



**IEEE Forced Oscillation TF, Dec. 2024**

# **Locating the Source of Oscillation with Two-Tier Dynamic Mode Decomposition Integrating Early-Stage Energy**

**Min-Seung Ko**

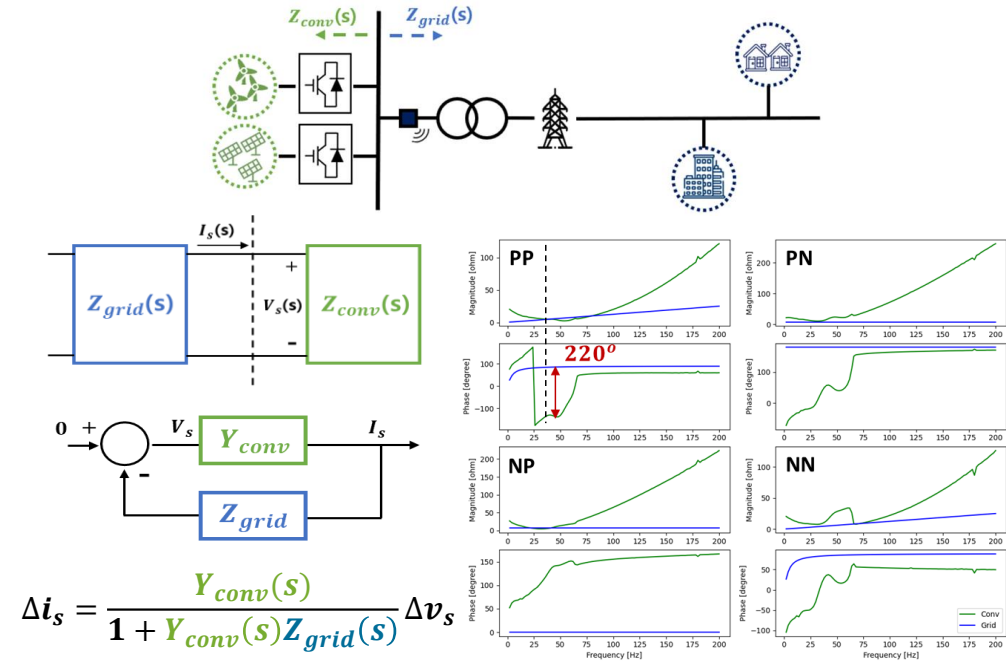
Postdoctoral Fellow supervised by Dr. Hao Zhu,  
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**Coauthor:** Dr. Wooyoung Shin, Dr. Kai Sun

**Advisor:** Dr. Kyeon Hur

# Overview

- Pre- and Post-Mortem Oscillation Framework
  - Pre-Mortem Oscillation Framework
    - How to prevent oscillation which may arise in various frequencies
    - Representative example is impedance-based oscillation analysis
    - Several actions such as adjustment of control gain, can be taken
  - Post-Mortem Oscillation Framework
    - Prior action is to find out the accurate oscillation source as soon as possible
    - Research objective: Generalizable OSL method for both forced and natural cases and both LFO and SSO cases using DMD
    - For more details, please refer to:
      - M. -S. Ko, W. Shin, K. Sun and K. Hur, "Locating the Source of Oscillation With Two-Tier Dynamic Mode Decomposition Integrating Early-Stage Energy," in IEEE Transactions on Power Systems, vol. 39, no. 4, pp. 5535-5547, July 2024







<Example of impedance-based analysis>

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 39, NO. 4, JULY 2024

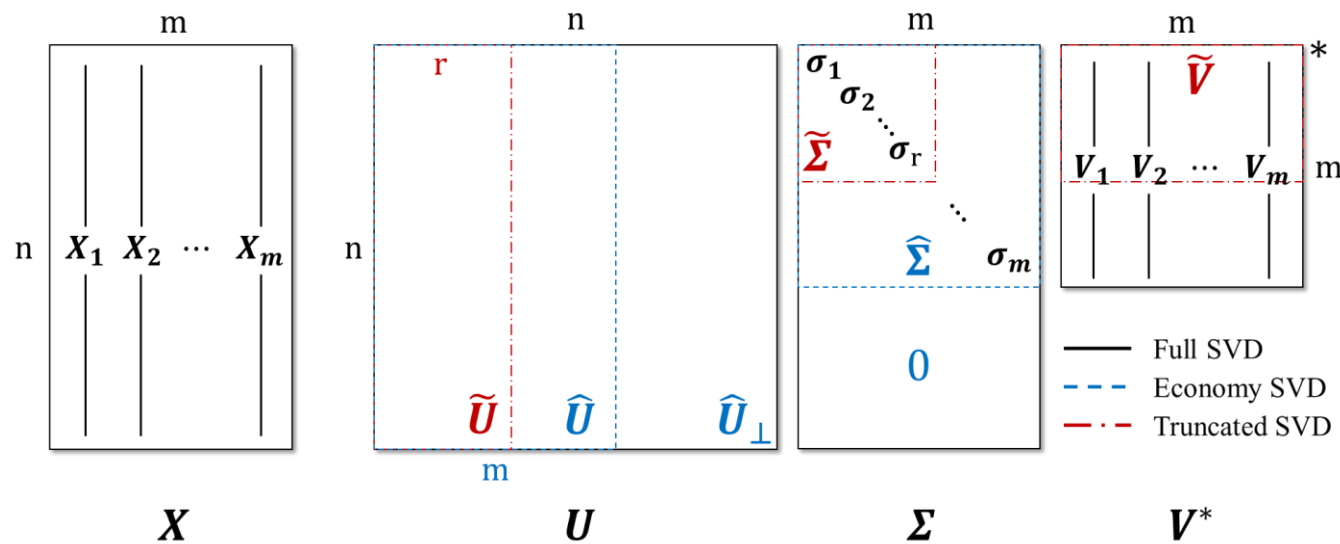
5535

## Locating the Source of Oscillation With Two-Tier Dynamic Mode Decomposition Integrating Early-Stage Energy

Min-Seung Ko , Graduate Student Member, IEEE, Wooyoung Shin , Member, IEEE, Kai Sun , Senior Member, IEEE, and Kyeon Hur , Senior Member, IEEE

# Preliminaries

- Mathematical formulation of Singular Value Decomposition (SVD)
  - Generalized eigen-decomposition of a square matrix into two unitary matrices and one diagonal matrix
  - SVD can be regarded as the data dimensionality reduction method, since key features of the correlation matrix can be drawn from large data



<Schematic diagram of SVD>

$$\begin{aligned}
 X &= U \Sigma V^* : \text{Full SVD} \\
 &= \hat{U} \hat{\Sigma} V^* : \text{Economy SVD} \\
 &\approx \tilde{U} \tilde{\Sigma} \tilde{V}^* : \text{Truncated SVD}
 \end{aligned}$$

where  $U = [u_1 u_2 \dots u_n]$ ,  $V = [v_1 v_2 \dots v_m]$ ,  
 $\Sigma = [\hat{\Sigma} \mathbf{0}]^T$ ,  $\hat{\Sigma} = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_m)$

Eigen-decomposition feature:

$$\begin{aligned}
 X^T X &= (\hat{U} \hat{\Sigma} V^T)^T (\hat{U} \hat{\Sigma} V^T) = V \hat{\Sigma}^2 V^T \\
 \Rightarrow X^T X V &= V \hat{\Sigma}^2
 \end{aligned}$$

# Preliminaries

\* S. L. Brunton and J. N. Kutz, Data-Driven Science and Engineering: Machine Learning, Dynamical Systems, and Control. Cambridge, U.K.: Cambridge Univ. Press, 2019.

## ➤ Derivation of Dynamic Mode Decomposition (DMD) as a spatiotemporal SVD\*

### ■ Eckart-Young's Theorem

- For the linearized objective system  $X' = AX$ , the best rank- $r$  approximation of  $X$  can be drawn from rank- $r$  truncated SVD of  $X$  as:

$$\operatorname{argmin}_{\tilde{X}} \|X - \tilde{X}\| \approx \tilde{U}\tilde{\Sigma}\tilde{V}^* \text{ s.t. } \operatorname{rank}(\tilde{X}) = r$$

$X$ : present measurement matrix  
 $X'$ : future measurement matrix  
 $A$ : best-fit linear operator

### ■ Derivation of DMD

- If we substitute the truncated SVD into the objective system, we can get

$$X' = AX \approx A\tilde{U}\tilde{\Sigma}\tilde{V}^* \rightarrow A = X'\tilde{V}\tilde{\Sigma}^{-1}\tilde{U}^*$$

- Then,  $\tilde{A}$ , which is the projection of  $A$  onto  $\tilde{U}$ , can be calculated as

$$\tilde{A} = \tilde{U}^* A \tilde{U} = \tilde{U}^* X' \tilde{V} \tilde{\Sigma}^{-1}$$

- If we let  $\Lambda$  and  $W$  as the eigenvalues and eigenvectors of  $\tilde{A}$ , i.e.,  $\tilde{A}W = W\Lambda$ ,  $X'$  can be modified as

$$X' = \tilde{U}W\Lambda W^{-1}\tilde{\Sigma}\tilde{V}^* = \Phi\Lambda\Gamma$$

$\Phi = \tilde{U}W$ : DMD mode  
 $\Gamma = W^{-1}\tilde{\Sigma}\tilde{V}^*$ : temporal distribution of mode amplitude  
 $\Lambda$ : empirical Ritz values (includes oscillation frequency information)

# Preliminaries

➤ Derivation of Dynamic Mode Decomposition (DMD) as a spatiotemporal SVD\*

(1) Objective System :

$$\begin{matrix} m-1 \\ \Delta V_{2\sim m} \\ \Delta \theta_{2\sim m} \\ X' \end{matrix} = \begin{matrix} 2l \\ A \end{matrix} \begin{matrix} m-1 \\ \Delta V_{1\sim m-1} \\ \Delta \theta_{1\sim m-1} \\ X \end{matrix}$$

(2) SVD:

$$\begin{matrix} m-1 \\ \Delta V_{1\sim m-1} \\ \Delta \theta_{1\sim m-1} \\ X \end{matrix} = \begin{matrix} 2l \\ U \end{matrix} \begin{matrix} 2l \\ \Sigma \end{matrix} \begin{matrix} m-1 \\ V^* \end{matrix}$$

(3) = (1)+(2):

$$A = X' V \Sigma^{-1} U^*$$

(7) = (5)+(6):

$$X' = \underbrace{U W}_{\Phi} \Lambda \underbrace{W^{-1} \Sigma V^*}_{\Gamma}$$

(4) Projection:

$$\tilde{A} = U^* A U$$

(5) = (3)+(4):

$$\tilde{A} = U^* X' V \Sigma^{-1}$$

(6) Eigen analysis:

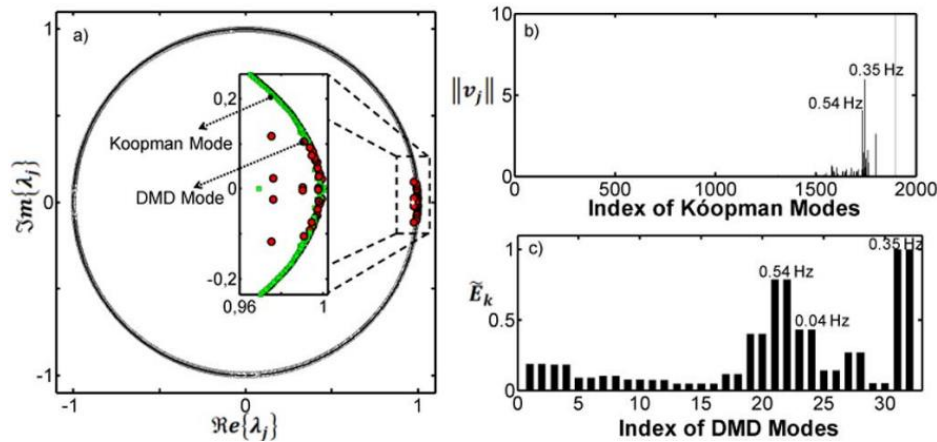
$$\tilde{A} = W \Lambda W^{-1}$$

This computation process is a specific case used for OSL

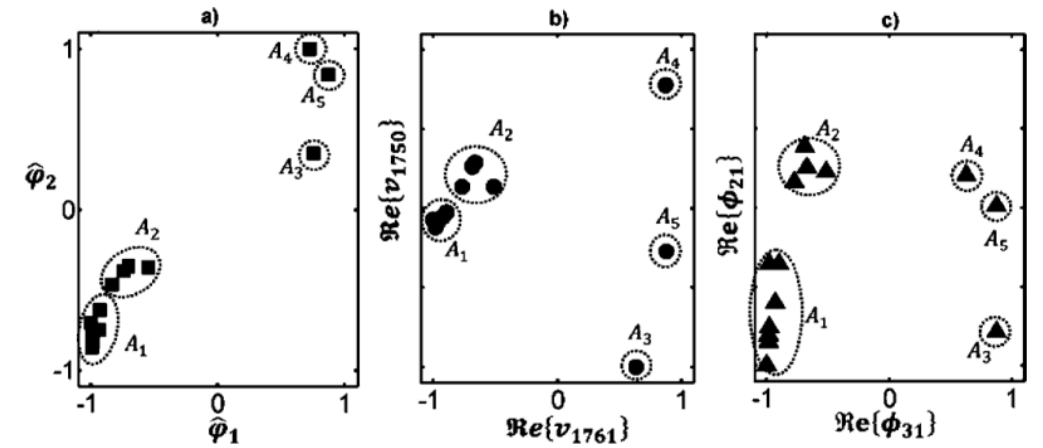
\* S. L. Brunton and J. N. Kutz, Data-Driven Science and Engineering: Machine Learning, Dynamical Systems, and Control. Cambridge, U.K.: Cambridge Univ. Press, 2019.

# DMD on Power System Oscillation

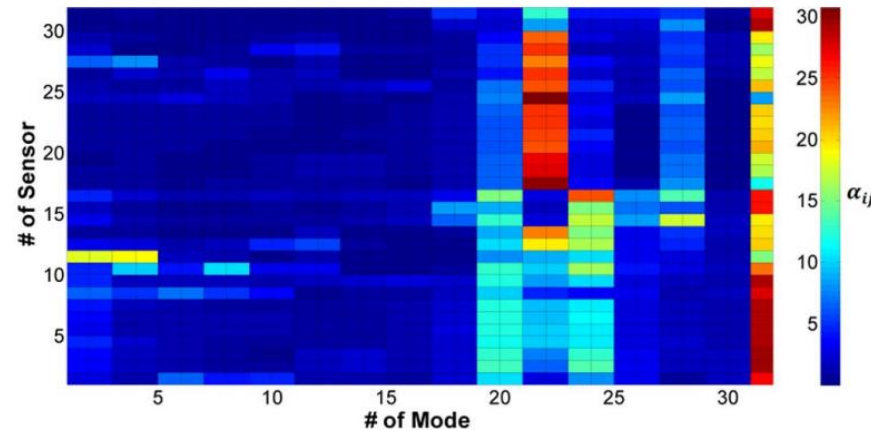
## ➤ Analysis of Power System Oscillation with DMD\*\*



<Estimated empirical Ritz value>



<Coherency identification with (a) POD, (b) Koopman, and (c) DMD>

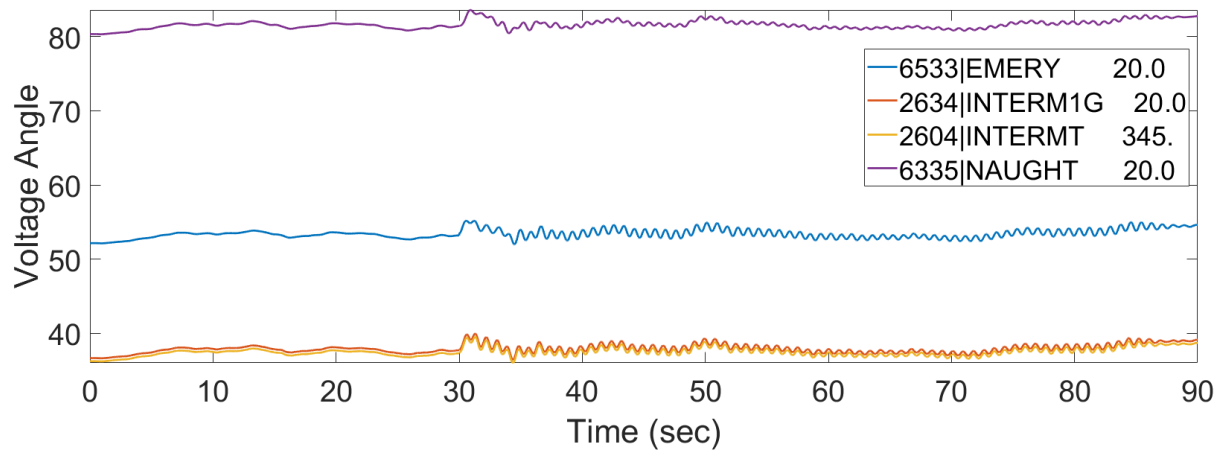


<Dominant mode analysis>

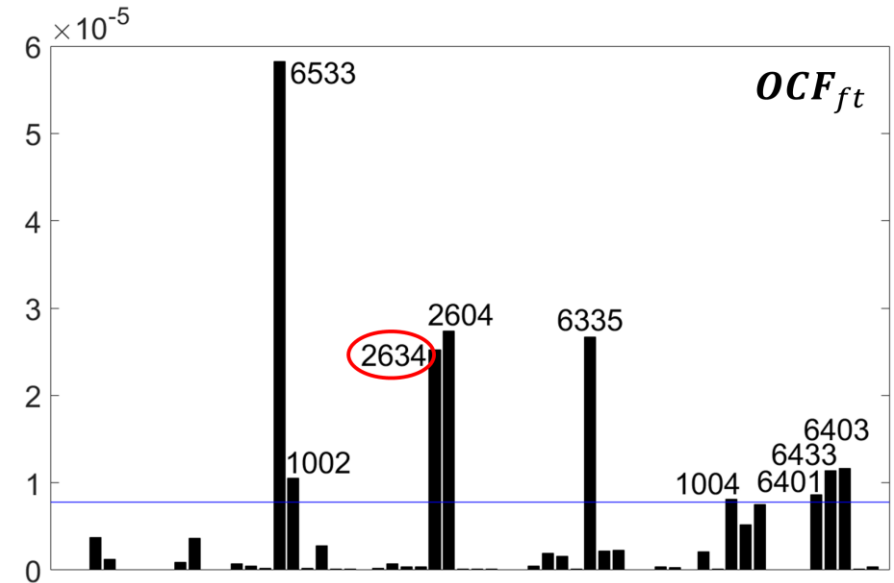
\*\* E. Barocio, et al., "A dynamic mode decomposition framework for global power system oscillation analysis," IEEE Trans. Power Syst., vol. 30, no. 6, pp. 2902–2912, Nov. 2015.

# DMD on Power System Oscillation

- Limitations of DMD on Oscillation Source Location (OSL)
  - No previous research to apply DMD for oscillation source location
    - Framework and details of DMD are not specified for OSL
  - DMD cannot be ensured to be effective for abnormal oscillation cases
    - (Category 2) Maximum oscillation is observed at non-source bus
    - (Category 3) Rectangular forcing signal injected
    - (Category 4) Measurements at the source bus do not exits
    - (Category 5) Other abnormal cases



<Voltage angle of NASPI case #2>

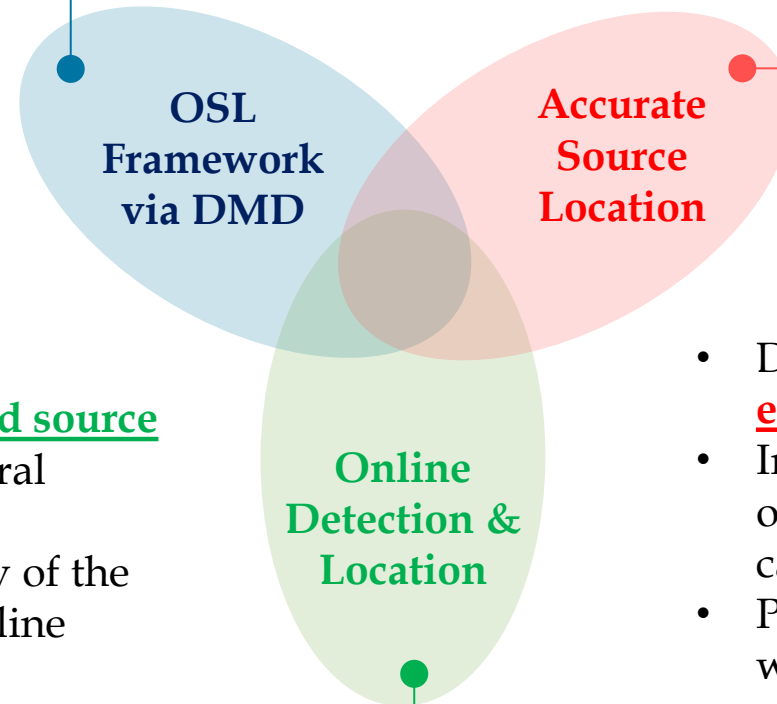


<Mode magnitude analysis of NASPI case #2>

# Contributions of the Proposed Method

- Extend the usage of DMD into oscillation source location
- Construct the oscillation monitoring framework via DMD
- Formulate the improved DMD

- Develop the oscillation detection and source location method using spatio-temporal analysis results from DMD
- Enhance the computational efficiency of the oscillation monitoring scheme for online application



- Develop the two-tier structure to identify the early-stage oscillatory energy
- Improve the location accuracy across various oscillation scenarios for both forced and natural cases
- Provide data preprocessing method to deal with the real oscillation case



# Improved DMD

## ➤ Modifying DMDc into improved DMD

- DMDc\*\*\* (DMD with control) inserts current control signals as an additional input to divide the control from the system dynamics
- Improved DMD for oscillation monitoring replaces the control signal with the initial state of the measurement matrix

\*\*\* J. L. Proctor, S. L. Brunton, and J. N. Kutz, "Dynamic mode decomposition with control," SIAM Journal on Applied Dynamical Systems 15.1 (2016): 142-161.

$$\mathbf{X}' = \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{X}_{ini} = [\mathbf{A} \ \mathbf{B}][\mathbf{X} \ \mathbf{X}_{ini}]^T = \mathbf{G}\mathbf{\Omega} \quad \mathbf{X}_{ini}: \text{initial state of the measurement matrix}$$

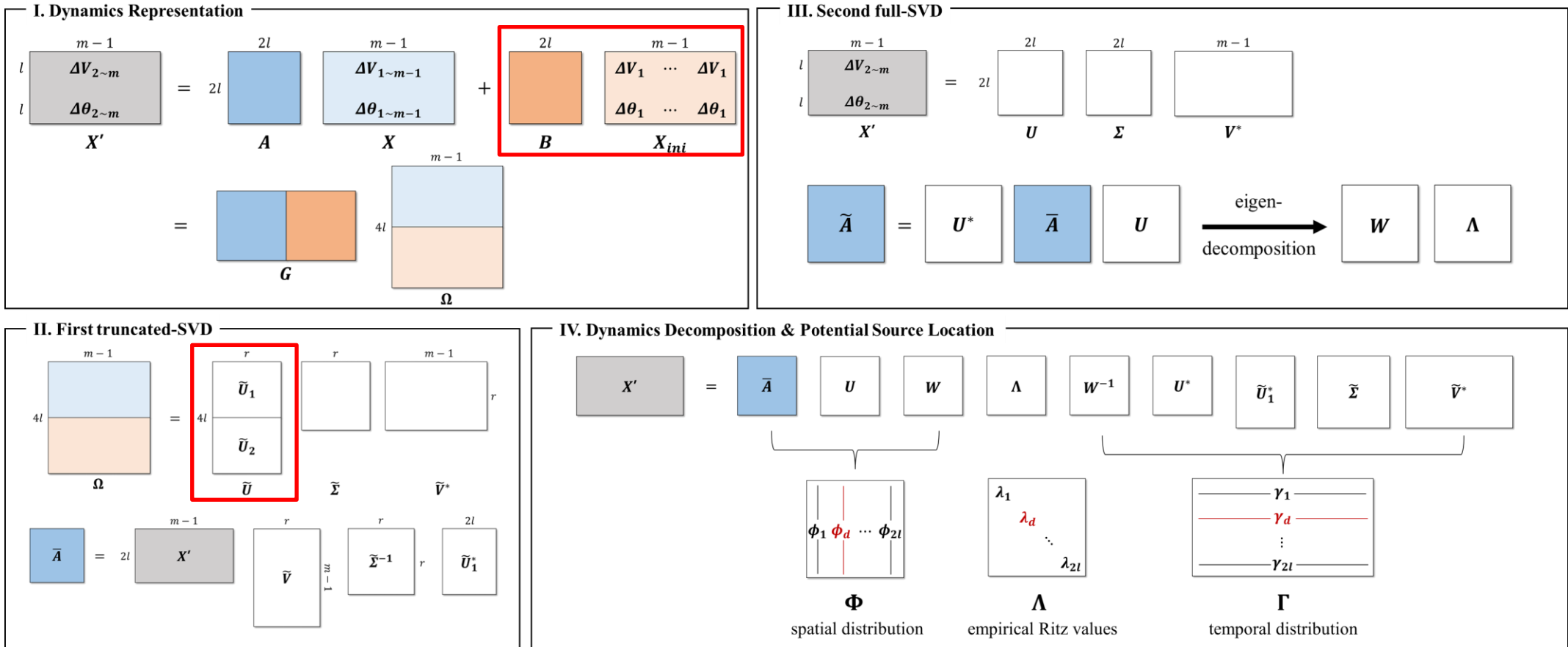
Assume that  $f(k)$  denotes a function describing the oscillation dynamics for every discrete time of the measurements,  $k \in (1, 2, \dots, m - 1)$ , the future matrix can be represented as

$$\begin{aligned} \mathbf{X}_{k+1} &= \mathbf{A}\mathbf{X}_k + \mathbf{B}\mathbf{X}_{ini} = \mathbf{X}_{ini} + f(k) \\ \Rightarrow \mathbf{X}_2 &= \mathbf{A}\mathbf{X}_{ini} + \mathbf{B}\mathbf{X}_{ini} = \mathbf{X}_{ini} + f(1) \\ \Rightarrow \mathbf{B}\mathbf{X}_{ini} &= (\mathbf{I} - \mathbf{A})\mathbf{X}_{ini} + f(1) \\ \Rightarrow \mathbf{A}(\mathbf{X}_k - \mathbf{X}_{ini}) &= f(k) - f(1) \end{aligned}$$

∴ The state matrix  $\mathbf{A}$  represents the linear relationship between the change of the states and change of the oscillation dynamics ( $\mathbf{A}$  can focus on the post-mortem modified dynamics,  $\mathbf{B}$  takes the basic dynamics driven by the initial state)

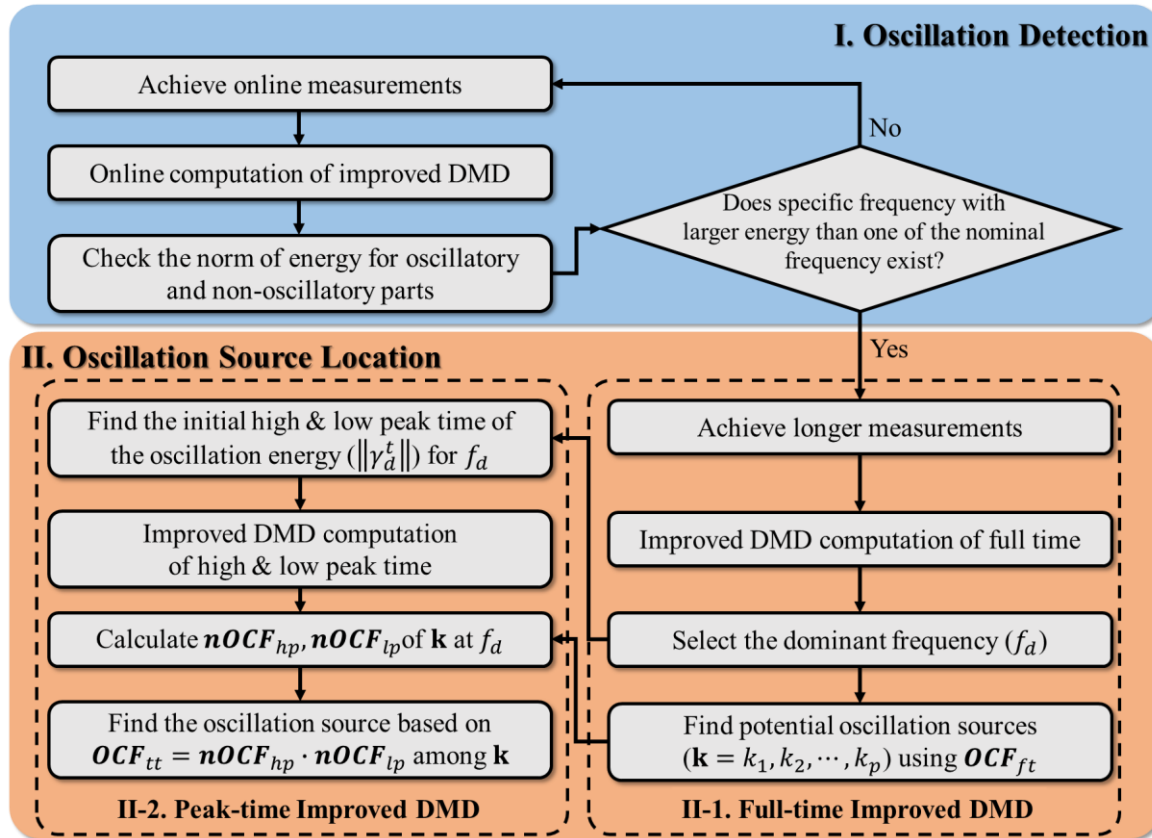
# Improved DMD

## ➤ Improved DMD Computation

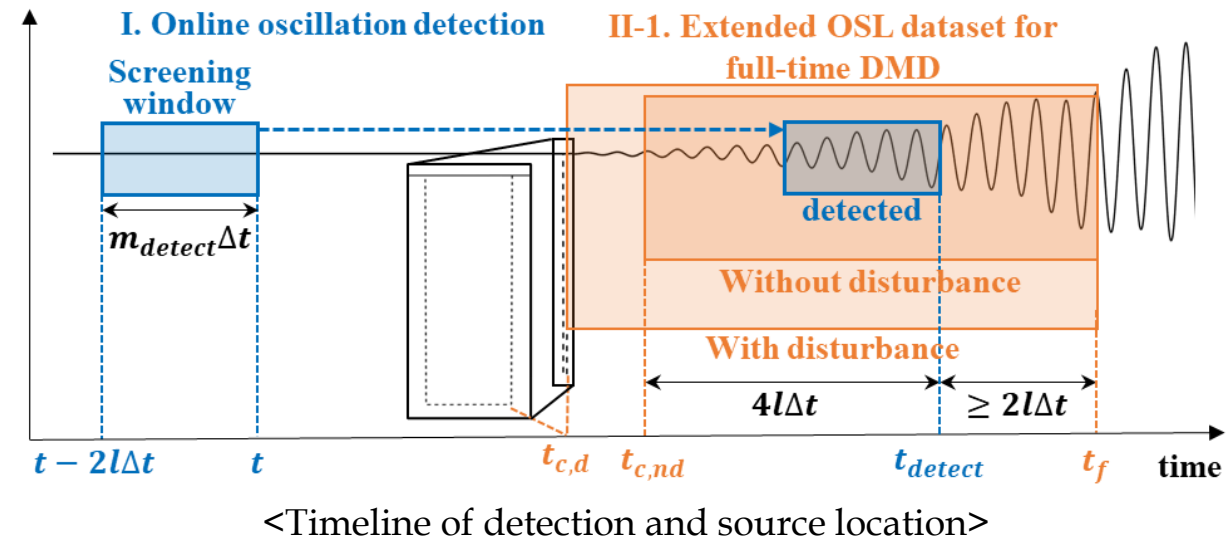


# Proposed Framework

## ➤ Framework Overview



<Oscillation detection & source location framework>

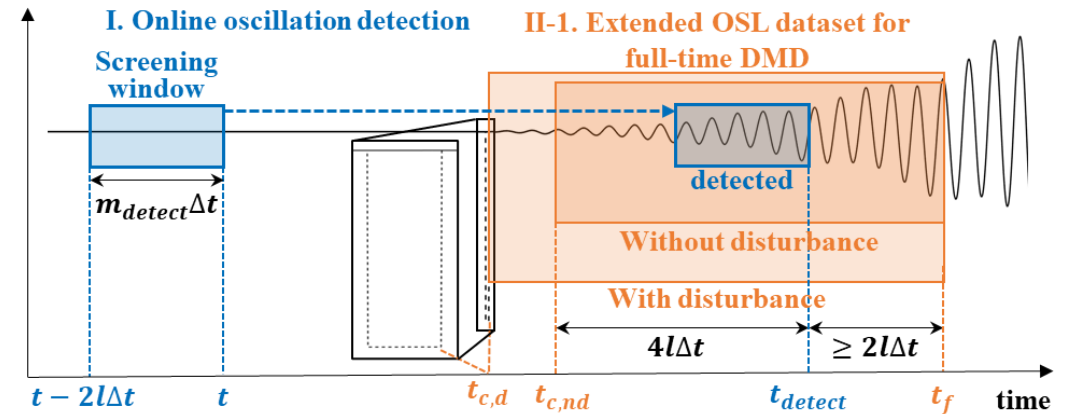


# Proposed Framework

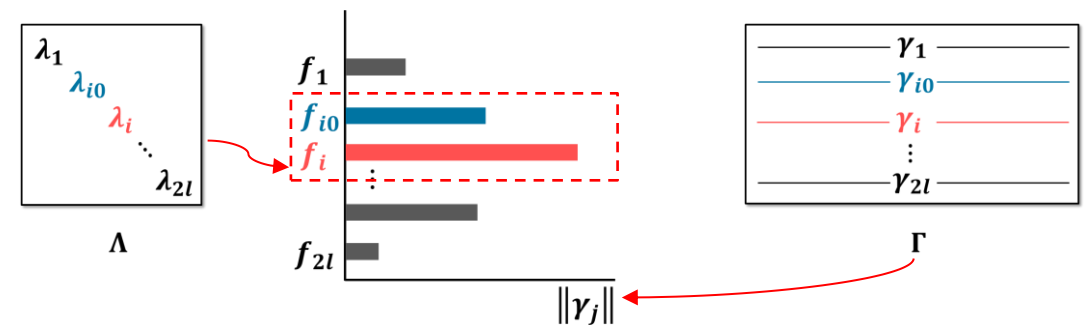
## ➤ Online Oscillation Detection

- Size of detection screening window
  - Length of the measurement matrix should be larger than rank  $r$  for the truncated SVD:  $m_{detect} \geq r = 2l$
  - The size needs to be as short as possible to guarantee online detection :  $m_{detect} = 2l$
  
- Oscillation detection
  - $\Lambda|_t$  and  $\Gamma|_t$  can be achieved by the DMD computation for the measurement from  $t - m_{detect}\Delta t$  to  $t$
  - Oscillation exists if the following condition is not met:
 
$$\forall i \in (1, 2, \dots, 2l) \text{ and } i \neq i_0, \|\gamma_{i_0}|_t\| > \alpha \cdot \|\gamma_i|_t\|$$

s. t.  $f_{i_0}^t = 0, \alpha = 3$



<Timeline of detection and source location>

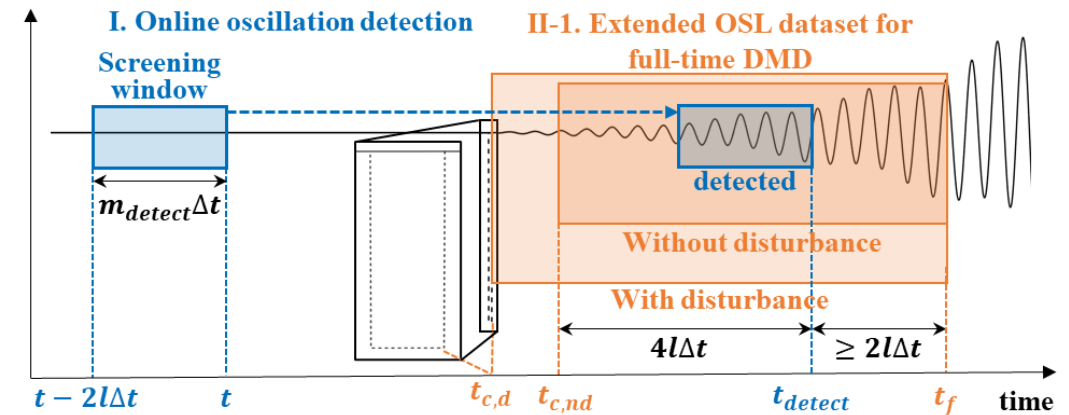


<Oscillation detection process using improved DMD>

# Proposed Framework

## ➤ Oscillation Source Location

- Size of location screening window
  - The screening window for OSL is extended for more detailed analysis
  - Minimum condition for  $t_f$ :  $t_f - t_{detect} \geq 2l\Delta t$
  - Critical time  $t_c$  is selected differently according to the existence of the disturbance<sup>1)</sup> in the measurements:  
With disturbance,  $t_{c,d}$  is equal to the fault clearing time. Without disturbance,  $t_{c,n} = t_{detect} - 4l\Delta t$



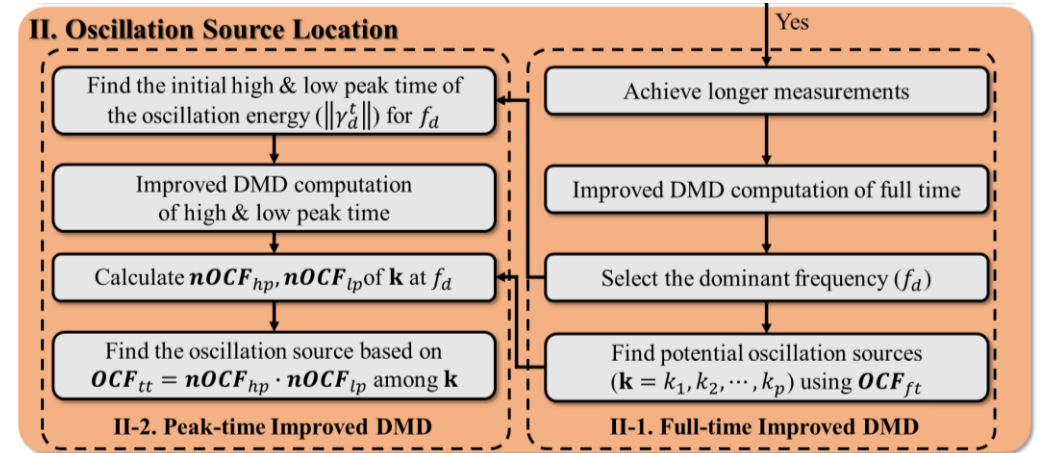
<Timeline of detection and source location>

<sup>1)</sup> Disturbance types of interest includes momentary sag, swell, and surge by short circuits, major equipment shutdown, etc., which mostly appear as impulse signals.

# Proposed Framework

## ➤ Oscillation Source Location

- Proposed source location framework
  - First stage: compute full-time improved DMD, specify potential oscillation sources
  - Second stage: oscillatory energy at the early stage is analyzed to identify the source
- First stage of OSL
  - With similar process to detection, find out dominant frequency  $f_d$
  - Select potential oscillation sources  $\mathbf{k} = [k_1, k_2, \dots, k_p]$  based on Oscillation Contribution Factor for full time ( $\mathbf{OCF}_{ft}$ )



<Process of OSL framework>

$$\mathbf{OCF}_{ft} = |\mathbf{MM}_v^{ft} \cdot \mathbf{MM}_\theta^{ft}|$$

$$\text{s. t. } \mathbf{MM}_v^{ft} = \frac{\text{Re}\{\phi_{d,v}\} \cdot \text{Im}\{\phi_{d,v}\}}{\sqrt{\text{Re}\{\phi_{d,v}\}^2 + \text{Im}\{\phi_{d,v}\}^2}},$$

$$\mathbf{MM}_\theta^{ft} = \frac{\text{Re}\{\phi_{d,\theta}\} \cdot \text{Im}\{\phi_{d,\theta}\}}{\sqrt{\text{Re}\{\phi_{d,\theta}\}^2 + \text{Im}\{\phi_{d,\theta}\}^2}}$$

# Proposed Framework

## ➤ Oscillation Source Location

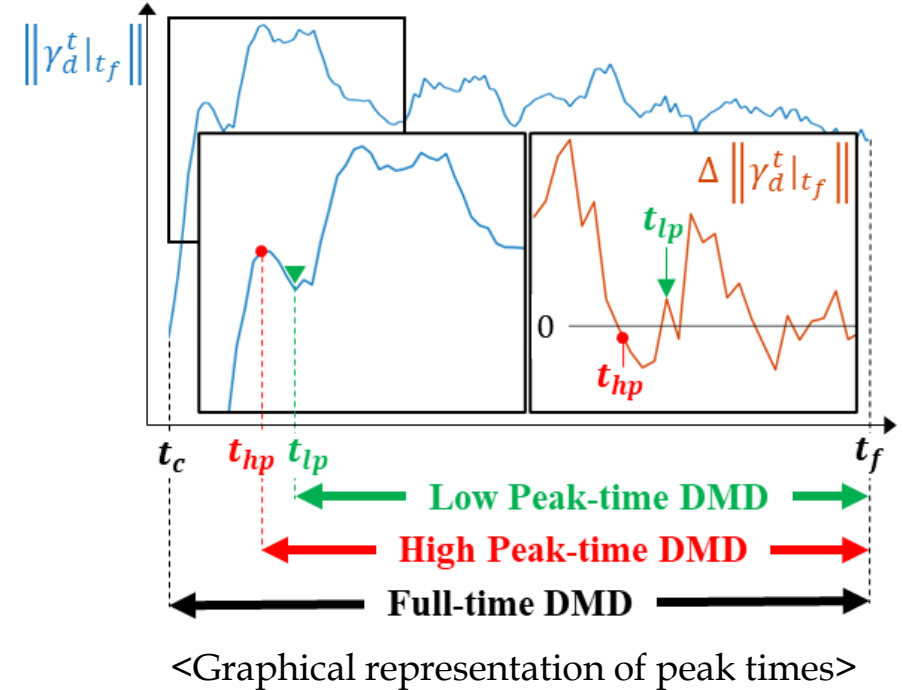
- Second stage of OSL
  - Early-stage oscillatory energy can be calculated by comparing the two peak-time DMD results subtracted by full-time DMD
  - High and low peak times are determined on the first swing of temporal oscillation energy,  $\|\gamma_d^t|_{t_f}\|$
  - The final oscillation contribution factor to identify the oscillation source is given by:

$$\mathbf{OCF}_{tt} = |\mathbf{nOCF}_{hp} \cdot \mathbf{nOCF}_{lp}|$$

$$\mathbf{nOCF}_{hp,lp} = [\mathbf{nOCF}_{k_1}^{hp,lp}, \mathbf{nOCF}_{k_2}^{hp,lp}, \dots, \mathbf{nOCF}_{k_p}^{hp,lp}],$$

$$\text{s. t. } \mathbf{nOCF}_k^{hp,lp} = \frac{\mathbf{OCF}_k^{hp,lp}}{\max(\mathbf{OCF}_{k \in k}^{hp,lp})},$$

$$\mathbf{OCF}_{hp,lp} = |(\mathbf{MM}_v^{ft} - \mathbf{MM}_v^{hp,lp}) \cdot (\mathbf{MM}_\theta^{ft} - \mathbf{MM}_\theta^{hp,lp})|$$



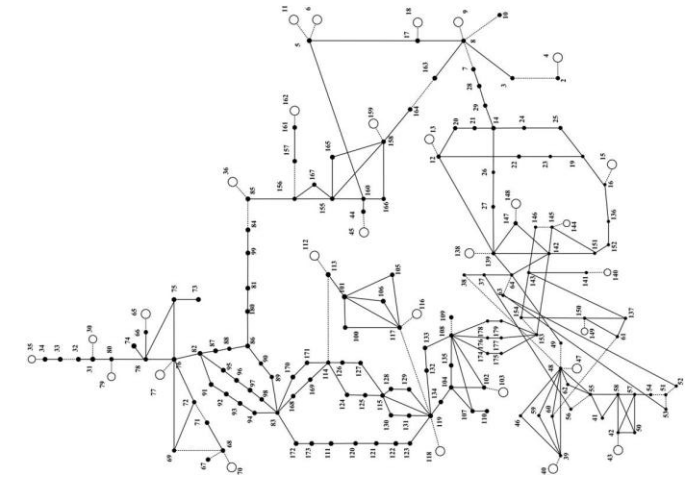
# Experimental Results on LFO

## ➤ Oscillation dataset

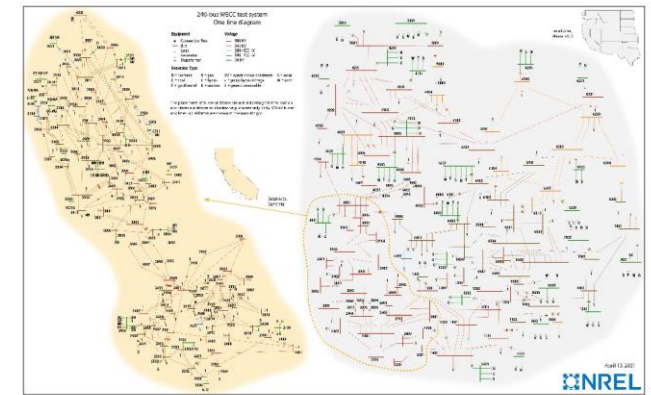
- Simulated cases (TSAT, WECC 179 bus system)
- 2021 NASPI contest cases (TSAT, WECC 240 bus system)
- Real oscillation cases (ISO-NE)
- Source: <https://web.eecs.utk.edu/~kaisun/Oscillation/>

## ➤ Overall Results

Category	Tested Cases	Oscillation Frequency	Oscillation Source	Estimated Frequency	Estimation Error	Estimated Source	Elapsed Location Time
Category 1	Sim 2F	0.86 Hz	Bus 79	0.8567 Hz	0.38%	Bus 79	0.41s
	Sim 6ND	1.408 Hz	Bus 45, 159	1.402 Hz	0.42%	Bus 45, 159	0.46s
	NASPI 7	0.379 Hz	Bus 2634	0.3751 Hz	1.03%	Bus 2634	0.57s
Category 2	Sim 3F	0.37 Hz	Bus 77	0.3680 Hz	0.54%	Bus 77	0.57s
	Sim 1ND	1.408 Hz	Bus 45	1.3991 Hz	0.63%	Bus 45	0.46s
	NASPI 2	1.19 Hz	Bus 2634	1.1885 Hz	0.12%	Bus 2634	0.52s
	NASPI 8	0.614 Hz	Bus 6333	0.6160 Hz	0.32%	Bus 6333	0.92s
Category 3	Sim 6F3	0.4 Hz	Bus 79	0.4025 Hz	0.63%	Bus 79	0.47s
	NASPI 12	0.37 Hz	Bus 6335	0.3739 Hz	1.05%	Bus 6335	0.63s
Category 4	NASPI 3	0.379 Hz	Bus 1131	0.3788 Hz	0.05%	Bus 1131 or 1032	0.86s
Category 5	NASPI 13	0.614 Hz	Bus 2619	0.6149 Hz	0.15%	Bus 2619	1.02s



(a)



(b)

<One-line diagram of (a) WECC 179 bus system and (b) WECC 240 bus system>

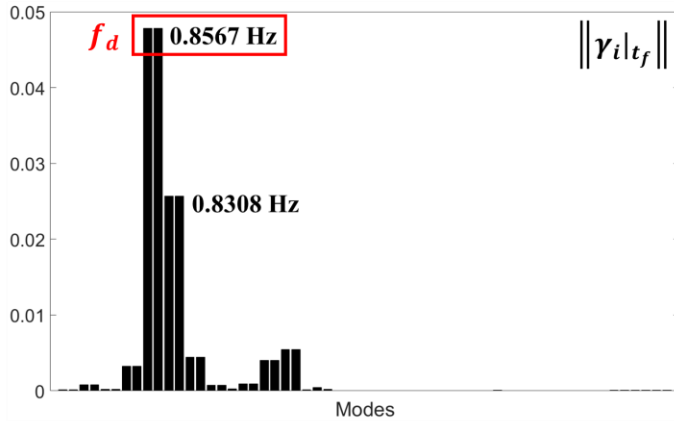
※ Sim: Simulated cases on WECC 179 bus system, NASPI: NASPI cases on WECC 240 bus system, F: Forced oscillation, N: Natural oscillation



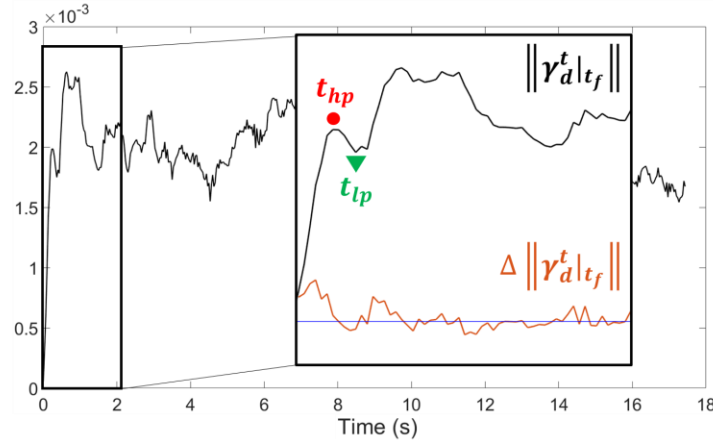
# Experimental Results on LFO

## ➤ OSL for Normal Oscillation Case

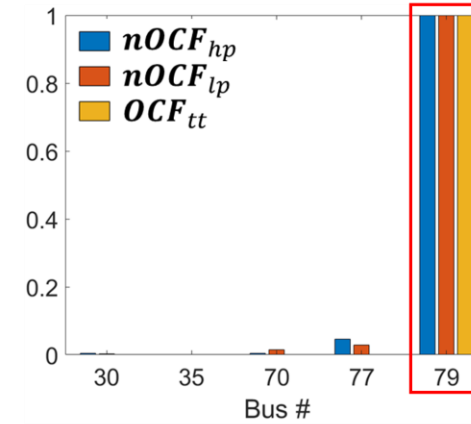
(a) Energy norm of each mode



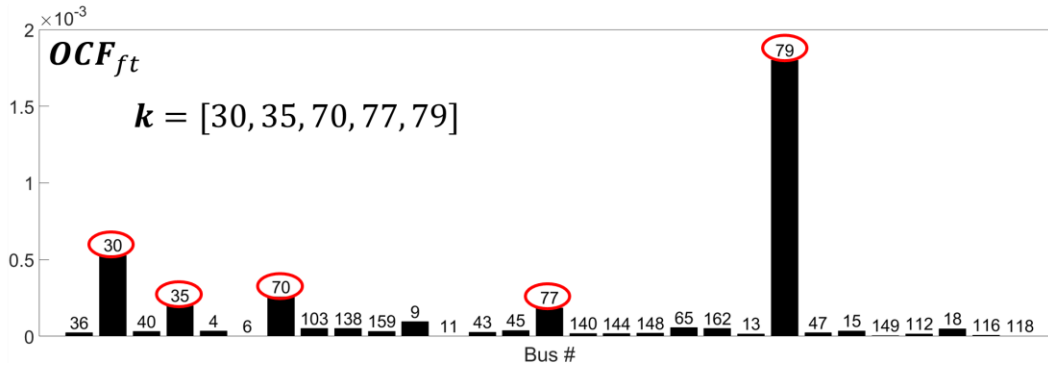
(b) Temporal energy distribution



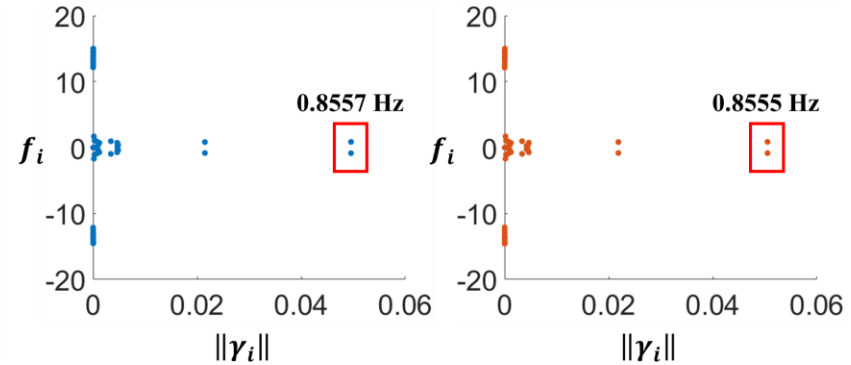
(e) Source Location



(c) Spatial energy distribution



(d) Determination of dominant frequency for peak DMDs

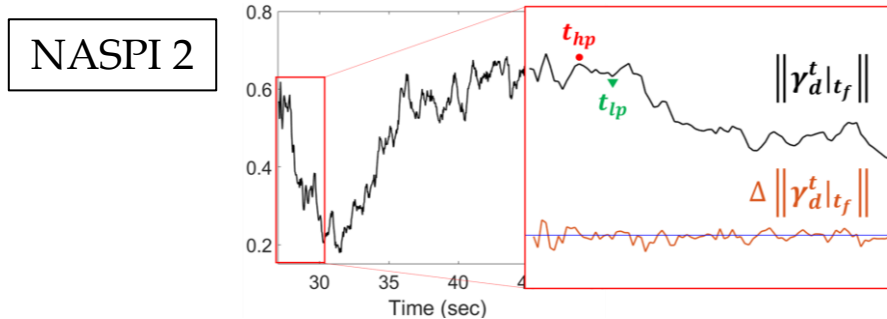


<Oscillation source location process for Sim 2F case>

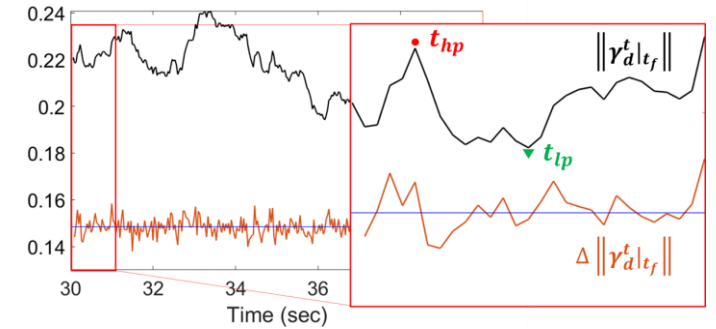
# Experimental Results on LFO

## ➤ OSL for Maximum Oscillation on Non-Source Bus

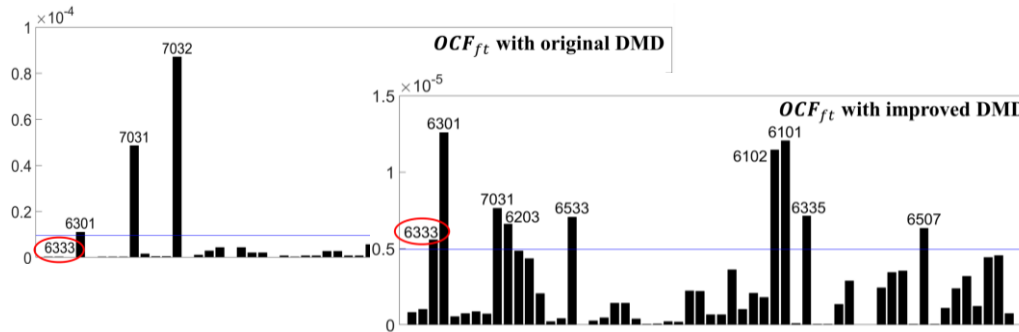
(a) Choice of peak times



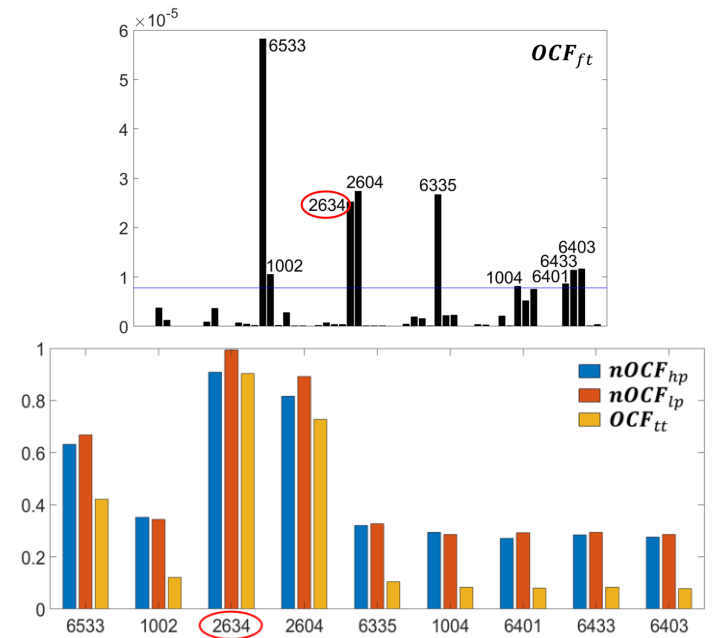
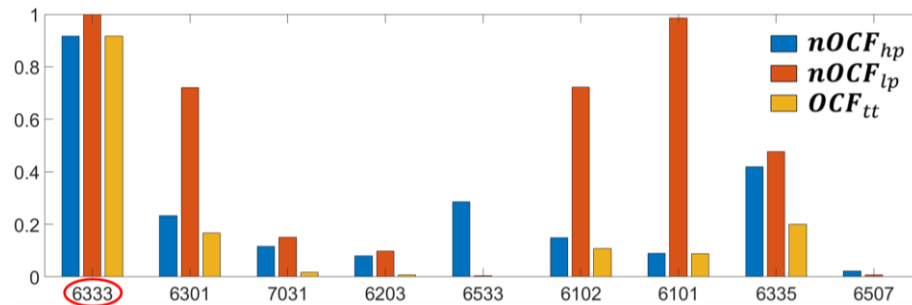
NASPI 8



(b)  $OCF_{ft}$

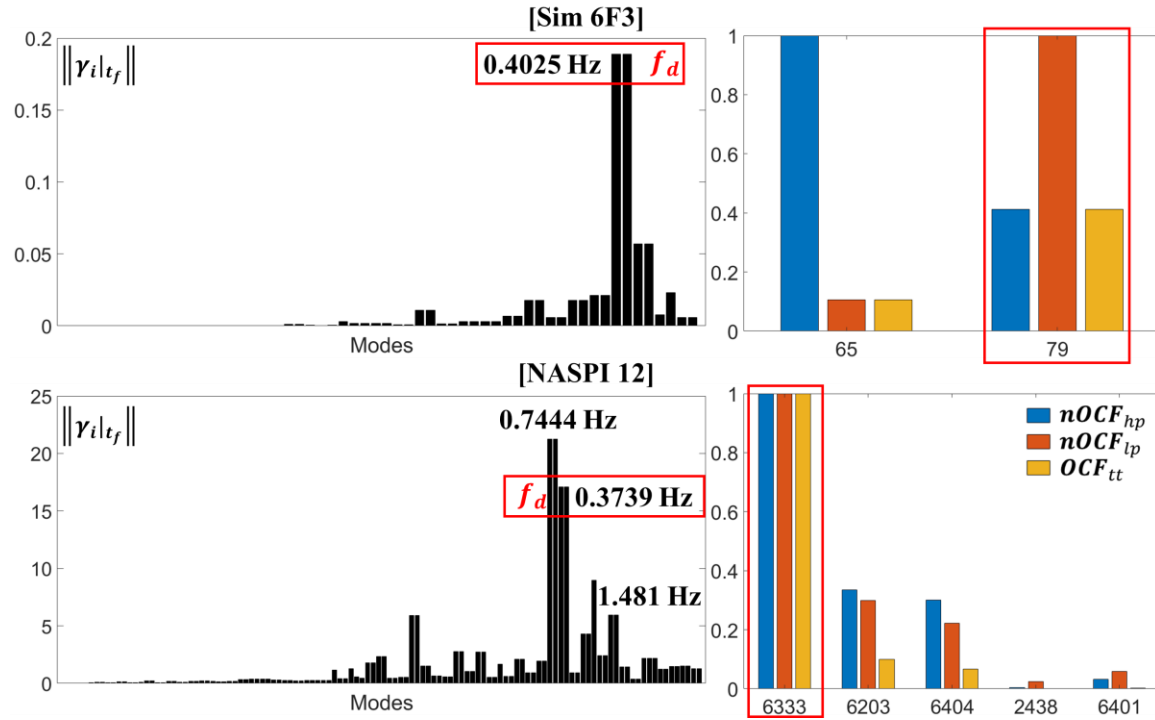


(c)  $OCF_{tt}$

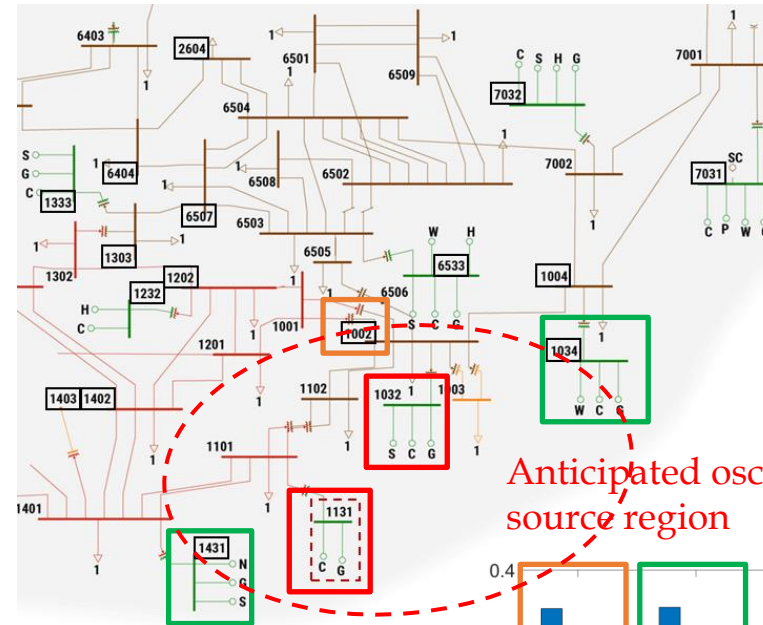


# Experimental Results on LFO

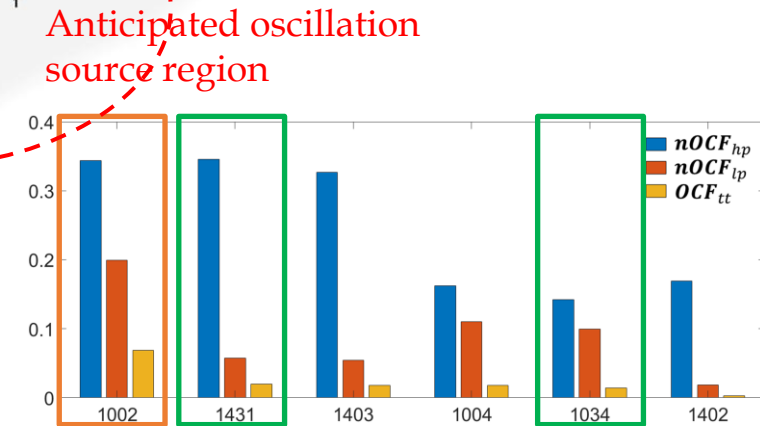
## ➤ OSL for Rectangular Signal Injection & Unmonitored Cases



<Schematic diagram of the simulation results for the third category cases>

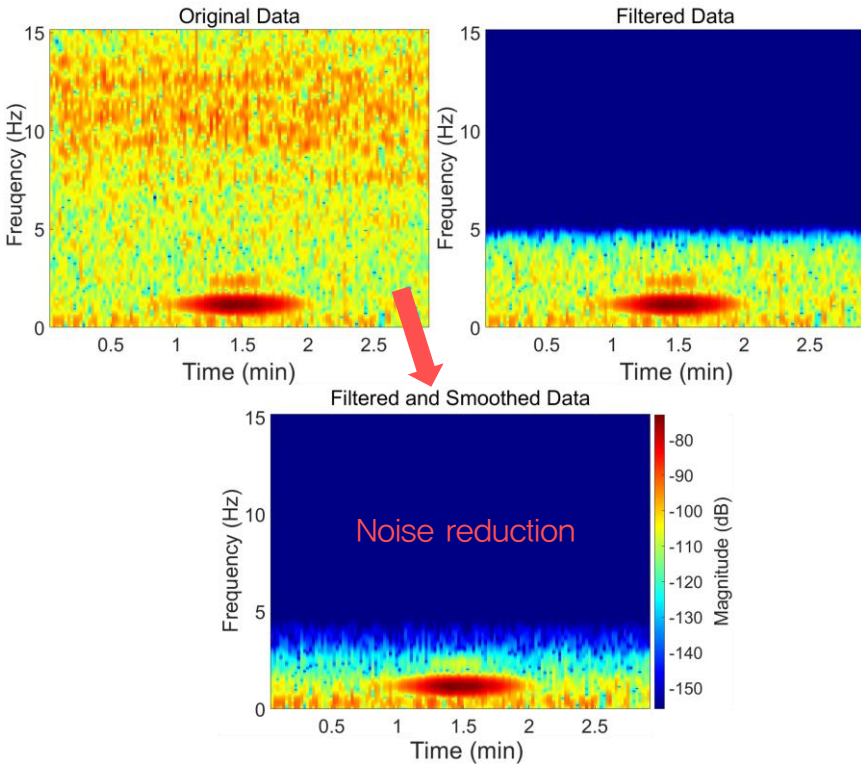


<Schematic diagram of the reduced WECC 240 bus system and values for NASPI 3 case>

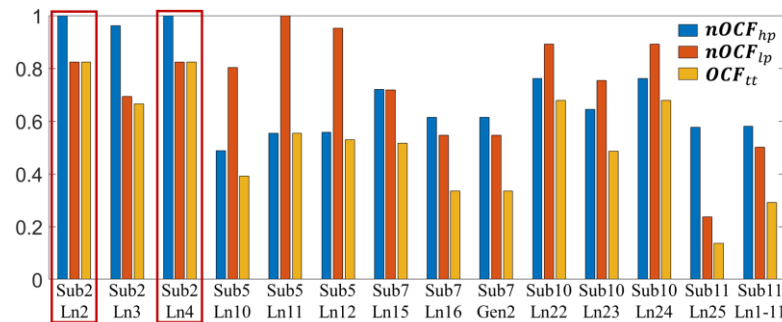
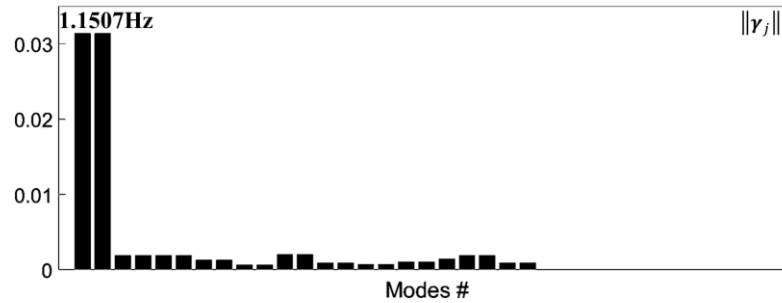


# Experimental Results on LFO

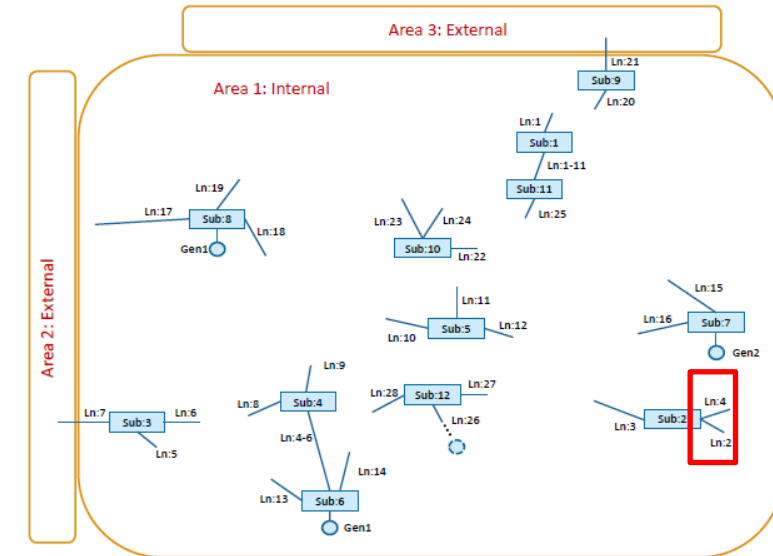
## ➤ OSL for Real Oscillation Case



<Power spectra for original, filtered, and filtered & smoothed data>



<Schematic diagrams of the dominant frequency and  $OCF_{tt}$  values>

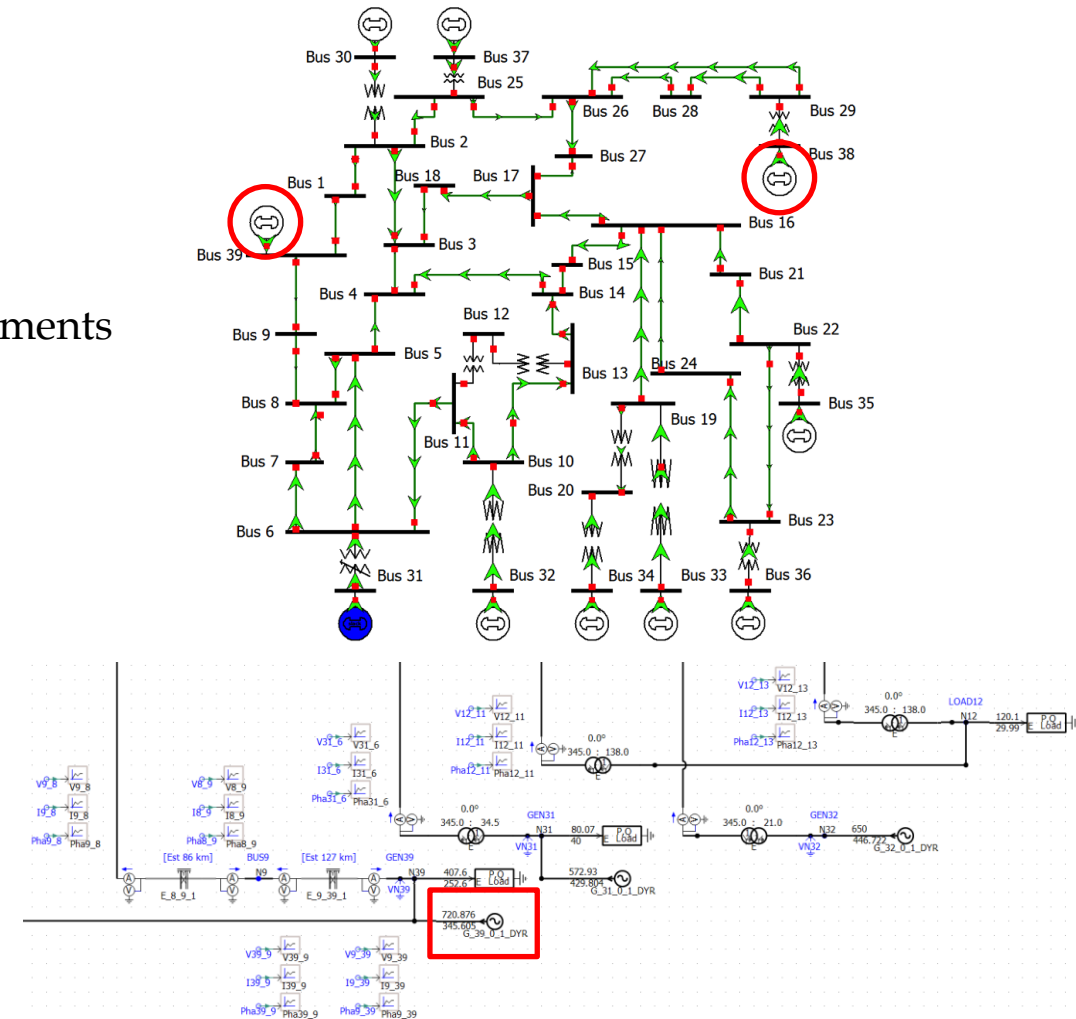
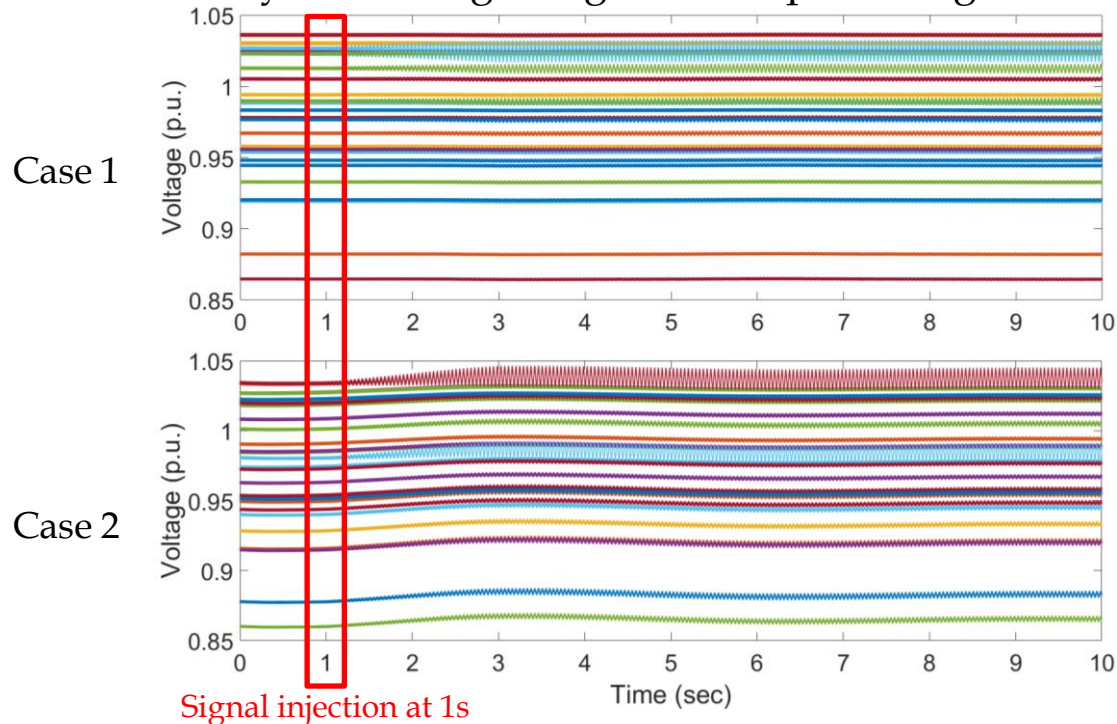


<ISO-NE map>

# Experimental Results on SSO

## ➤ Oscillation dataset

- Simulated cases (PSCAD, IEEE 39 bus system)
  - Case 1: 23 Hz forcing signal injection in Bus 38
  - Case 2: 23 Hz forcing signal injection in Bus 39
  - Assume only line voltage magnitude & phase angle measurements



# Experimental Results on SSO

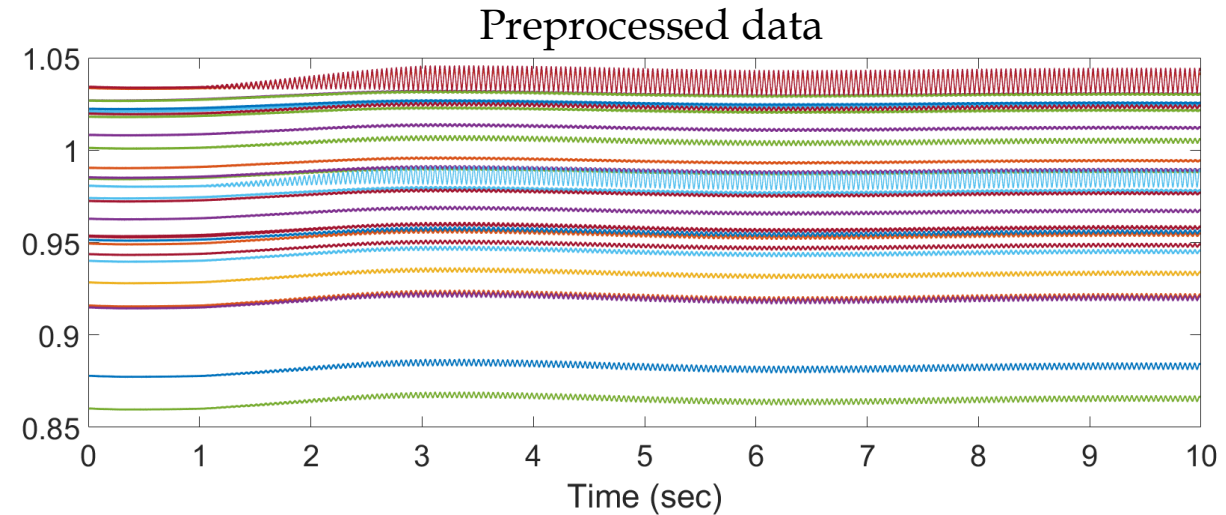
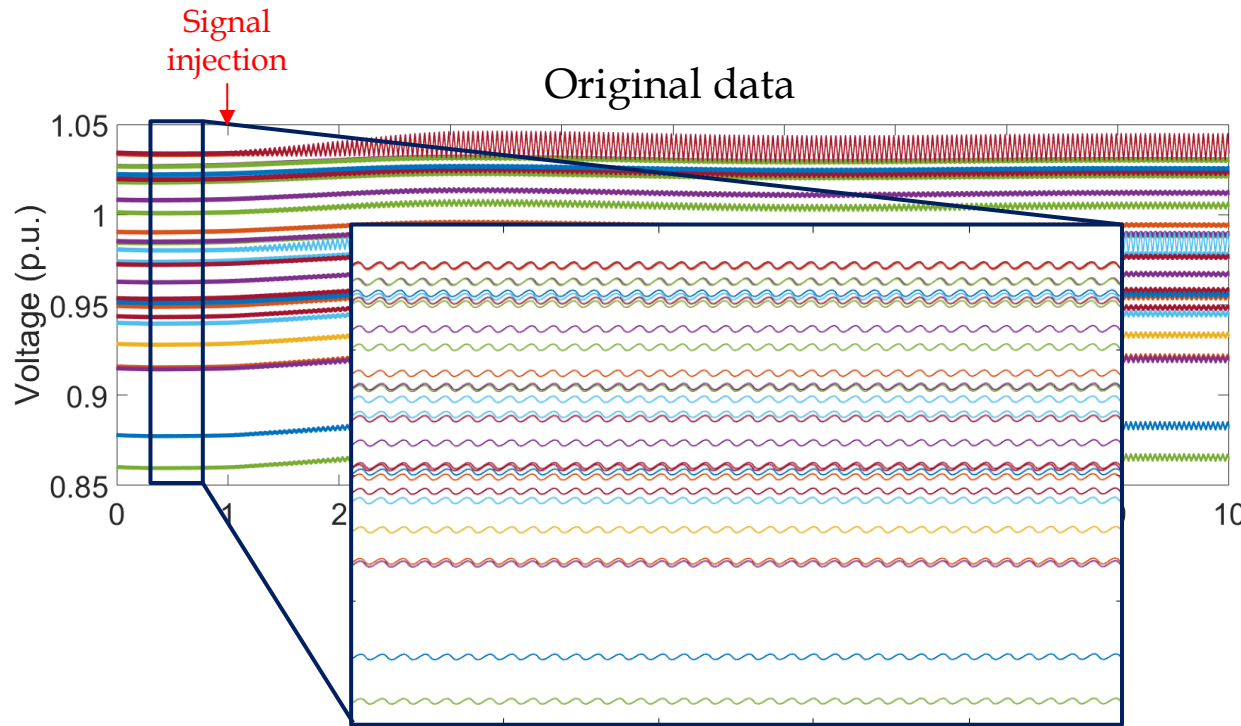
## ➤ Differences from LFO Case

### ■ Problem

- Difference with LFO in interested frequency and measuring frequency ( $F_s = 10000$  Hz)
- Ripples observed in all time span

### ■ Strategy

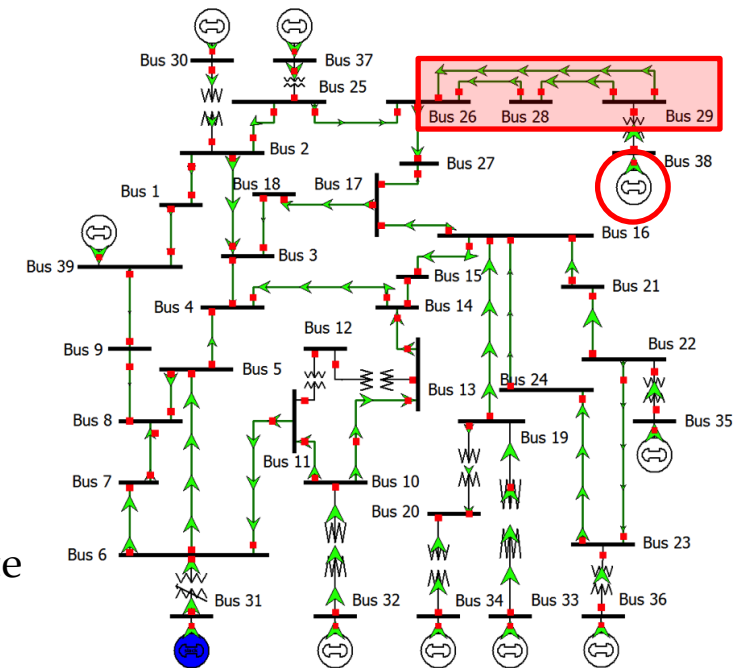
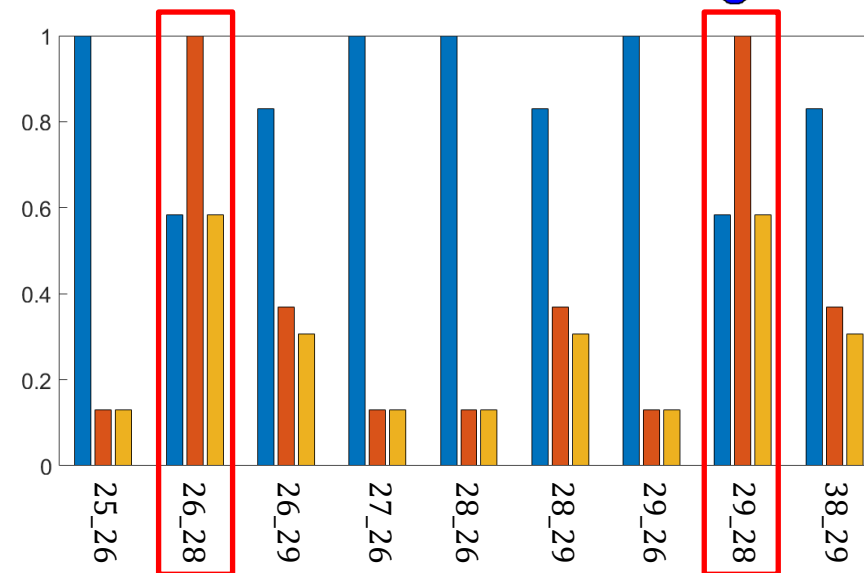
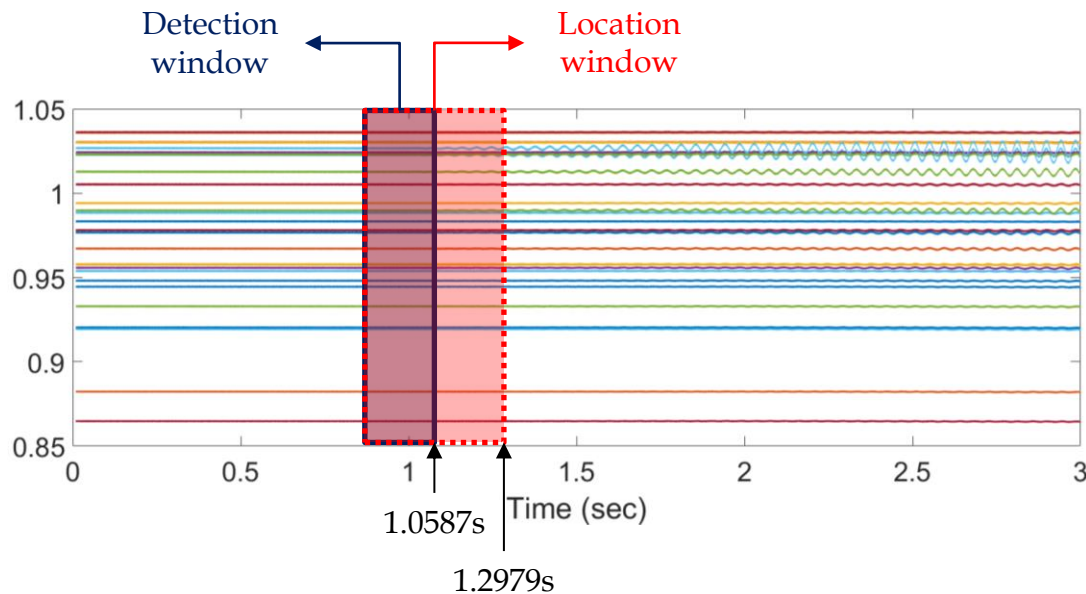
- Preprocessing is required for simulation data: 60 Hz LPF & Smoothing
- Extension of window size:  $m_{detect} = 2l \rightarrow 20l$  (1.5s for detection, 3s for OSL in SSO case)



# Experimental Results on SSO

## ➤ OSL Results for SSO Case 1 (Bus 38 Injection)

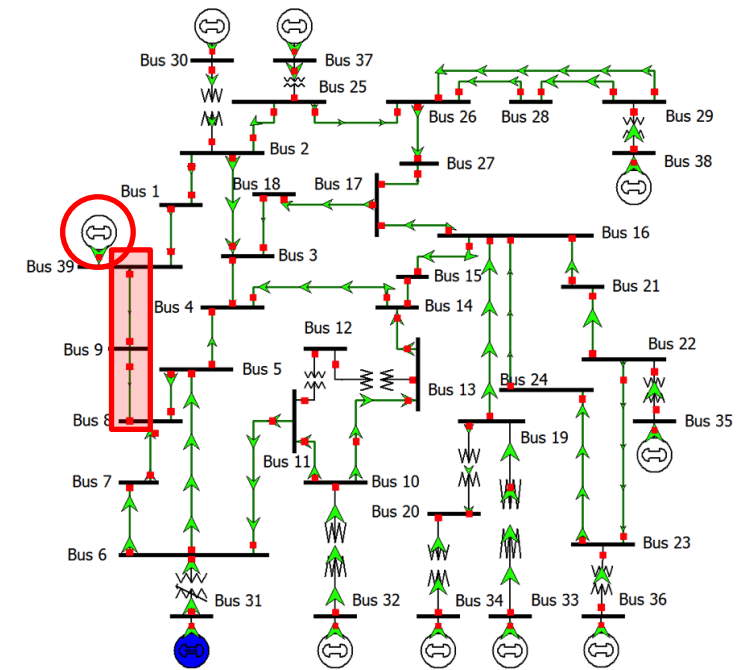
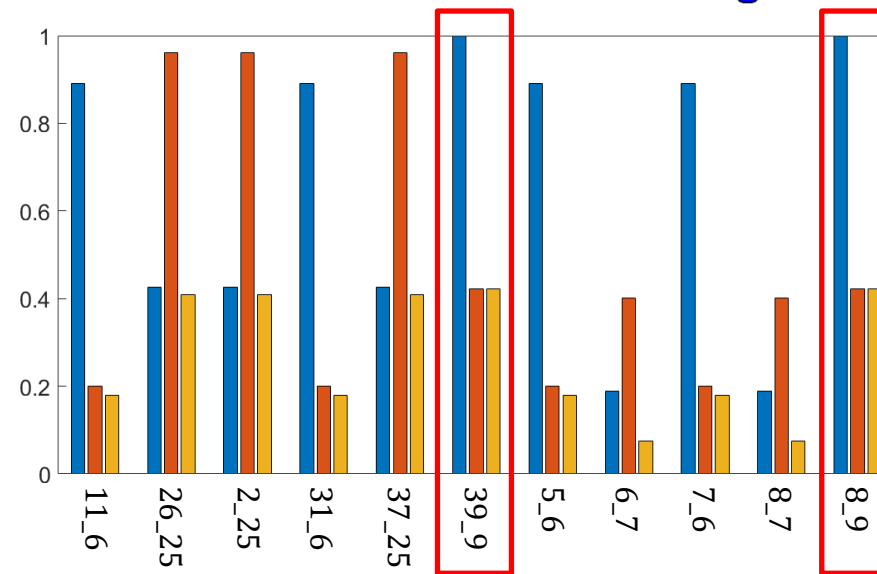
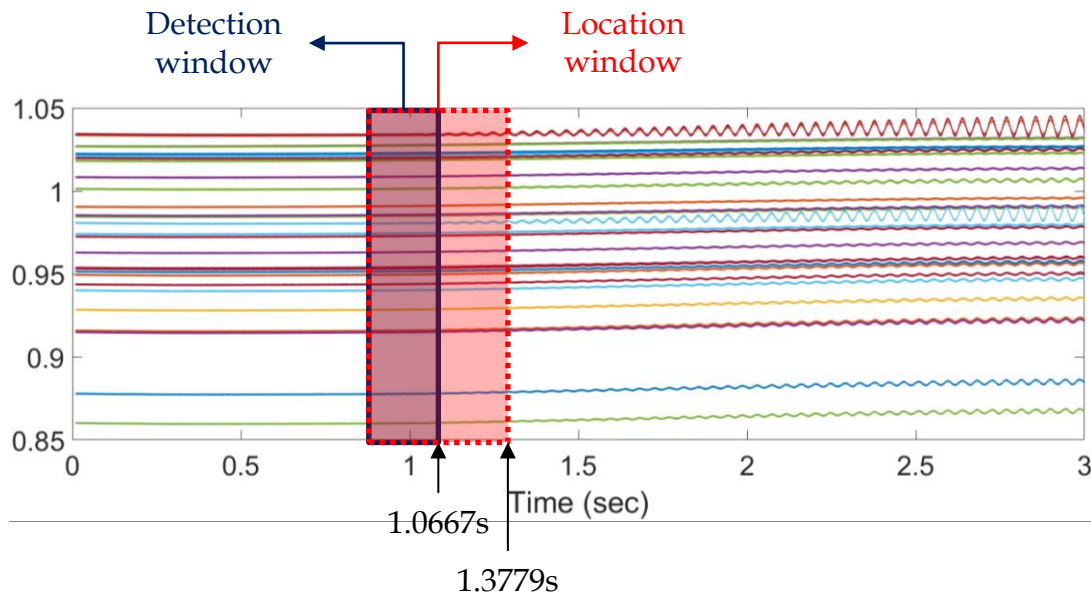
- Oscillation detection
  - Detected at 1.0587s with 24.75 Hz estimated frequency
- Source Location
  - Calculation starts at 1.2979s with computation time 1.9s
  - Largest  $OCF_{tt}$  values in lines 26\_28 and 28\_29 → Gen 38 as the oscillation source



# Experimental Results on SSO

## ➤ OSL Results for SSO Case 2 (Bus 39 Injection)

- Oscillation detection
  - Detected at 1.0667s with 24.00 Hz estimated frequency
- Source Location
  - Calculation starts at 1.3779s with computation time 3.3s
  - Largest  $OCF_{tt}$  values in lines 39\_9 and 8\_9 → Gen 39 as the oscillation source





# Conclusion

- OSL Framework with Improved DMD and Two-Tier Structure
  - **Improved DMD** including the initial state of the measurements as an additional input to enhance the clarity of the oscillation analysis
  - **Online oscillation detection** and followed oscillation **source location** framework
  - **Two-tier structure** with full-time and peak-time DMDs for enhancing accuracy of the source location on various oscillation categories
  - Validation of the proposed method on **forced & natural LFO** and **forced SSO** cases