked as \( \delta \).