Objectives of Voltage Stability Analysis (VSA)

• **Proximity**: how close is the system to voltage instability?
  – Operating/planning stability margin (distance to instability) may be measured in terms of physical quantities, e.g., load level, MW flow through a critical interface, and var reserve
  – The most appropriate measure for any given situation depends on the specific system and the intended use of the margin
  – Consideration must be given to possible contingencies (line outages, loss of a generating unit or a var source, etc.)

• **Mechanism**: 
  – How and why does instability occur?
  – What are the key factors contributing to instability?
  – What are the voltage-weak areas?
  – What measures are most effective in improving voltage stability?
Methods of VSA

- **Dynamic Analysis** considers DEs with time handled explicitly
  - Enhanced time-domain simulations (more accurate and longer)

- **Static (Steady-state) Analysis** considers only AEs and time is handled implicitly
  - Powerflow based techniques

- **Quasi-Dynamic Analysis** considers AEs with time handled explicitly
  - Quasi-dynamic fast time-domain simulations
    - Fast dynamics are ignored
    - Equations are algebraic and solved every time the variables associated with slow dynamics are changed
Dynamic Analysis

• Advantages:
  – Captures the events and chronology leading to voltage instability
  – Accurately replicates the actual dynamics of voltage instability if accurate simulation models are used.
  – Performance of system and individual devices is provided

• Disadvantages:
  – Substantial data requirements: credibly modeling voltage-related devices; beyond those for transient stability simulations (accurate for 10 sec.)
  – Needs to simulate for long times (up to tens of minutes)
  – Simulations do not readily provide sensitivity or stability margin information

• Applications:
  – Essential for studies involving the coordination of controls and protections
  – Short-term voltage stability analysis
  – Postmortem studies
  – Benchmarking of simplified studies (steady-state analysis)
Kundur’s Example 14.2

• Models:
  – 6 transformers (1 ULTC)
  – 3 shunt capacitor (buses 7, 8 & 9)
  – Detailed G2 and G3 with thyristor exciters
  – 1 over-excitation limiter (OXL) with G3
  – Load 11: 50% Impedance + 50% Current
  – Load 8: a) constant P&Q; b) induction motor; c) constant Q + thermostatic P

• Load levels:
  1. 6655MW+1986Mvar
  2. 6755MW+2016Mvar
  3. 6805MW+2031Mvar

Figure E14.4 Test system

Figure E14.6 OXL characteristic
Constant P&Q load at bus 8

- **Load level 1:**
  - The ULTC of T6 restores bus 11 voltage at about 40s

- **Load level 2:**
  - While the ULTC of T6 tries to restore bus 11 voltage, the field current limit of G3 is met and the OXL ramps the field current down starting around 180s.

- **Load level 3:**
  - The field current of G3 reaches its limit at about 50s
  - Bus 11 voltage drops with each tap movement of the ULTC of T6
  - The voltage settles when the ULTC reaches its limit and stops
**Induction motor load at bus 8**

- The motor stalls at about 65s, draws rapidly increased reactive power and leads to voltage collapse.

![Graphs showing motor speed, active power, reactive power, and bus voltage over time.](image)

*Figure E14.11* Response of voltage magnitude at bus 11 with (a) constant MVA load and (b) induction motor load at bus 8; system load at level 2
Thermostatically controlled load at bus 8

- The load controller increases the conductance to restore the load and results in a lower bus 11 voltage

**Figure E14.13** Response of voltage magnitude at bus 11 with TC load and constant resistance load at bus 8
Static Analysis

• **Approach:** capturing *snapshots (equilibrium points)* of system conditions at various time frames along the time-domain trajectory
  - At each of these time frames, assume $\frac{dx}{dt}=0$, and $x$ takes values appropriate to the specific time frame
  - Consequently, the overall system equations reduce to purely algebraic equations, and can be solved by powerflow based techniques

• **Advantages:**
  - Can provide insight into state, proximity to, and mechanism of instability
  - Faster in computation and able to study a large number of system conditions and multiple contingencies

• **Disadvantages:**
  - No dynamics; time trajectory is not available, so there is no indication whether a snapshot (equilibrium) can be reached.

• **Applications**
  - Planning and operation studies involving many system conditions and contingencies
  - Online system security analysis
  - Identifying causes of instability and selecting remedial actions
Sensitivity and Modal Analysis on Power Flow Model

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
= 
\begin{bmatrix}
J_{P\theta} & J_{PV} \\
J_{Q\theta} & J_{QV}
\end{bmatrix}
\begin{bmatrix}
\Delta \theta \\
\Delta V
\end{bmatrix}
\]

where

\(\Delta P\) = incremental change in bus real power
\(\Delta Q\) = incremental change in bus reactive power injection
\(\Delta \theta\) = incremental change in bus voltage angle
\(\Delta V\) = incremental change in bus voltage magnitude

Elements of the Jacobian matrix give the sensitivity between power flow and voltage changes.

Let \(\Delta P=0\),

\[
\Delta Q = J_R \Delta V \\
\Delta V = J_R^{-1} \Delta Q
\]

\[
J_R = [J_{QV} - J_{Q\theta} J_{P\theta} J_{PV}^{-1}]
\]

\(J_R\) is the reduced Jacobian matrix of the system. It represents the linearized relationship between the incremental changes in bus voltage magnitudes and bus reactive power injections.
• Voltage stability characteristics of the system can be identified by computing the eigenvalues and eigenvectors of $J_R$

$$J_R = \xi \Lambda \eta$$

where

$\xi$ = right eigenvector matrix of $J_R$
$\eta$ = left eigenvector matrix of $J_R$
$\Lambda$ = diagonal eigenvalue matrix of $J_R$

$$J_R^{-1} = \xi \Lambda^{-1} \eta$$

$$\Delta V = \xi \Lambda^{-1} \eta \Delta Q = \sum_i \frac{\xi_i \eta_i}{\lambda_i} \Delta Q$$

where

$$v = \Lambda^{-1} q \quad v_i = \frac{1}{\lambda_i} q_i$$

$v = \eta \Delta V$ is the vector of modal voltage variations (note: $\eta=\xi^{-1}$)
$q = \eta \Delta Q$ is the vector of modal reactive power variations
For a 3-bus system:

\[
\begin{bmatrix}
1/\lambda_1 & 1/\lambda_2 & 1/\lambda_3 \\
\eta_{11} & \eta_{12} & \eta_{13} \\
\eta_{21} & \eta_{22} & \eta_{23} \\
\eta_{31} & \eta_{32} & \eta_{33}
\end{bmatrix}
\begin{bmatrix}
\Delta Q_1 \\
\Delta Q_2 \\
\Delta Q_3
\end{bmatrix}
= \begin{bmatrix}
\eta_{11} & \eta_{12} & \eta_{13} \\
\eta_{21} & \eta_{22} & \eta_{23} \\
\eta_{31} & \eta_{32} & \eta_{33}
\end{bmatrix}
\begin{bmatrix}
\Delta V_1 \\
\Delta V_2 \\
\Delta V_3
\end{bmatrix}
\]

For mode 1:

\[
\frac{1}{\lambda_1} (\eta_{11} \Delta Q_1 + \eta_{12} \Delta Q_2 + \eta_{13} \Delta Q_3) = (\eta_{11} \Delta V_1 + \eta_{12} \Delta V_2 + \eta_{13} \Delta V_3)
\]

Or

\[
\frac{1}{\lambda_1} q_1 = v_1
\]

- If \( \lambda_i > 0 \), the \( i^{th} \) modal voltage and the \( i^{th} \) modal reactive power variations are along the same direction, indicating that the system is voltage stable
- If \( \lambda_i < 0 \), the \( i^{th} \) modal voltage is unstable
- If \( \lambda_i = 0 \), the \( i^{th} \) modal voltage collapses since any small change in that modal reactive power causes infinite change in the modal voltage
- The magnitude of \( \lambda_i \) determines the proximity to instability
Bus Participation Factors

\[
\frac{\partial V_k}{\partial Q_k} = \sum_i \frac{\xi_{ki} \eta_{ik}}{\lambda_i}
\]

- The relative participation of bus \( k \) in mode \( i \) is given by
  \[
P_{ki} = \xi_{ki} \eta_{ik}
\]
- \( P_{ki} \) determines the contribution of \( \lambda_i \) to the V-Q sensitivity at bus \( k \)
- Bus participation factors determine the critical buses and areas associated with each mode
- The size of bus participation in a given mode indicates the effectiveness of remedial actions applied at that bus in stabilizing the mode
- Localized modes: very few buses with large participations
- Not localized modes: many buses have small but similar degree of participations.
- In practice, it is seldom necessary to compute more than 5-10 of the smallest eigenvalues to identify all critical modes.
- Also see participation factors of branches and generators in Kundur’s 14.3.3
Continuation Powerflow (CPF) Analysis

• The powerflow Jacobian matrix becomes singular at the voltage stability limit (nose point)
• Conventional powerflow algorithms are prone to divergence problems at operating conditions near the stability limit
• The continuation powerflow (CPF) overcomes this problem by reformulating the powerflow equations so that they remain well-conditioned at all possible loading conditions
• Allows the solution of the powerflow problem for stable as well as unstable equilibrium points
• The method described is based on the work of Ajjarapu and Christy published in 1991
  – Locally-parameterized continuation method, which belongs to a general class of methods for solving nonlinear algebraic equations known as path-following methods
From an initial solution A, a tangent predictor is used to estimate B for a specified pattern of load increase.

Then, a corrector step determines the exact solution C using a conventional powreflow analysis with the system load assumed to be fixed.

The voltages for a further increase in load are then predicted based on a new tangent predictor.

If the new estimated load D is now beyond the maximum load on the exact solution, a corrector step with loads fixed would not converge; therefore, a corrector step with a fixed voltage at the monitored bus is applied to find the exact solution E.

As the voltage stability limit is reached, to determine the exact maximum load, the size of load increase has to be reduced gradually during the successive predictor step.

See mathematical formulation in Kundur’s 14.3.5.

**Figure 14.10** A typical sequence of calculations in a continuation power-flow analysis.
Complementary use of conventional and continuation powerflow methods

• Continuation method of powerflow analysis is robust and flexible
• Ideally suited for solving powerflow problem with convergence difficulties.
• However, it is very slow and time-consuming
• The best overall approach for computing powerflow solutions up to and beyond the critical point is to use the two methods in a complementary manner
  – Usually the conventional methods (N-R or Fast Decoupled) can be sued to provide solutions right up to the critical point
  – The continuation methods become necessary only if solutions exactly at and past the critical point are required
Prevention of Voltage Collapse

See Kundur’s Chapter 14.4

• Application of var compensating devices
  – Ensure adequate stability margin (MW and Mvar distances to instability) by proper selection of schemes
  – Selection of sizes, ratings and locations of the devices (especially for dynamic reactive reserves, e.g. synchronous condensers, STATCOM and SVCs) based on a detailed study
  – Design criteria based on maximum allowable voltage drop following a contingency are often not satisfactory from voltage stability viewpoint
  – Important to recognize voltage control areas and weak boundaries (buses with high participation factors associated with a voltage instability mode).

• Control of transformer tap changers
  – Can be controlled either locally or centrally
  – Where tap changing is detrimental, a simple method is to block tap changing when the source side sags and unblock when voltage recovers
  – Use the knowledge of load characteristics to improve the control schemes
  – Microprocessor-based ULTC controls
Prevention of Voltage Instability (cont’d)

• Control of network voltage and generator reactive output
  – Improvement on AVRs, e.g. adding load (or line drop) compensation
  – Secondary coordinated outer loop voltage control (e.g. the hierarchical, automatic two/three-layer voltage control)

• Coordination of protections/controls
  – Ensure adequate coordination based on dynamic simulation studies
  – Tripping of equipment to protect from overloaded conditions should be the last resort. The overloaded conditions could be relieved by adequate control measures before isolating the equipment.

• Under-voltage load shedding (UVLS)
  – To cater for unplanned or extreme situations; analogous to UFLS
  – Provide a low-cost means of preventing widespread system collapse
  – Particularly attractive if conditions leading to voltage instability are of low probability but consequences are high
  – Characteristics and locations of the loads to be shed are more important for voltage problems than for frequency problems
  – Should be designed to distinguish between faults, transient voltage dips, and low voltage conditions leading to voltage collapse