How to Study and Distinguish Forced Oscillations

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Natural and Forced Oscillations

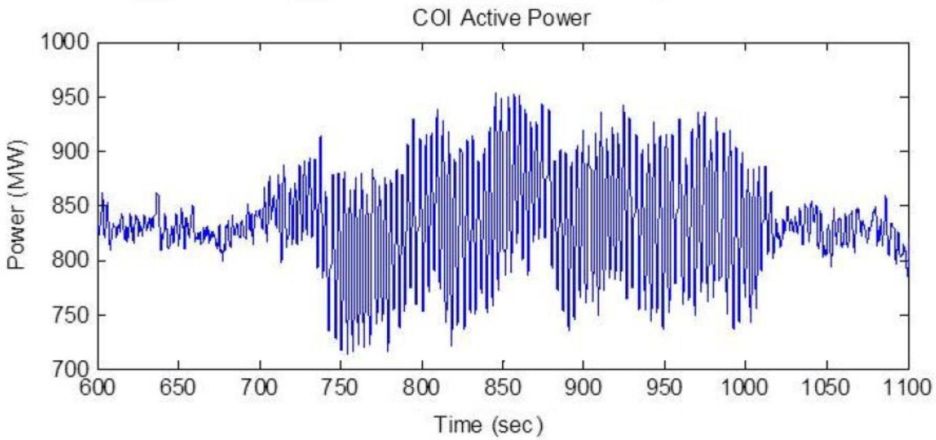
- Two types of oscillations are widely observed
- Natural/Free oscillation - Oscillations due to undamped system modes
- Forced oscillation - Oscillations from periodic sources external to the system

$$\dot{x} = Ax \qquad x(t) = \sum_{i=1}^{n} \varphi_i c_i e^{\lambda_i t}$$ Natural

$$\dot{x} = Ax + f(t)$$ $f(t)$ is periodic — Forced

Oscillation Type Distinguishing

- Why should we distinguish oscillation type?
  - Different control measures for different oscillation types
  - Natural oscillation: Increase the damping ratio of the critical mode and the oscillation will decay
  - Forced oscillation: Remove the external disturbance
- Why is this problem difficult?
  - The approach should be measurement-based, or it is not online applicable
  - Both oscillation types show sustained oscillation with constant amplitude in steady state
  - Actual oscillation waveforms are complicated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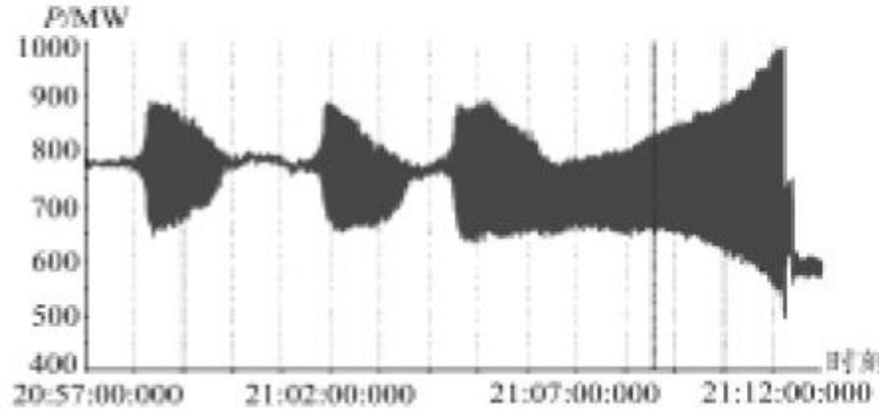
Power (MW)
1000
950
900
850
800
750
700
600
650
700
750
800
850
900
950
1000
1050
1100
Time (sec)


P/MW
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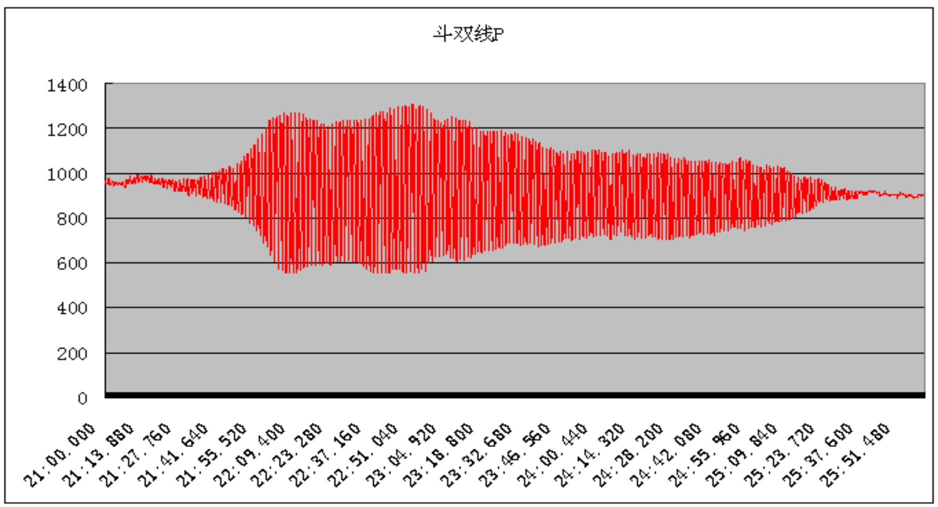
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23:32.680
23:46.560
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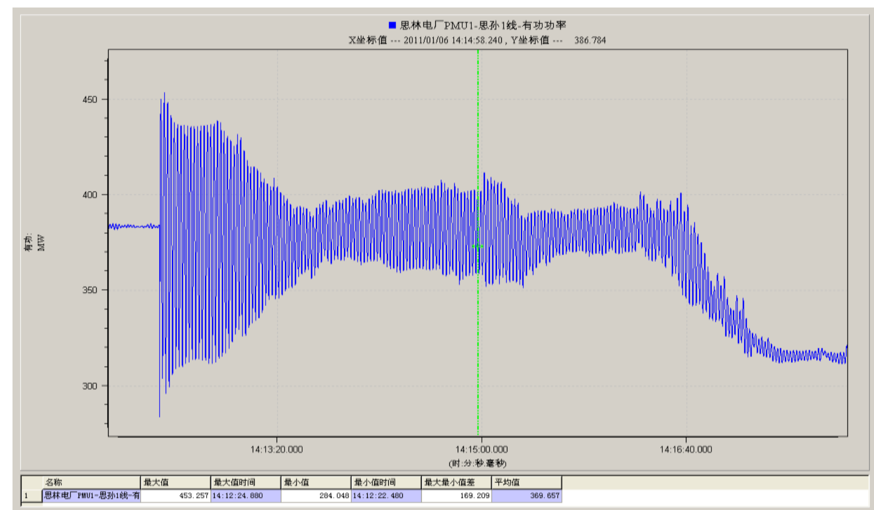
思林电厂PMU1-思孙1线-有功功率
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名称
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369.657

How to Study Forced Oscillation

- Natural oscillation
  - Determined by system features
  - Eigenvalue analysis or modal analysis
  - Time-domain simulation
- Forced oscillation
  - Determined by both external disturbances and system features
  - Time-domain simulation method is applicable, but not capable of analytic and quantitative analysis
  - Extended modal analysis

Extended Modal Analysis

Linearized system $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$

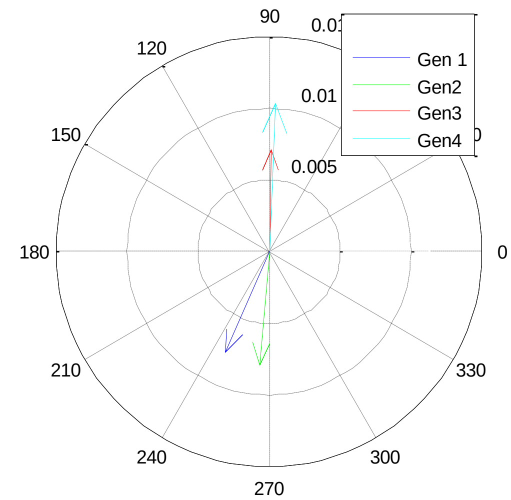
Modal transformation $\boldsymbol{x} = \boldsymbol{\Phi}\boldsymbol{z}$

Uncoupled system $\dot{\boldsymbol{z}} = \boldsymbol{\Lambda}\boldsymbol{z} + \boldsymbol{\Phi}^{-1}\boldsymbol{B}\boldsymbol{u}$

Natural oscillation

$\boldsymbol{u} = 0$

$$\begin{cases} z_r = z_{r0}e^{\lambda_r t} = \boldsymbol{\Psi}_r^T \boldsymbol{x}_0 e^{\lambda_r t} \\ z_r^* = z_{r0}^* e^{\lambda_r^* t} = \boldsymbol{\Psi}_r^{*T} \boldsymbol{x}_0 e^{\lambda_r^* t} \end{cases}$$



$$x_i(t) = \sum_{r=1}^{n-1} |\phi_{ir}||z_{r0}| e^{-\alpha_r t}\left[e^{\mathrm{j}(\omega_{dr}t + \gamma_{ir} + \theta_r)} + e^{-\mathrm{j}(\omega_{dr}t + \gamma_{ir} + \theta_r)}\right]$$

$$= 2\sum_{r=1}^{n-1} |\phi_{ir}||z_{r0}| e^{-\alpha_r t} \cos(\omega_{dr}t + \gamma_{ir} + \theta_r)$$

$|\phi_{ir}|\angle\gamma_{ir}$

Mode shape

Extended Modal Analysis

Forced oscillation

$$\begin{bmatrix} \dot{\boldsymbol{z}} \\ \dot{\boldsymbol{z}}^* \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Lambda} & \\ & \boldsymbol{\Lambda}^* \end{bmatrix} \begin{bmatrix} \boldsymbol{z} \\ \boldsymbol{z}^* \end{bmatrix} + \boldsymbol{\Phi}^{-1}\boldsymbol{B}\boldsymbol{u}$$

Assumption: only one sinusoidal disturbance $\boldsymbol{Bu} = \Delta \boldsymbol{P}_T = \Delta \boldsymbol{P}_{Tm} \sin \omega t$

Focus on steady state response

$$\Delta \boldsymbol{P}_T = \Delta \tilde{\boldsymbol{P}}_{Tm} e^{\mathrm{j}\omega t}$$

Complex phasor representation

$$\begin{cases} z_r = \dfrac{\tilde{\boldsymbol{\Psi}}_r^T \Delta \tilde{\boldsymbol{P}}_{Tm}}{\mathrm{j}\omega - \tilde{\lambda}_r} e^{\mathrm{j}\omega t} \\ z_r^* = \dfrac{\tilde{\boldsymbol{\Psi}}_r^{*T} \Delta \tilde{\boldsymbol{P}}_{Tm}}{\mathrm{j}\omega - \tilde{\lambda}_r^*} e^{\mathrm{j}\omega t} \end{cases}$$

$$\begin{aligned} \boldsymbol{x}(t) &= \sum_{r=1}^{n} \tilde{\boldsymbol{\Phi}}_r z_r + \sum_{r=1}^{n} \tilde{\boldsymbol{\Phi}}_r^* z_r^* \\ &= \sum_{r=1}^{n} \left[\frac{\mathrm{j}\omega(\tilde{\boldsymbol{\Phi}}_r \tilde{\boldsymbol{\Psi}}_r^T + \tilde{\boldsymbol{\Phi}}_r^* \tilde{\boldsymbol{\Psi}}_r^{*T}) - (\tilde{\boldsymbol{\Phi}}_r \tilde{\boldsymbol{\Psi}}_r^T \tilde{\lambda}_r^* + \tilde{\boldsymbol{\Phi}}_r^* \tilde{\boldsymbol{\Psi}}_r^{*T} \tilde{\lambda}_r)}{(\mathrm{j}\omega - \tilde{\lambda}_r)(\mathrm{j}\omega - \tilde{\lambda}_r^*)} \Delta \tilde{\boldsymbol{P}}_{Tm} e^{\mathrm{j}\omega t} \right] \end{aligned}$$

Steady State Response of Forced Oscillation

The *r*th mode in the *i*th state variable $x_{i,r}(t) = B_{i,r}\sin(\omega t - \varphi_{i,r})$

$$B_{i,r} = \sqrt{\frac{(a/\omega_{nr}^2)^2 + (v_r b/\omega_{nr})^2}{(1-v_r^2)^2 + (2\zeta_r v_r)^2}}\,|\tilde{p}_l| \qquad \varphi_{i,r} = \arctan\frac{2\zeta_r v_r a - (1-v_r^2)v_r b\omega_{nr}}{a(1-v_r^2) + 2\zeta_r v_r^2 b\omega_{nr}}$$

$$a = -(\tilde{\lambda}_r^* \tilde{\phi}_{ir}\tilde{\psi}_{lr} + \tilde{\lambda}_r \tilde{\phi}_{ir}^* \tilde{\psi}_{lr}^*)$$

$$b = \tilde{\phi}_{ir}\tilde{\psi}_{lr} + \tilde{\phi}_{ir}^*\tilde{\psi}_{lr}^*$$

ζ_r Damping ratio

$\omega_{nr} = |\lambda_r|$ Natural frequency

$v_r = \omega / \omega_{nr}$ Frequency ratio

Frequency ratio inducing the largest amplitude

$$v_r^2 = \sqrt{1 + 2(1-2\zeta_r^2)\left(\frac{a}{b\omega_{nr}}\right)^2 + \left(\frac{a}{b\omega_{nr}}\right)^4} - \left(\frac{a}{b\omega_{nr}}\right)^2$$

$\zeta_r \approx 0$ Poorly damped

$v_r \approx 1$ Resonance

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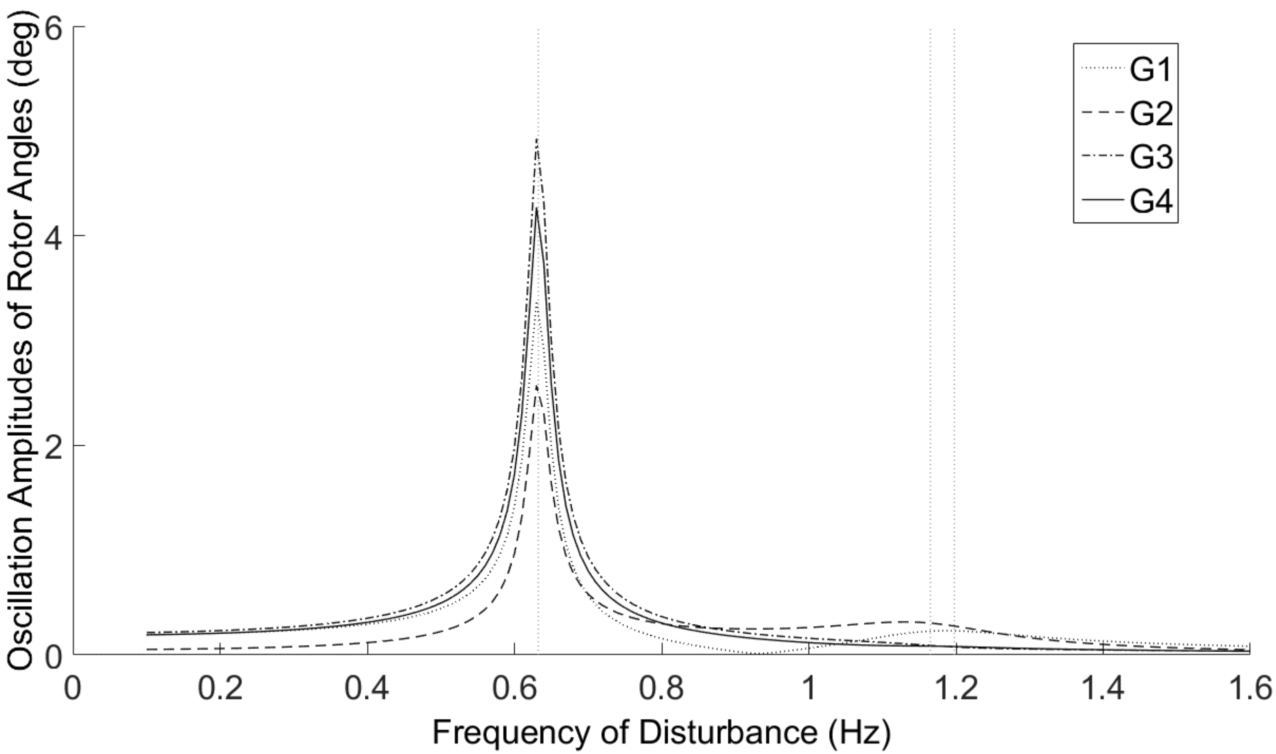
Resonance - When the frequency of the external disturbance is close to the frequency of a poorly damped mode, the system oscillates with a large amplitude, which is often larger than that of the disturbance



Easy to Understand

Steady State Response at Resonance

When resonance with a poorly damped mode, the mode dominates the response

$$x_i(t) \approx \frac{\left|\tilde{\phi}_{ir}\right|\left|\tilde{\psi}_{lr}\right|\left|\tilde{p}_l\right|}{\zeta_r \omega_{nr}} \sin\left(\omega_{nr} t + \gamma_{ir} + \sigma_{lr}\right)$$

ζ_r Damping ratio

$\tilde{\phi}_{ir} = \left|\tilde{\phi}_{ir}\right| e^{j\gamma_{ir}}$ Right eigenvector

$\tilde{\psi}_{lr} = \left|\tilde{\psi}_{lr}\right| e^{j\sigma_{lr}}$ Left eigenvector

i State variable

l Location of disturbance

Oscillation Amplitude

$$x_i(t) \approx \frac{\left|\tilde{\phi}_{ir}\right|\left|\tilde{\psi}_{lr}\right|\left|\tilde{p}_l\right|}{\zeta_r \omega_{nr}} \sin\left(\omega_{nr} t + \gamma_{ir} + \sigma_{lr}\right)$$

Oscillation amplitude is affected by

Amplitude of disturbance	$\left|\tilde{p}_l\right|$
Location of disturbance	$\left|\tilde{\psi}_{lr}\right|$
Damping ratio of mode	ζ_r

- Larger amplitude of disturbance induces larger amplitude of oscillation
- Larger $\left|\tilde{\psi}_{lr}\right|$, which means the location of disturbance has stronger controllability on the mode, induces larger amplitude of oscillation
- Smaller damping ratio induces larger amplitude of oscillation

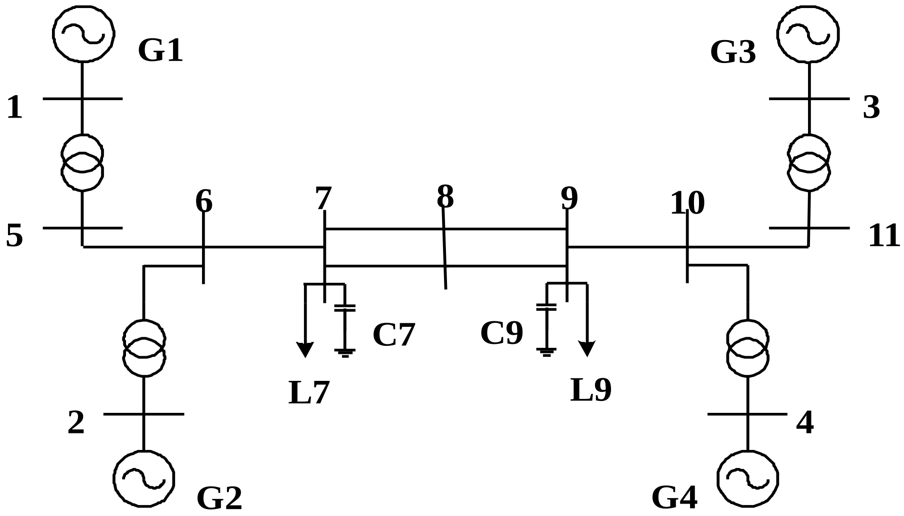
Mode Shape

$$x_i(t) \approx \frac{\left|\tilde{\phi}_{ir}\right|\left|\tilde{\psi}_{lr}\right|\left|\tilde{p}_l\right|}{\zeta_r \omega_{nr}} \sin\left(\omega_{nr} t + \gamma_{ir} + \sigma_{lr}\right)$$

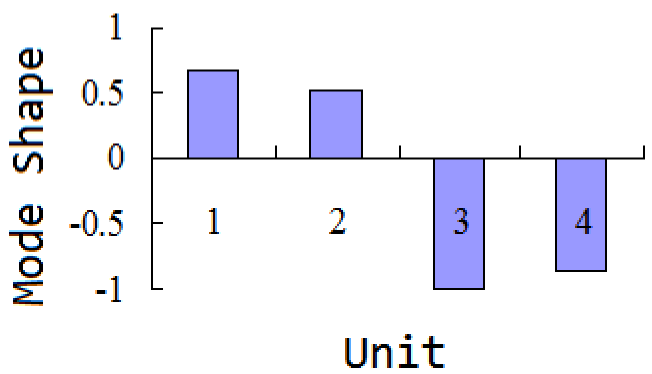
- Relative amplitude and phase of different state variables is also determined by the right eigenvector $\left|\tilde{\phi}_{ir}\right| \angle \gamma_{ir}$
- Same as natural oscillation
- Not affected by the location of disturbance

At resonance, the mode shape of forced oscillation converges to system mode shape, which brings difficulties in oscillation type distinguishing and source location

Simulations

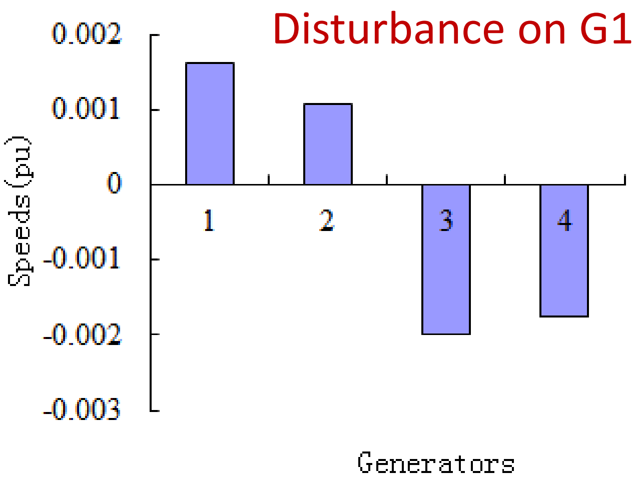


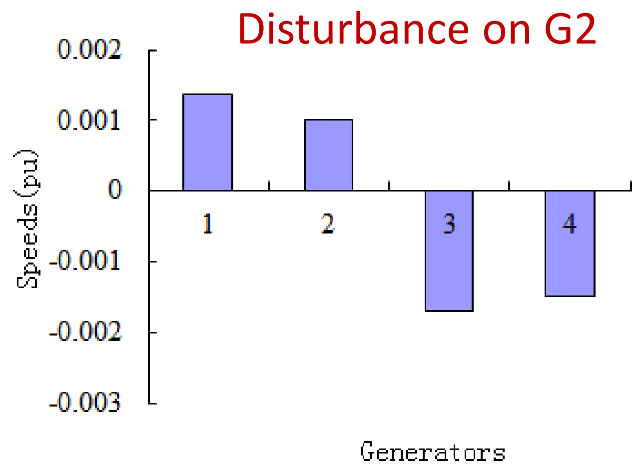
Inter-area mode
Right eigenvector

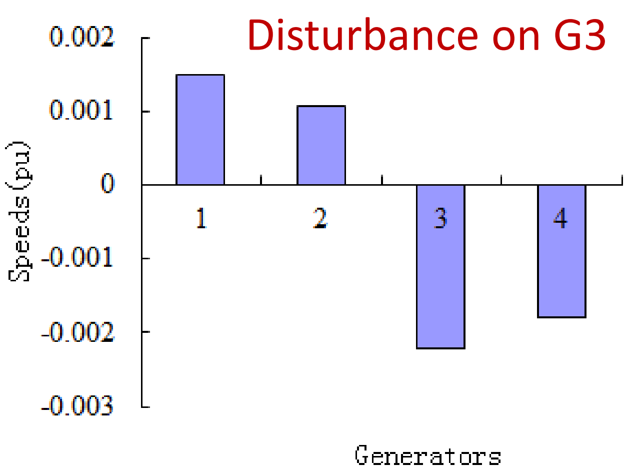


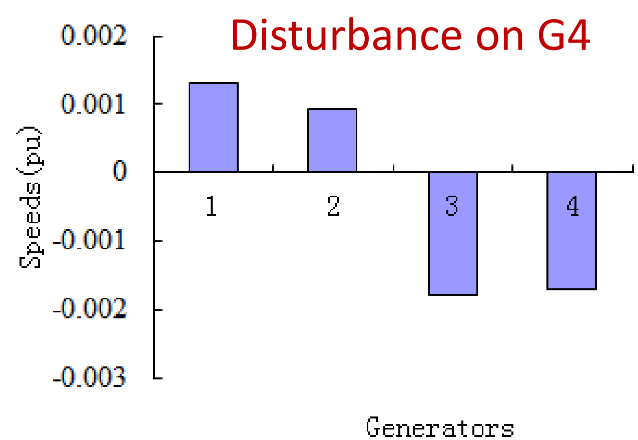
$$\zeta = 2.15\%$$

Simulations









How to Distinguish Forced Oscillation

- Distinguishing natural and forced oscillations based on system measurements is critical for control measure decisions
- In steady state, both natural and forced oscillations show sustained oscillations with nearly constant amplitude
  - A damped oscillation is obviously natural oscillation – Not Critical
  - Oscillation with negative damping will converge to constant-amplitude oscillation due to nonlinearities such as saturations and limits in actual systems
- Mode shapes are similar when resonance
- Not easy to distinguish

Fundamental Differences

- Influence factor
  - Natural oscillation - system features
  - Forced oscillation - system features + external disturbances
  - The steady state waveform of natural oscillation is mainly sinusoidal
  - If the external disturbance is non-sinusoid, the forced oscillation waveform will also deviate a lot from sinusoid
  - An obvious non-sinusoidal waveform is a sufficient but unnecessary indicator of forced oscillation
- Intrinsic system damping
  - Natural oscillation - zero or negative
  - Forced oscillation – positive
  - How to obtain the intrinsic damping from its outward performance?

1. Harmonic Content of Steady State Waveform

- Obvious non-sinusoidal waveform in steady state is a sufficient but unnecessary condition of forced oscillation
- Harmonic content
  - Harmonic content higher than a given threshold is an indicator of non-sinusoidal waveform, and forced oscillation

$$h = \frac{m_2 + m_3}{m_1} > h_m$$

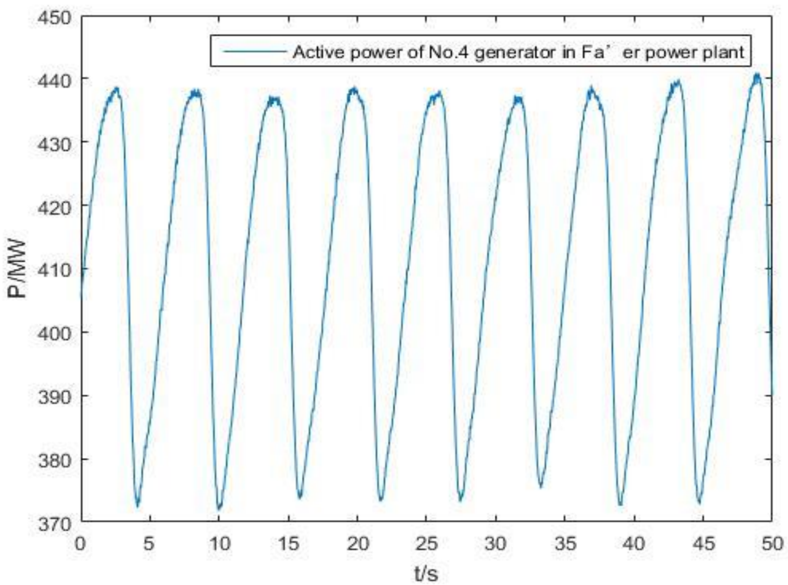
m_1 Amplitude of fundamental

m_i Amplitude of ith harmonic

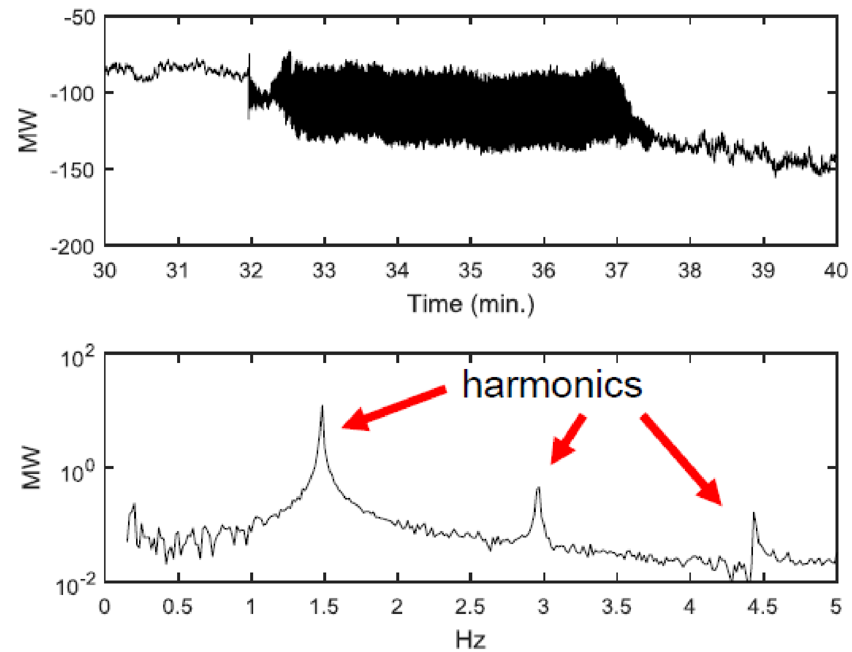
  - A recommended value of the threshold is 0.11, which is the harmonic content index for a triangle wave

Simple but Practical

Examples



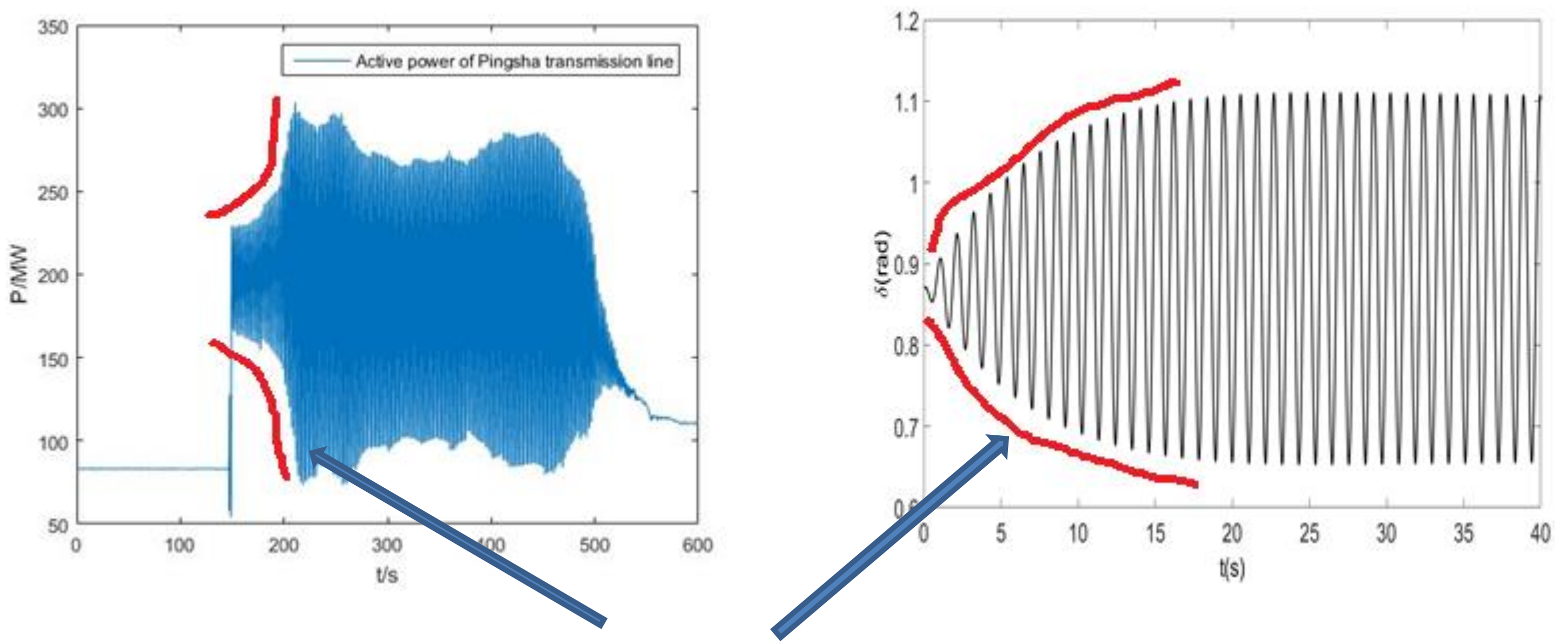
h=0.34>0.11
Forced
oscillation



WECC FO, 2015
From Dan

2. Features of Start Up Waveform

- Different intrinsic system dampings result in different features of start up waveform



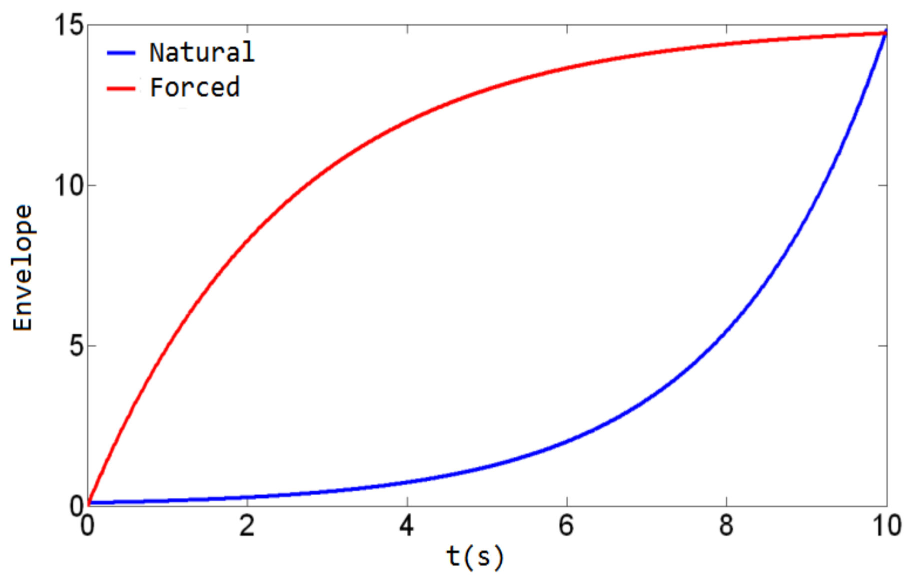
Start up: the stage when the amplitude increases

Features of Start Up Waveform

- The envelope of start up waveform $Ae^{\sigma t} + B$

Natural oscillation $A > 0, B = 0, \boxed{\sigma > 0}$

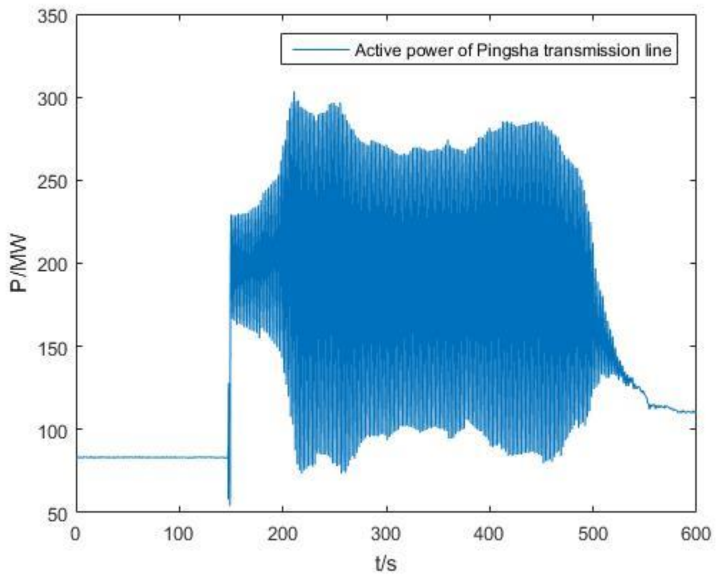
Forced oscillation $A < 0, B = -A, \boxed{\sigma < 0}$

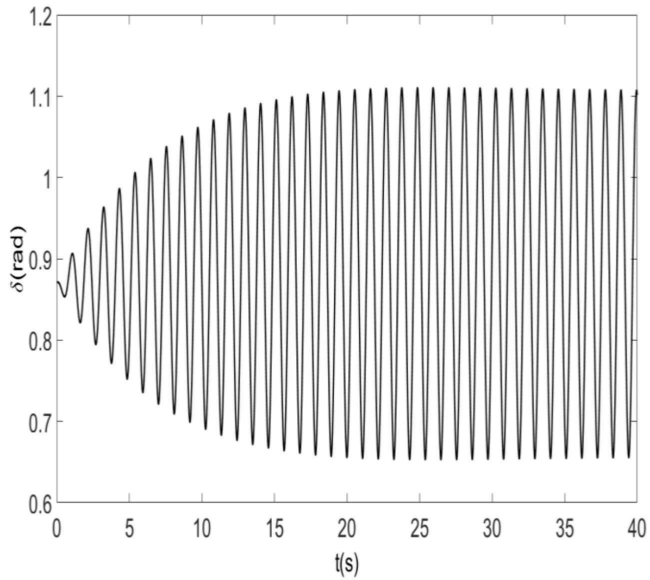


Features of Start Up Waveform

- The envelope of start up waveform $Ae^{\sigma t} + B$
- the difference is σ
- Steps
  - Peak-peak value X_i
  - increment of peak-peak value $Y_i = X_i - X_{i-1} = A(\mathrm{e}^{\sigma T} - 1)\mathrm{e}^{(i-1)\sigma T}$
  - logarithm $Z_i = \ln Y_i = (\ln(A(\mathrm{e}^{\sigma T} - 1)) - \sigma T) + \sigma T \cdot i$
  - Linear fitting to get the slope $S = \sigma T$
  - S>ε: natural oscillation, S<-ε: forced oscillation

Examples



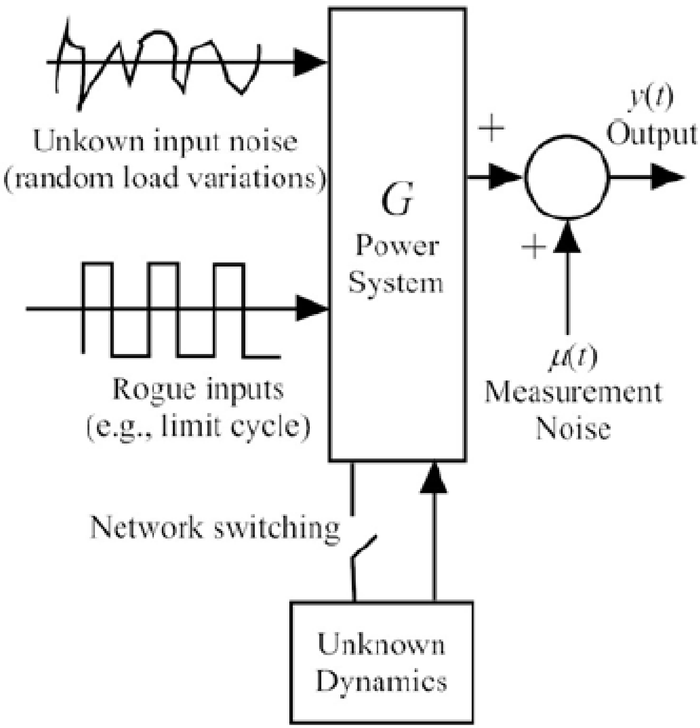


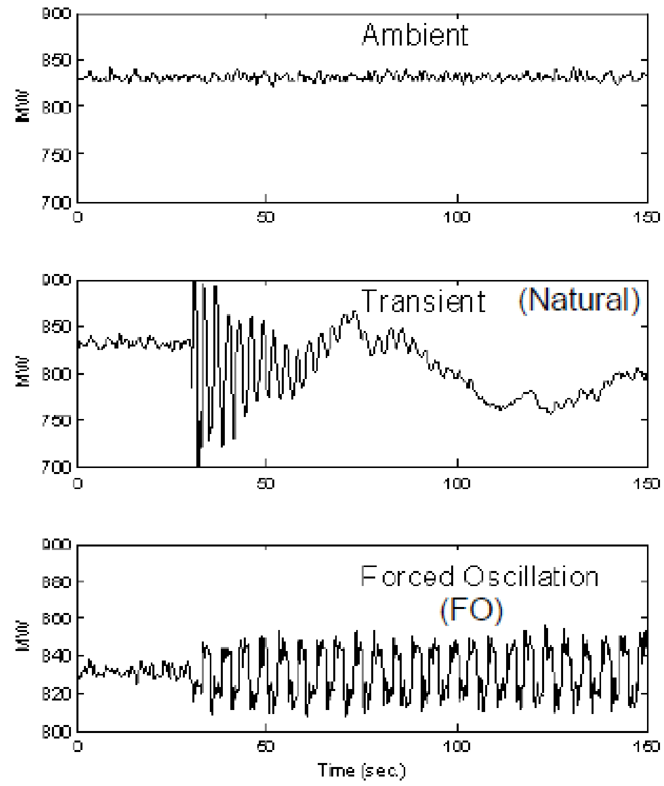
Measured
S=0.14>0, Natural

Simulated
S=-0.06<0, Forced

3. Spectral Methods (From Ruichao and Dan)

- The actual system response contains three components
- The intrinsic damping is contained in the ambient component





Steady State Response

- Natural (undamped)

$$\hat{y}_r(t) = 2\left|\underline{c}_r\underline{u}_n\underline{v}_n\underline{x}(0)\right|\cos\left(\omega_n t + \angle\left(\underline{c}_r\underline{u}_n\underline{v}_n\underline{x}(0)\right)\right) \Longrightarrow \text{Transient}$$

$$+\sum_{l=1}^{M}\left[q_l(t) \circledast \left[2\left|\underline{c}_r\underline{u}_n\underline{v}_n\underline{b}_{2l}\right|\cos(\omega_n t + \angle\underline{c}_r\underline{u}_n\underline{v}_n\underline{b}_{2l})\right]\right] \Longrightarrow \textbf{Noise 1: Random Sinusoidal Noise}$$

$$+\sum_{l=1}^{M}\left[q_l(t) \circledast \left[\sum_{\substack{i=1\\ i\neq n,n^*}}^{N} \underline{c}_r\underline{u}_i\underline{v}_i\underline{b}_{2l}e^{\lambda_i t}\right]\right] \Longrightarrow \text{Noise 2: Colored Noise}$$

- Forced

$$\hat{y}_r(t) = \sum_{m=1}^{\infty}\left(\left(\sum_{i=1}^{N}\underline{c}_r\underline{u}_i\underline{v}_i\underline{b}_1 e^{\lambda_i t}\right) \circledast |A_m|\cos(m\omega_0 t + \angle A_m)\right) \Longrightarrow \text{Forced}$$

$$+\sum_{l=1}^{M}\left(q_l(t) \circledast \left(\sum_{i=1}^{N}\underline{c}_r\underline{u}_i\underline{v}_i\underline{b}_{2l}e^{\lambda_i t}\right)\right) \Longrightarrow \text{Colored Noise}$$

Signal-Noise Separation

- First separate the whole response into signal and noise
- Power spectral density (PSD) of natural oscillation

$$S_{\hat{y}_r\hat{y}_r}(\omega_n) = S_{\hat{y}_{Sr}\hat{y}_{Sr}}(\omega_n) + S_{\hat{y}_{Nr}\hat{y}_{Nr}}(\omega_n)$$

$$S_{\hat{x}_{Sr}\hat{x}_{Sr}}(\omega_n) = \left|2\pi \underline{c}_r \underline{u}_n \underline{v}_n \underline{x}(0)\right|^2 \delta(0)^2$$

PSD of signal/transient component

$$S_{\hat{y}_{Nr}\hat{y}_{Nr}}(\omega_n) \cong \left|2\pi \underline{c}_r \underline{u}_n\right|^2 \left[\sum_{l=1}^{M} \left|\underline{v}_n \underline{b}_{2l}\right|^2 S_{q_l q_l}(\omega)\right] \delta(0)^2$$

PSD of noise, dominated by sinusoidal noise

- For two different measurements

$$\alpha_S = \frac{S_{\hat{y}_{S1}\hat{y}_{S1}}(\omega_n)}{S_{\hat{y}_{S2}\hat{y}_{S2}}(\omega_n)} = \frac{\left|\underline{c}_1 \underline{u}_n\right|^2}{\left|\underline{c}_2 \underline{u}_n\right|^2}$$

Equal

$$\alpha_N = \frac{S_{\hat{y}_{N1}\hat{y}_{N1}}(\omega_n)}{S_{\hat{y}_{N2}\hat{y}_{N2}}(\omega_n)} = \frac{\left|\underline{c}_1 \underline{u}_n\right|^2}{\left|\underline{c}_2 \underline{u}_n\right|^2}$$

Signal-Noise Separation

- Power spectral density (PSD) of forced oscillation

$$S_{\hat{y}_r\hat{y}_r}(\omega_0) = S_{\hat{y}_{Sr}\hat{y}_{Sr}}(\omega_0) + S_{\hat{y}_{Nr}\hat{y}_{Nr}}(\omega_0)$$

$$S_{\hat{y}_{rf}\hat{y}_{rf}}(\omega_0) = \left|\sum_{i=1}^{N} \frac{2\pi A_1 \underline{c_r}\,\underline{u_i}\,\underline{v_i}\,\underline{b_1}}{j\omega_0 - \lambda_i}\right|^2 \delta(0)^2$$

PSD of signal/forced component

$$S_{\hat{y}_{rn}\hat{y}_{rn}}(\omega_0) = \sum_{l=1}^{M}\left|\sum_{i=1}^{N} \frac{\underline{c_r}\,\underline{u_i}\,\underline{v_i}\,\underline{b_{2l}}}{j\omega_0 - \lambda_i}\right|^2 S_{q_l q_l}(\omega_0)$$

PSD of noise

- For two different measurements

$$\alpha_S = \frac{S_{\hat{y}_{S1}\hat{y}_{S1}}(\omega_n)}{S_{\hat{y}_{S2}\hat{y}_{S2}}(\omega_n)}$$

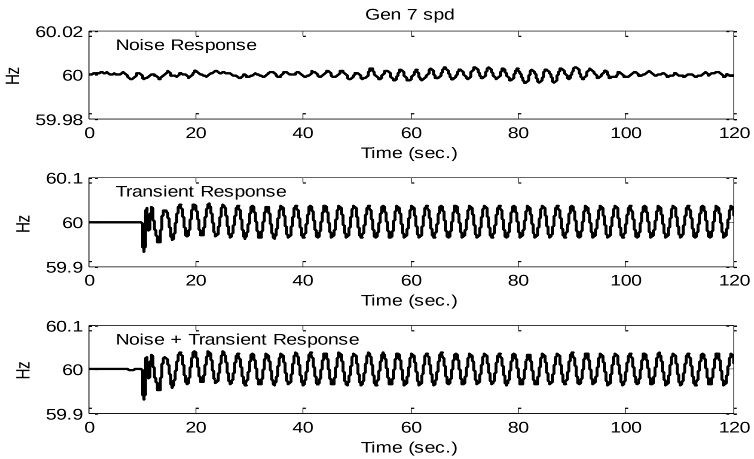
NOT Equal

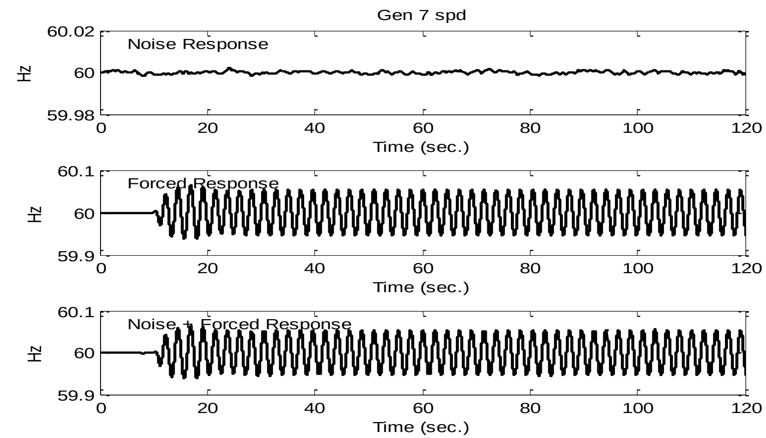
$$\alpha_N = \frac{S_{\hat{y}_{N1}\hat{y}_{N1}}(\omega_n)}{S_{\hat{y}_{N2}\hat{y}_{N2}}(\omega_n)}$$

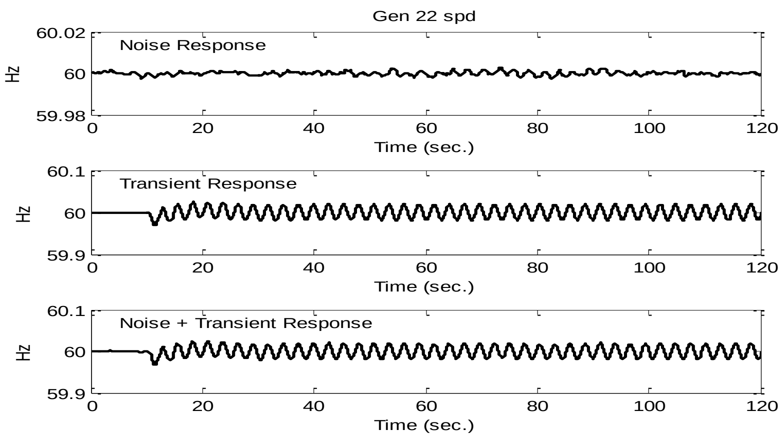
Examples

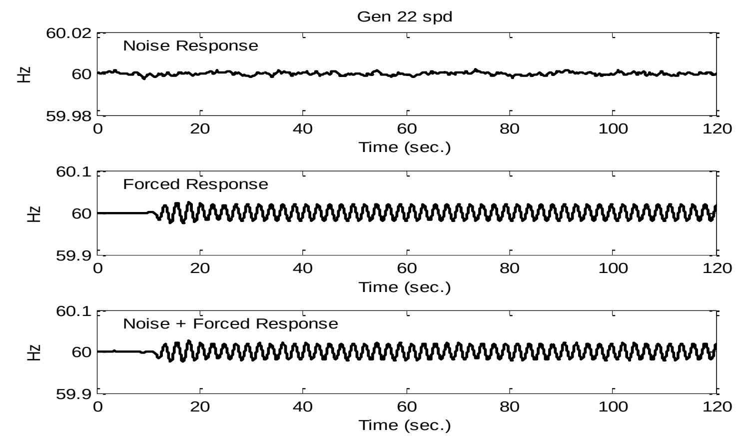
Natural $\alpha_S = 3.80$ and $\alpha_N = 3.85$

Forced $\alpha_S = 7.28$ and $\alpha_N = 0.91$









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Cross-Spectrum Difference Function

- To detect the existence of the "random sinusoid" component
- cross-spectrum difference function

$$S_r[\Omega] \triangleq \tilde{Y}^*_{rw_1}\tilde{Y}_{rw_2} - \tilde{Y}^*_{rw_2}\tilde{Y}_{rw_3}$$

$\tilde{Y}_{rw_i}$ the scaled DFT of the signal over window i

- Cross-spectrum index

$$C_{rg}[\Omega] \triangleq \frac{\left|\mathrm{E}\left\{S_r^*[\Omega]S_g[\Omega]\right\}\right|^2}{\mathrm{E}\{S_r^*[\Omega]S_r[\Omega]\}\ \mathrm{E}\left\{S_g^*[\Omega]S_g[\Omega]\right\}}$$

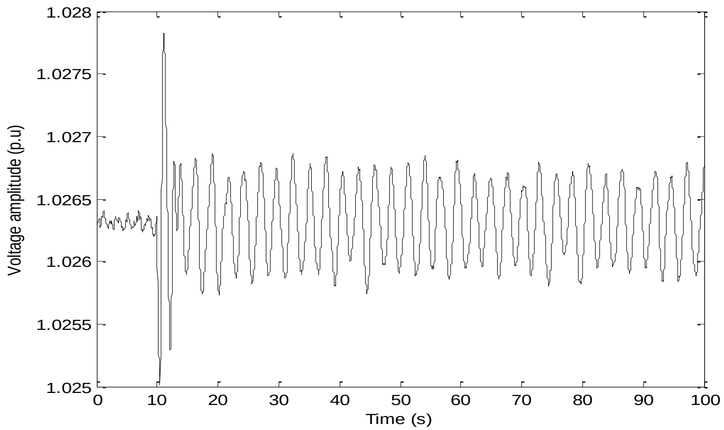
r, g: different measurement channels

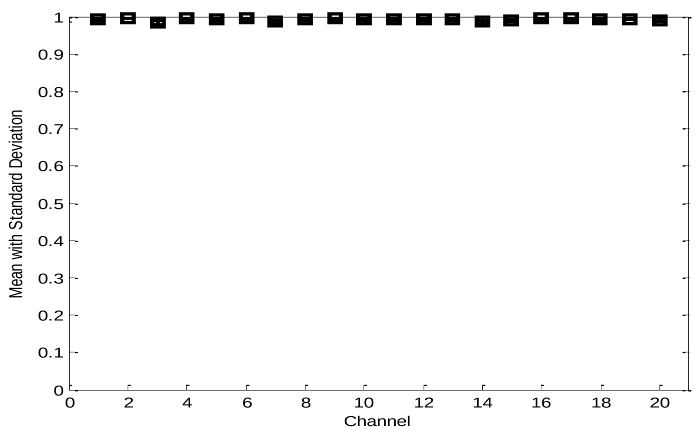
- Criterion

$C_{rg}[\Omega] = 1$ Natural $C_{rg}[\Omega] < 1$ Forced

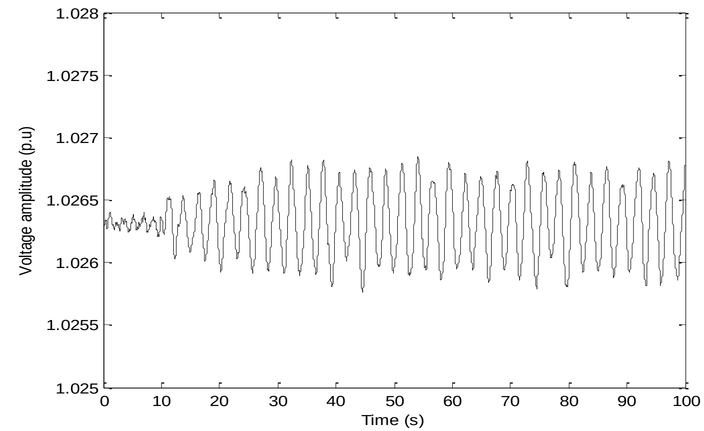
Examples

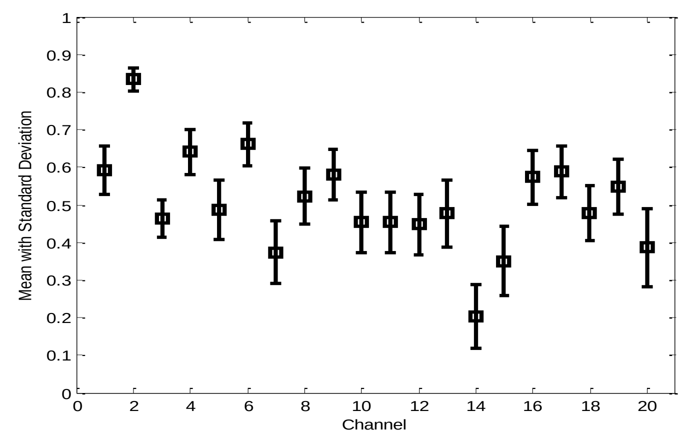
Natural





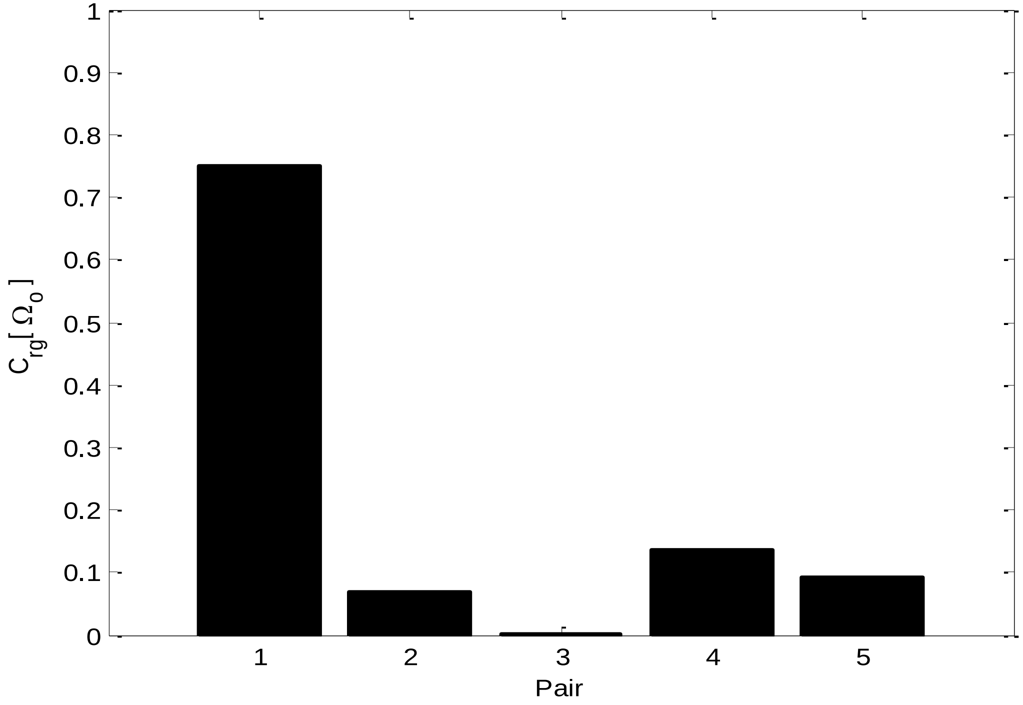
Forced





Examples

An actual forced oscillation incident in the western North American power system



Conclusions

- Some methods for online distinguishing natural and forced oscillations are proposed
- The problem is NOT well solved
- Due to the complexity of actual oscillation curves, many methods, though have solid theoretical foundations and perform well with simulation results, do not perform well with actual records
- More practical approaches are still needed

Thanks!

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