Power System Simulation Using the Multi-Stage Adomian Decomposition Method

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***Abstract*—This paper proposes a new approach for power system transient stability simulation, which is based on a semi-analytical solution (SAS) of power system differential-algebraic equations. In this paper, an SAS is derived using the Adomian decomposition method as a closed-form explicit function of symbolic variables such as time, the initial state and other variables on system conditions, and hence it can directly give a power system's dynamic trajectory being accurate for a certain time window. Unlike a traditional numerical integration based simulation approach, the proposed new approach offline derives an SAS and online evaluates the SAS by plugging in values of symbolic variables for a series of time windows making up the desired simulation period. This paper further studies the maximum length of the time window for an SAS being accurate and proposes a divergence indicator for simulation using adaptive time windows. Implementation of this new approach on parallel computers is also studied. The new approach is validated through contingency simulation of the IEEE 10-generator 39-bus system with detailed generator models.**

***Index Terms*— Adomian decomposition method; parallel computing; power system simulation; semi-analytical solution; transient stability**

I. INTRODUCTION

TIME-DOMAIN simulation of a power system following a contingency for transient stability analysis needs to solve the initial value problem (IVP) of nonlinear differential equations (DEs) about the system state over a simulation period. Numerical integration methods, either explicit or implicit, are traditionally employed but their iterative computations could be time-consuming for a multi-machine power system because its model is essentially a set of DEs nonlinearly coupled through sine functions. For accurate numerical integration, a very small integration step, typically less than one millisecond, is usually required. Thus, a large number of computations are needed even for a typical 10-second simulation period. Also, numerical instability may become another concern with explicit integration methods like the 4th-order Runge–Kutta method (R-K 4), which is currently widely applied in simulation software. Implicit integration methods like the trapezoidal method overcome numerical instability by introducing implicit algebraic equations, which have to be solved through numerical iterations by, e.g., the Newton-Raphson method, and thus, the computational complexity is significantly increased.

Intuitively, to solve a power system's DEs, if the analytical solution of the IVP about each state variable could be found as an explicit, closed-form function about symbolic variables including time, the initial state and other variables on the system operating condition, such a function would directly give the state value at any time instant without conducting time-consuming computations or iterations through all integration steps as R-K 4 does. However, for nonlinear power system DEs, such an analytical solution being accurate for any simulation time period does not exist in theory. Thus, a compromise is to find an approximate analytic solution, named a semi-analytical solution (SAS), which keeps accuracy for a certain length of time window (denoted by T), and can be repeatedly used over a series of such windows until those windows make up a desired simulation period. If an SAS is derived beforehand, then solving the IVP becomes simply evaluating the SAS, i.e. plugging in values of symbolic variables, which can be extremely fast compared to numerical integration. If online evaluation of the SAS for each window T takes a short computation time τ, the T/τ indicates how many times the SAS-based power system simulation can be faster than the wall-clock time.

The true solution of nonlinear power system DEs may be approached by summating infinite terms of some series expansion. An SAS can be defined as the sum of a finite number of terms that is accurate over window T. Such a series expansion can be derived using the Adomian decomposition method (ADM) proposed by George Adomian in the 1970s [1]. The method applies the sum of infinite Adomian polynomials to approach any nonlinear expression. Compared to other decomposition methods like Taylor series expansion, the ADM is able to keep nonlinearity of the system model. Reference [2] utilizes the ADM's semi-analytical feature to analyze inter-area oscillation. Reference [3] proves that the convergence of Adomian polynomials is equivalent to the convergence of the SAS given by the ADM. Our preliminary work reported in [4] and [5] has tested the feasibility of using ADM to solve the DEs of multi-machine power systems having all generators represented by the 2nd-order classic model. The results show that the ADM may give the solution as accurate as the R-K 4 within a proper window T. The contributions of this paper include the following. First, it applies the multi-stage ADM (M-ADM) [6] to transient stability simula-

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tion with detailed generator models so as to validate the capability of the ADM-based SAS for practical power system simulation. Second, a two-stage SAS-based approach is proposed for online simulation: the offline stage applies the ADM to derive an SAS about the symbolized time, initial state variables and other variables on system conditions; the online stage repeatedly evaluates the SAS for a series of time windows making up the desired simulation period. Third, the paper studies the maximum time window for an SAS being accurate, and accordingly proposes a divergence indicator and a method to choose its threshold. By the comparison of that divergence indicator to its threshold, a variable-length time window is allowed for better performance. The development of high performance computing (HPC) has provided opportunities for improving the speed of power system simulation [7], [8]. The paper also studies parallelization of this SAS-based new approach for online power system simulation using parallel computers.

The rest of the paper is organized as follows. Section II briefly introduces the ADM, applies the ADM to derive an SAS for power system DEs, studies the length of the time window of accuracy, and proposes the divergence indicator for SAS evaluation using a variable-length time window. Section III presents the flowchart, detailed steps and online implementation of an SAS-based two-stage power system simulation approach using the M-ADM. Section IV validates the new approach using the IEEE 10-machine 39-bus system with detailed generator models, tests the time performances and discusses the parallelization of computation tasks in simulation. Finally, conclusions are drawn in Section V.

II. Solving Power System DEs Using the ADM

A. Adomian Decomposition Method

Consider a nonlinear dynamic system, e.g., a power system, with M state variables modeled by nonlinear DE (1).

$$\begin{gathered}\dot{\mathbf{x}}(t)=\mathbf{f}(\mathbf{x}(t))\\ \mathbf{x}(t)=\left[x_1(t)\quad x_2(t)\quad \dots\quad x_M(t)\right]^T\\ \mathbf{f}(\cdot)=\left[f_1(\cdot)\quad f_2(\cdot)\quad \dots\quad f_M(\cdot)\right]^T\end{gathered}\tag{1}$$

To solve $\mathbf{x}(t)$, the first step of the ADM is to apply Laplace transform $\mathscr{L}[\cdot]$ to transform (1) into an algebraic equation (AE) about complex frequency s [9], [10], and then solve $\mathscr{L}[\mathbf{x}]$ to obtain (2).

$$\mathscr{L}[\mathbf{x}]=\frac{\mathbf{x}(0)}{s}+\frac{\mathscr{L}[\mathbf{f}(\mathbf{x})]}{s}\tag{2}$$

$$\mathbf{x}(t)=\sum_{n=0}^{\infty}\mathbf{x}_n(t)\tag{3}$$

$$f_i(\mathbf{x})=\sum_{n=0}^{\infty}A_{i,n}(\mathbf{x}_0,\mathbf{x}_1,\dots,\mathbf{x}_n),\quad i=1\cdots M\tag{4}$$

Assume that $\mathbf{x}(t)$ can be decomposed as (3). Then, use (4) to decompose each $f_i(\cdot)$, i.e. $\mathbf{f}(\cdot)$'s i-th element, as a sum of infinite Adomian polynomials given by (5), where λ is called a grouping factor [11].

$$A_{i,n}=\frac{1}{n!}\left[\frac{\partial^n}{\partial\lambda^n}\,f_i\left(\sum_{i=0}^{n}\mathbf{x}_i\lambda^i\right)\right]\Bigg|_{\lambda=0}\tag{5}$$

Matching the terms of $\mathbf{x}(t)$ and $\mathbf{f}(\cdot)$ with the same index [12], we can easily derive recursive formulas (6) and (7) for $\mathscr{L}[\mathbf{x}_n]$ ($n\geq0$), where $\mathbf{A}_n$=[$A_{1,n}$, … $A_{M,n}$]^T.

$$\mathscr{L}[\mathbf{x}_0]=\mathbf{x}(0)/s\tag{6}$$

$$\mathscr{L}[\mathbf{x}_{n+1}]=\mathscr{L}[\mathbf{A}_n]/s\qquad n\geq 0\tag{7}$$

By applying an inverse Laplace transform $\mathscr{L}^{-1}[\cdot]$ to both sides of (6) and (7), we can obtain $\mathbf{x}_n(t)$ for any n. An SAS of (1) is defined as the sum of first N terms of $\mathbf{x}_n(t)$:

$$\mathbf{x}^{SAS}(t)=\sum_{n=0}^{N-1}\mathbf{x}_n(t)\tag{8}$$

B. Deriving an ADM-based SAS of Power System DEs

$$\begin{cases}\dot{\delta}_k=\omega_k-\omega_R\\ \dot{\omega}_k=\dfrac{\omega_R}{2H_k}(P_{mk}-P_{ek}-D_k\dfrac{\omega_k-\omega_R}{\omega_R})\\ \dot{e}'_{qk}=\dfrac{1}{T'_{d0k}}\left[E_{fdk}-e'_{qk}-(x_{dk}-x'_{dk})i_{dk}\right]\\ \dot{e}'_{dk}=\dfrac{1}{T'_{q0k}}\left[-e'_{dk}+(x_{qk}-x'_{qk})i_{qk}\right]\end{cases}\tag{9}$$

$$\begin{aligned}&E_k=e'_{dk}\sin\delta_k+e'_{qk}\cos\delta_k+j(e'_{qk}\sin\delta_k-e'_{dk}\cos\delta_k)\\ &I_{tk}=i_{Rk}+ji_{Ik}\overset{\text{def}}{=}\mathbf{Y}_k^*\mathbf{E}\\ &P_{ek}=e_{qk}i_{qk}+e_{dk}i_{dk}\\ &i_{qk}=i_{Ik}\sin\delta_k+i_{Rk}\cos\delta_k,\quad i_{dk}=i_{Rk}\sin\delta_k-i_{Ik}\cos\delta_k\\ &e_{qk}=e'_{qk}-x'_{dk}i_{dk},\qquad\qquad e_{dk}=e'_{dk}+x'_{qk}i_{qk}\\ &V_k=\sqrt{e_{dk}^2+e_{qk}^2}\end{aligned}\tag{10}$$

For a power system having K synchronous generators, consider the 4th-order two-axis model (9) to model each generator with saliency ignored [13]. All generators are coupled through nonlinear AEs in (10) about the network. In (9) and (10), ω_R is the rated angular frequency; δ_k , ω_k, H_k and D_k are respectively the rotor angle, rotor speed, inertia and damping coefficient of the machine k; $\mathbf{Y}_k$ is the kth row of the reduced admittance matrix $\mathbf{Y}$; $\mathbf{E}$ is the column vector of all generator's electromotive forces (EMFs) and E_k is the kth element; P_{mk} and P_{ek} are the mechanical and electric powers; E_{fdk} is the internal field voltage; e'_{qk}, e'_{dk}, i_{qk}, i_{dk}, T'_{q0k}, T'_{d0k}, x_{qk}, x_{dk}, x'_{qk} and x'_{dk} are transient voltages, stator currents, open-circuit time constants, synchronous reactances and transient reactances in q- and d-axes, respectively; V_k is the terminal bus voltage magnitude.

In addition, consider the following first-order exciter and governor models [14]:

$$\dot{E}_{fdk}=\frac{1}{T_{Ak}}\left[-E_{fdk}+K_{Ak}(V_{refk}-V_k)\right]\tag{11}$$

$$\dot{P}_{mk} = \frac{1}{T_{gk}}(-P_{mk} + P_{refk} - \frac{\omega_k - \omega_R}{R_k}) \tag{12}$$

where T_{Ak} and K_{Ak} are respectively the time constant and gain in voltage regulation with the exciter, V_{refk} is the reference voltage value, T_{gk} is total time constant of the governor and turbine, P_{refk} is the setting point of the mechanical power output, R_k is the speed regulation factor.

In the following context, the 4th-order model is utilized as an example to illustrate the derivation of an SAS for simplicity of description. A similar procedure is applied to the 6th-order DE model in (9), (11) and (12) and other DE models. Substitute AEs (10) into DEs (9) to eliminate i_{qk}, i_{dk} and P_{ek}. Then, the differential-algebraic equations (9) and (10) are transformed into the form of (1), where state vector $\mathbf{x} = \begin{bmatrix}\delta_1 & \omega_1 & e'_{q1} & e'_{d1} & \ldots & \delta_K & \omega_K & e'_{qK} & e'_{dK}\end{bmatrix}^T$ has M=4K state variables as the elements. Then, an SAS of this set of DEs can be derived by formulas (6) and (7), as illustrated below about the generator speed ω of a single-machine infinite-bus (SMIB) system modeled by (9). Assume that the infinite bus has voltage V_∞=1 pu. Let $\mathbf{x}=[\delta, \omega, e'_q, e'_d]^T$ and $\mathbf{f}=[f_1, f_2, f_3, f_4]^T$, which are the nonlinear functions in four DEs. From (3),

$$\delta(t) = \sum\nolimits_{n=0}^{\infty} \delta_n(t) \qquad \omega(t) = \sum\nolimits_{n=0}^{\infty} \omega_n(t)$$
$$e'_d(t) = \sum\nolimits_{n=0}^{\infty} e'_{d,n}(t) \quad e'_q(t) = \sum\nolimits_{n=0}^{\infty} e'_{q,n}(t) \tag{13}$$

Then, equation (2) about ω becomes

$$\mathscr{L}[\omega] = \frac{\omega(0)}{s} + \frac{\mathscr{L}[f_2(\delta, \omega, e'_q, e'_d)]}{s} \tag{14}$$

From (4) and (5), the first two Adomian polynomials for f_2 are given in

$$A_{2,0} = \frac{\omega_R}{2H}\left[-e'_{q,0}\gamma_1 - e'_{d,0}\gamma_2 + \gamma_1\gamma_2(x'_d - x'_q) + P_m - D\frac{\omega_0 - \omega_R}{\omega_R}\right]$$

$$\begin{aligned} A_{2,1} = \frac{\omega_R}{2H}[&Y_o x'_d(e'_{q,1}\gamma_2 + e'_{d,1}\gamma_1) - \delta_1 Y_\infty^2(x'_d - x'_q)\cos 2\delta_0 \\ &- 2Y_\infty(e'_{q,0}e'_{q,1} + e'_{d,0}e'_{d,1}) - Y_\infty(e'_{d,1}\cos\delta_0 - e'_{q,1}\sin\delta_0) \\ &- x'_d Y_o\gamma_4 + Y_o Y_\infty \delta_1 x'_q \gamma_4 + Y_\infty\delta_1\gamma_3 - Y_o x'_q(e'_{d,1}\gamma_1 + e'_{q,1}\gamma_2) - \frac{D\omega_1}{\omega_R}] \end{aligned}$$

where $\gamma_1 = Y_o e'_{q,0} - Y_\infty \sin\delta_0$, $\gamma_2 = Y_o e'_{d,0} + Y_\infty\cos\delta_0$,

$$\gamma_3 = e'_{q,0}\cos\delta_0 + e'_{d,0}\sin\delta_0,\ \gamma_4 = e'_{d,0}\cos\delta_0 + e'_{q,0}\sin\delta_0 \tag{15}$$

where Y_o and Y_∞=|Y_∞|∠β are respectively the admittances from the generator's EMF to the ground and to the infinite bus. Note that E_{fd}, which is constant in this 4th-order DE model, only explicitly appears in the Adomian polynomials about e'_q.

Since the accuracy of an SAS defined by (8) only lasts for a limited time window T [15]—[18], a multi-stage strategy, i.e. the M-ADM [6], [19] —[21], is adopted to extend the accuracy of the same SAS to an expected simulation period by these two steps:

Step-1: Partition the simulation period into sequential windows of T each able to keep an acceptable accuracy of the SAS.

Step-2: Evaluate the SAS at desired time points in the first T using the given initial state and the values of other symbolic variables; starting from the second window T, evaluate the SAS by taking the final state of the previous T as the initial state.

As long as the final state of each window is accurate enough, the accuracy of the next window will be ensured. To apply this approach to simulate a contingency, we may first perform the numerical approach until the contingency is cleared to obtain the initial state for the IVP about the post-contingency simulation period, and then the M-ADM can be performed.

C. *Convergence and Time Window of Accuracy for an SAS*

This subsection studies the convergence and time window of accuracy of the ADM-based SAS. First, consider an SMIB system having a 2nd-order classical model generator connected to the infinite bus by an impedance. Thus, Y_o is zero and the EMF E of the generator has a constant magnitude so as to eliminate two DEs on e'_d and e'_q in (9). System parameters and initial conditions are listed in Table I. Mechanical power P_m determines the operating condition. V_∞ is the voltage magnitude of the infinite bus, whose phase angle is considered zero. $\delta(0)$ and $\omega(0)$ are the initial rotor angle and speed of the generator, which are initial state variables.

TABLE I
PARAMETERS OF THE SMIB SYSTEM

H	D	Y_∞= |Y_∞|∠β	Y_o	P_m
3 s	0 s	0.9∠90°pu	0 pu	0.8 pu
|E|	V_∞	ω_R	$\delta(0)$	$\omega(0)$
1.1 pu	1	377 rad/s	0.06 rad	2.05 rad/s

Fig. 1 plots the trajectories of six different SASs with N=3 to 8, respectively, and compare them with the numerical integration result from the R-K 4.

$$\delta(t) = \sum_{n=0}^{4}\delta_n = 11.39t^8 + 4.50t^7 + 1353.32t^6 + 361.02t^5$$
$$-240.09t^4 - 47.72t^3 + 20.81t^2 + 2.05t + 0.06$$

where
$$\delta_0 = 0.06,$$
$$\delta_1 = 20.81t^2 + 2.05t,$$
$$\delta_2 = -241.61t^4 - 47.72t^3,$$
$$\delta_3 = 1184.67t^6 + 350.95t^5 + 1.52t^4,$$
$$\delta_4 = 11.39t^8 + 4.50t^7 + 168.66t^6 + 10.07t^5 \tag{16}$$

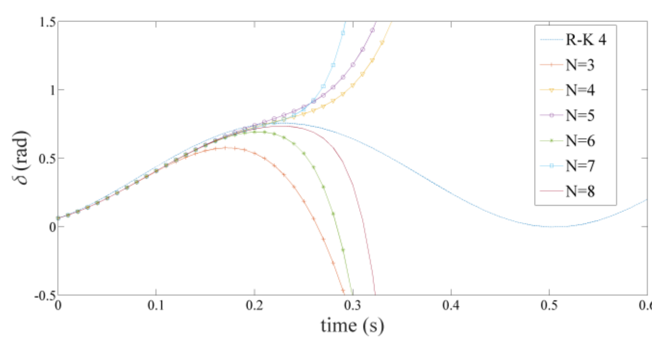


Fig. 1. Comparison of SASs with numerical result.

For N=5, 5 terms of the SAS are given in (16) as an example and its trajectory and the trajectories of individual terms are shown in Fig. 2. In Fig. 2, $T_{\max}$ denotes a limit of the time window of accuracy. Also define the absolute value of the last term, i.e.$|x_{N-1}|$, as a divergence indicator I_D, which is close to zero within $T_{\max}$ and sharply increases the magnitude, otherwise. $T_{\max}$ can be estimated by selecting an appropriate threshold $I_{D,\max}$ for I_D. For instance in Fig. 2, $I_{D,\max}$ is set at 0.01 rad to determine $T_{\max}$. There are two observations from Fig. 2:

- The SAS from the ADM matches well the R-K 4 result within 0.2s, i.e. a time window of accuracy.
- The higher order of a term, the less contribution it has and the faster it diverges to infinity. The last term δ_4 diverges quickly outside 0.2s.

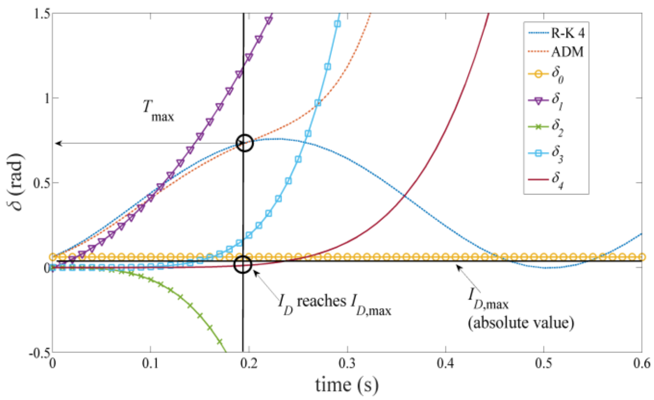


Fig. 2. Different terms of the SAS and the time window of accuracy.

TABLE II
$T_{\max}$ VS. TIME CONSTANTS OF THE SYSTEM

No.	H_3(s)	T_1(s)	T_2(s)	$T_{\max}$(s)
1	4.5	0.9510	0.5516	0.2546
2	4.0	0.9438	0.5280	0.2342
3	3.5	0.9369	0.5014	0.2131
4	3.0	0.9304	0.4718	0.1905
5	2.5	0.9241	0.4365	0.1662
6	2.0	0.9183	0.3961	0.1410
7	1.5	0.9128	0.3479	0.1137
8	1.0	0.9076	0.2881	0.0845

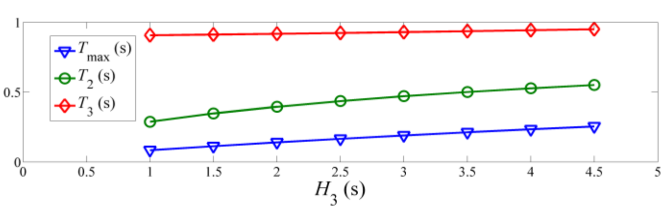


Fig. 3. Relationships between $T_{\max}$, T_2, T_1 and H_3.

To unveil the relation between $T_{\max}$ and time constants of a multi-machine system, the IEEE 3-generator 9-bus system in [22] is studied. Gradually decrease H_3, the inertia of generator 3, from 4.5 s to 1.0 s while keeping the other two unchanged at original 23.64 s and 6.4 s, such that eight system models are yielded as shown in Table II. Because the system has two oscillation modes and their oscillation periods T_1 and T_2 may be important time constants influencing $T_{\max}$, T_1 and T_2 are estimated from each linearized model of the system and are listed in Table II. A three-phase fault at bus 7 cleared by tripping line 5-7 is simulated on each model by both the R-K 4 and the ADM with N=3 (using the post-fault state from the R-K 4 as its initial state). Using 0.01 rad as $I_{D,\max}$, the estimated $T_{\max}$ for each model is given in the table. Fig. 3 illustrates that T_1, T_2 and $T_{\max}$ monotonically increase with H_3. The bigger time constant T_1 does not change significantly with H_3. Fig. 4 shows values of $T_{\max}$ for H_3=1.5 s, 3 s and 4.5 s, beyond which the ADM result starts diverging from the R-K 4 result. A hypothesis for a multi-machine power system is that $T_{\max}$ is mainly influenced by the smallest time constant.

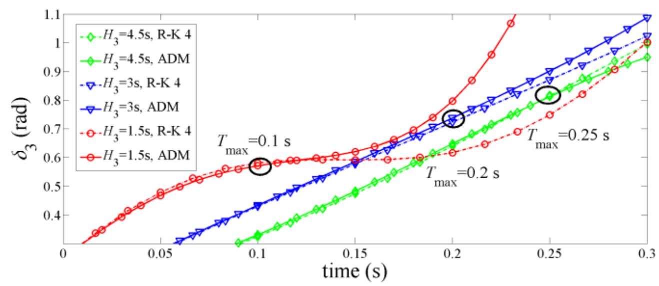


Fig. 4. $T_{\max}$'s with respect to selected H_3's.

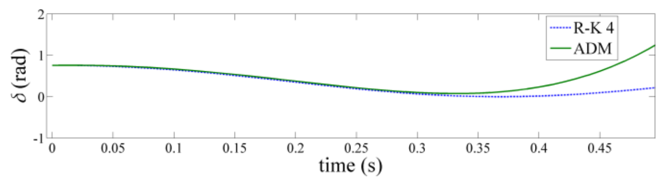


Fig. 5 Using an initial state with $\omega(0)$=0 rad/s and $\delta(0)$=0.76 rad.

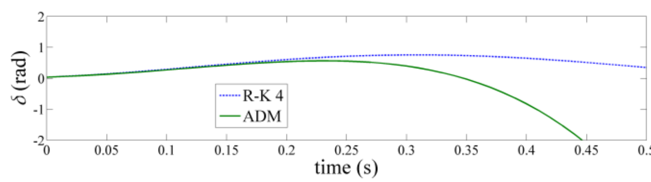


Fig. 6. Using an initial state with $\omega(0)$=1.38 rad/s and $\delta(0)$=0.04 rad.

If the initial state varies, the time of accuracy may change as well. For the above SMIB system, different values of $\delta(0)$ and $\omega(0)$ will lead to different $T_{\max}$'s. As illustrated by Figs.5 and 6, the SAS evaluated starting from an initial state with $\omega(0)$=0 rad/s and $\delta(0)$=0.76 rad keeps its accuracy for a time window around 0.25s while for a larger $\omega(0)$=1.38 rad/s and $\delta(0)$=0.04 rad, the window of accuracy may reduce to below 0.2 s.

For a general multi-machine system, it can be difficult to analyze how $T_{\max}$ changes with $I_{D,\max}$ about a state variable. However, we may analyze their relationship on the above SMIB system first to help gain an insight on their relationship for a multi-machine system. Consider a 3-term SAS of rotor angle δ, whose last term δ_2 has this expression

$$\delta_2 = c_1 t^4 + c_2 t^3 \tag{17}$$

where

$$c_1 = \frac{\omega_0^2 V_\infty |Y_\infty E|^2 \sin(\beta - \delta(0))}{96H^2}\left[|E|\cos\beta + V_\infty\cos(\beta-\delta(0)) - \frac{P_m}{|Y_\infty E|}\right]$$

$$c_2 = \frac{-\omega_0 V_\infty |Y_\infty E|(\omega(0)-\omega_R)\sin(\beta-\delta(0))}{12H}$$

Define divergence indicator I_D as δ_2 and let t=$T_{\max}$ and δ_2=$I_{D,\max}$ in (17) to obtain

$$I_{D,\max} = c_1 T_{\max}^4 + c_2 T_{\max}^3 \tag{18}$$

$T_{\max}$ has 4 roots as given in

$$T_{\max} = -\frac{c_2}{4c_1} \pm \frac{p_4}{2} \mp \frac{\sqrt{p_5 \pm p_6}}{2} \text{ or } -\frac{c_2}{4c_1} \pm \frac{p_4}{2} \pm \frac{\sqrt{p_5 \pm p_6}}{2} \tag{19}$$

where

$$p_1 = -27c_2^2 I_{D,\max}, \quad p_2 = p_1 + \sqrt{4\cdot(12c_1 I_{D,\max})^3 + p_1^2}$$

$$p_3 = -\frac{4I_{D,\max}}{\sqrt[3]{2/p_2}} + \frac{\sqrt[3]{p_2/2}}{3c_1}, \quad p_4 = \sqrt{\frac{c_2^2}{4c_1^2} + p_3}$$

$$p_5 = \frac{c_2^2}{2c_1^2} - p_3, \quad p_6 = \frac{-c_2^3}{4c_1^3 p_4}$$

Since $T_{\max}$>0, the smallest positive root should be selected as an estimate of $T_{\max}$. For a multi-machine system, equations (18) and (19) can also be applied to approximately analyze the relationship of $I_{D,\max}$ and $T_{\max}$ for state variables of each machine by means of an SMIB equivalent about that machine against the rest of the system.

The studies above show that, for an SAS, its $T_{\max}$ depends on time constants of the system, the initial state starting the evaluation and the contingency as well. Therefore, we may either choose a fixed time window less than the most conservative $T_{\max}$ observed offline based on many simulations on probable contingency scenarios or allow the window T to change adaptively as long as divergence indicator I_D remains below a preset threshold $I_{D,\max}$ for each state variable.

D. Evaluating an SAS Using an Adaptive Time Window

The convergence of the SASs for a general nonlinear system is still an open question [23], and no sufficient condition for convergence has been proved yet. Reference [24] gives a necessary condition, i.e. the satisfaction of a ratio test: $\|\mathbf{x}_{n+1}\|_2 < \alpha \|\mathbf{x}_n\|_2$ holds for n=0, 1, …, N-1, where 0<α<1 is a constant depending on the system. However, α is difficult to derive analytically for a high-dimensional system.

This paper proposes a practical approach for evaluation of an ADM-based N-term SAS using an adaptive time window. The approach compares divergence indicator I_D with a preset threshold $I_{D,\max}$ to adaptively judge the end of the current window for evaluation and proceed to the next window until the entire simulation period is made up. $I_{D,\max}$ is estimated by the following procedure for a list of scenarios that each have a contingency simulated under a specific operating condition:

Step-1: For each scenario, use the post-contingency state from the R-K 4 as the initial state to run the M-ADM using a small enough fixed time window T.

Step-2: Find the maximum per unit absolute value that the last SAS term, i.e. $|x_{k,N-1}|$, of any state variable may reach over the entire simulation period. Use that value as a guess of $I_{D,\max}$.

Step-3: Add a small random variation to the post-contingency state and repeat *Step-2* for a number of times. Take the smallest guess of $I_{D,\max}$.

Step-4: After finishing *Steps 1-3* for all contingencies, choose the smallest $I_{D,\max}$ as the final threshold.

Remarks: 1) *Step-2* on guessing an $I_{D,\max}$ may exclude $\delta_{k,N-1}$, i.e. the last SAS term for each rotor angle δ_k, since its divergence can be detected through the divergence of the last SAS term of ω_k; 2) *Step-2* finds the maximum value of all last terms rather than the minimum value in order to provide a necessary condition for convergence rather than an over-conservative, sufficient condition causing loss of the advantage of using an adaptive time window; 3) the random variation in *Step-3* is added to make the $I_{D,\max}$ more independent of the post-contingency state, which may be around 1% as case studies in Section IV do.

The above procedure can be performed offline for potential contingencies and operating conditions. Based on our tests, $I_{D,\max}$ does not vary significantly with contingencies, so in practice, the list of scenarios does not have to be large to find an effective $I_{D,\max}$.

III. SAS-BASED SCHEME FOR POWER SYSTEM SIMULATION

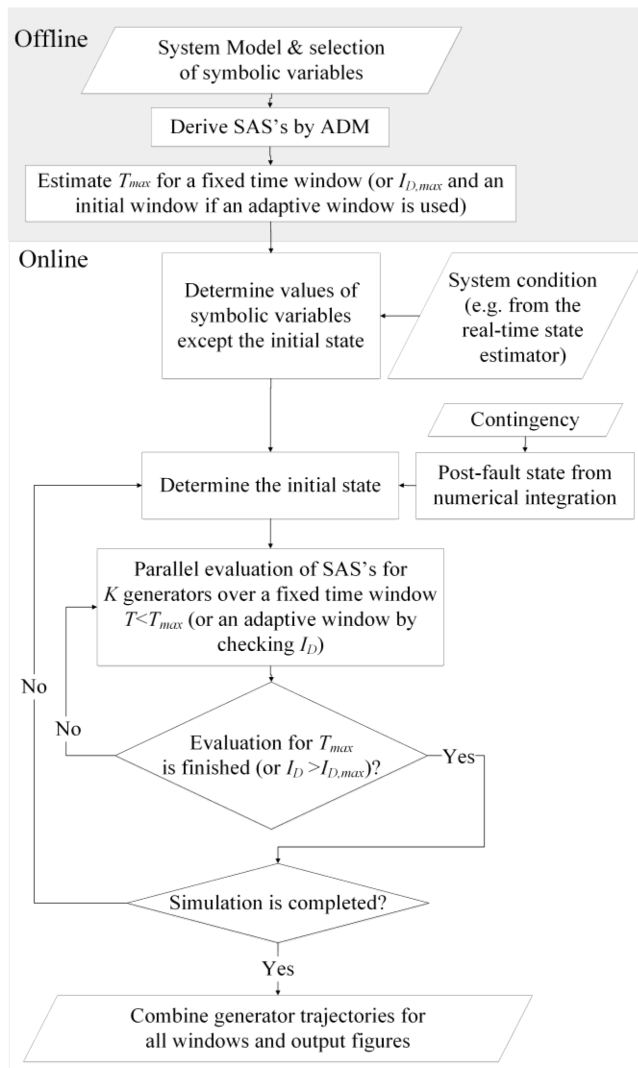


Fig. 7. Flowchart of the proposed approach.

A two-stage scheme is presented for power system simulation using the M-ADM, which comprises an offline stage to derive the SASs and an online stage to evaluate the SASs as shown in Fig. 7.

A. Offline Stage

Assuming a constant impedance load at each bus, an SAS is derived by the ADM for each generator with symbolic variables from, e.g., one of these two groups:

- ***Group-1***: Time, the initial state, and the operating condition (e.g. generator outputs and load impedances)
- ***Group-2***: *Group-1* plus selected symbolized elements (symbolized parameters of system that subject to changes) in the system admittance matrix.

Group-1 assumes a specific post-contingency system topology (i.e. a constant system admittance matrix) but relaxes the system operating condition so as to enable one SAS to simulate for multiple loading conditions. *Group-2* additionally relaxes selected elements in the admittance matrix and hence enables one SAS suitable for simulating multiple contingencies. Other symbolic variables can also be added as undetermined parameters but the more symbolic variables the more complex expression of the SAS. All SASs derived in the offline stage will be saved in storage for later online use.

If an adaptive time window for SAS evaluation is used, the offline stage also needs to estimate $I_{D,\max}$. The detailed implementation of estimating $I_{D,\max}$ is illustrated in section IV. If a fixed window is adopted, T can be chosen less than the minimum $T_{\max}$ estimated by a procedure similar to that for the determination of $I_{D,\max}$ using a list of scenarios.

B. Online Stage

For a specific contingency scenario, this stage evaluates the corresponding SAS's of every generator consecutively over time windows T, fixed or adaptive, until making up the expected simulation period. The first time window needs to know the post-contingency initial system state, which can be obtained from numerical integration for the fault-on period until the fault is cleared. Starting from the second window, the initial state takes the final state of the previous window.

If an adaptive time window is applied, an initial window may be chosen less than the estimated $T_{\max}$ for a fixed window. Then, during each window, the divergence indicator I_D for each state variable is calculated and compared with the threshold $I_{D,\max}$ acquired in the offline stage in order to decide when to proceed to the next window, i.e. the end of the current window. Thus, even if the initial window is not small enough, comparison of I_D and $I_{D,\max}$ will enable self-adaptive adjustment of the window.

Within each window, because SAS's are independent expressions, their evaluations can be performed simultaneously on parallel computers. In expression, each SAS is the sum of terms in this form

$$C \cdot \underbrace{x_i \ldots x_j}_{h} t^n \underbrace{f_k(x_k) \ldots f_l(x_l)}_{m} \quad \text{where } f(\cdot) \text{ is } \sin(\cdot) \text{ or } \cos(\cdot) \tag{20}$$

Where C is a constant which depends on system parameters, t is time, i, j, k and l are integer indices of state variables. For different numbers of SAS terms and different systems, the ranges of h, m and n are different. For the IEEE 39-bus system with 3 SAS terms tested in Section IV, h=0,…,3, n=0,1,2 and m=0,…,4. Expression (20) is defined as one Computing Unit (CU) in this paper. All such CUs can be evaluated simultaneously on parallel processors to accelerate the online stage.

The proposed SAS-based approach may be applied for fast power system simulation in the real-time operating environment: in the offline stage, an SAS is derived that symbolizes a group of uncertain parameters like *Group-2*; then, in the online stage, whenever the real-time state estimation is finished (typically, every 1 to 3 minutes) to give the current power-flow solution and network topology, the SAS will be evaluated to provide simulation results on a given contingency. However, if a change on the network topology or any parameter about the operating condition is detected in real time by, e.g., the SCADA system [25] and makes the most recent state estimation result invalid, the SAS evaluation should wait until the state estimator gives a new estimation result. Thus, online power system simulation using the proposed approach can be performed synchronously with real-time state estimation.

IV. Case Studies on the IEEE 39-bus System

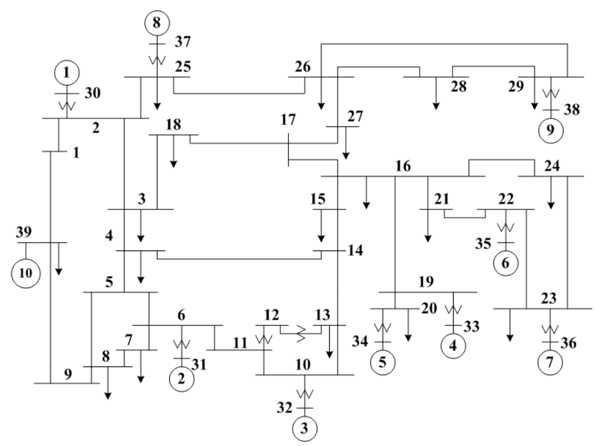


Fig. 8. IEEE 10-generator 39-bus system.

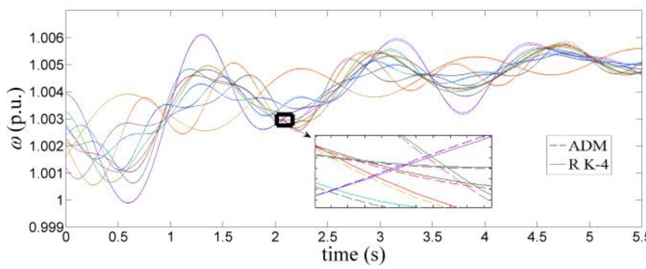
IEEE 10-generator, 39-bus system, as shown in Fig. 8, is used to validate the SAS-based approach for power system simulation. Generator 39 has the largest inertia and its rotor angle is defined as the reference. The proposed two-stage scheme is tested using both a fixed time window and an adaptive time window. To achieve the fastest simulation, only one data point on each trajectory is evaluated within each time window, which is enough for transient stability assessment.

A. Fixed Time Window

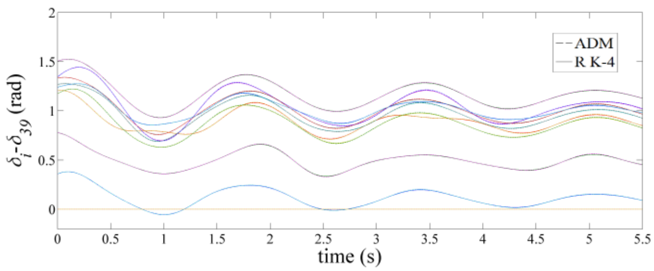
A permanent three-phase fault lasting for 0.08 s is applied to line 3-4 at bus 3. We preset $I_{D,\max}$=0.005 p.u. (per unit) for all state variables except for the rotor angle. If all generators are represented by the 4th-order model in (9), our tests show that when an SAS with 2 terms is evaluated over a time window of 0.002 s, the largest 2nd SAS term of the state variables is 0.0047 p.u. $<I_{D,\max}$, which means $T_{\max}\approx$0.002 s for a 2-term SAS. Fig. 9 gives the results from the M-ADM (dash lines) using a 0.001 s window and the results from the R-K 4 (solid lines) with a 0.001 s integration step, which are identical. If the time window and integration step are both increased to

0.01 s (>T_{max}), the simulation results from the R-K 4 and M-ADM have slight, noticeable differences as shown in Fig. 10.

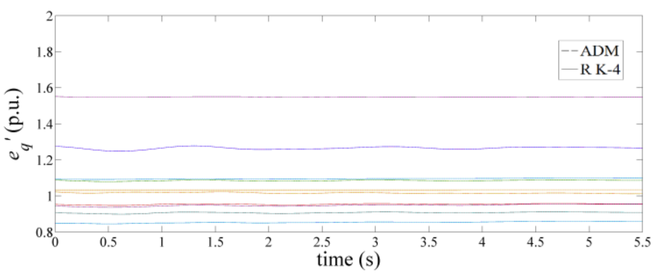
Although including more terms is expected to increase T_{max} as indicated by Fig. 1, using an SAS with 3 terms does not extend T_{max} significantly in this case. For example, use a 0.01 s time window to run a 3-term SAS for the same contingency, there are still obvious mismatches between the R-K 4 and M-ADM results. Moreover, a 3-term SAS has a more complex expression, so it takes longer to evaluate than a 2-term SAS.



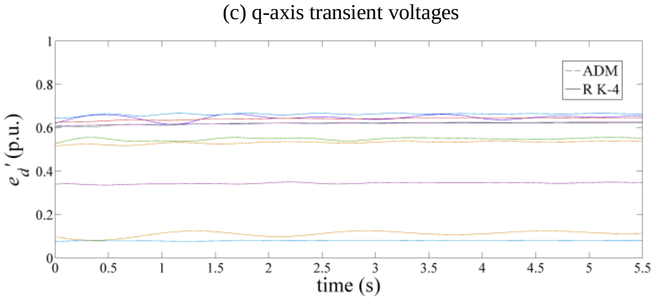
(a) Rotor speeds



(b) Rotor angles

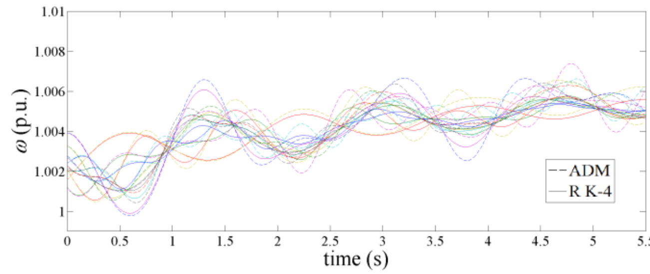


(c) q-axis transient voltages

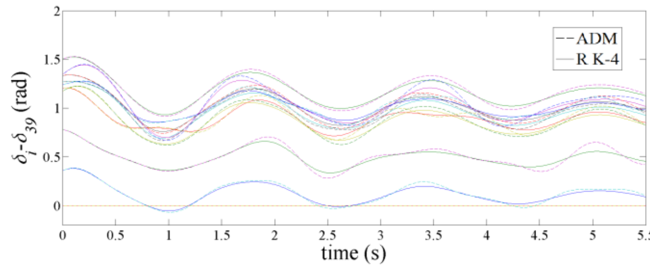


(d) d-axis transient voltages

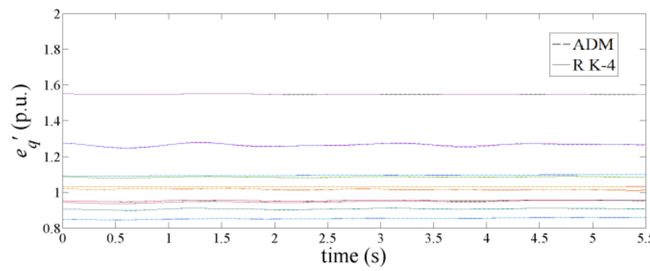
Fig. 9. Comparison of the simulation results given by the R-K 4 and the 2-term SAS using a fixed time window of 0.001 s.



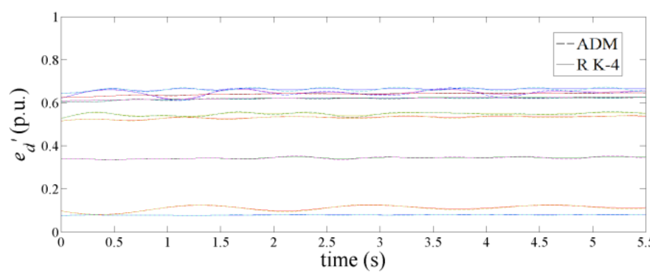
(a) Rotor speeds



(b) Rotor angles



(c) q-axis transient voltages



(d) d-axis transient voltages

Fig. 10. Comparison of the simulation results given by the R-K 4 and the 2-term SAS using a fixed time window of 0.01 s.

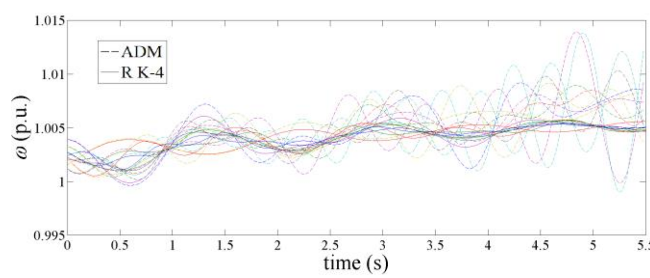
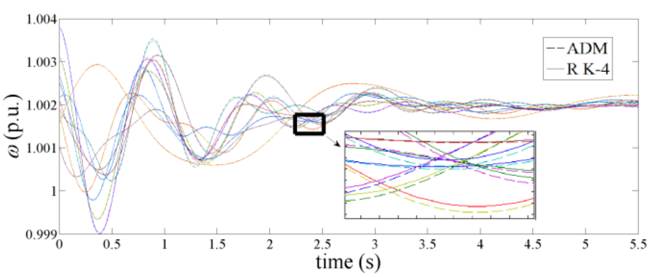


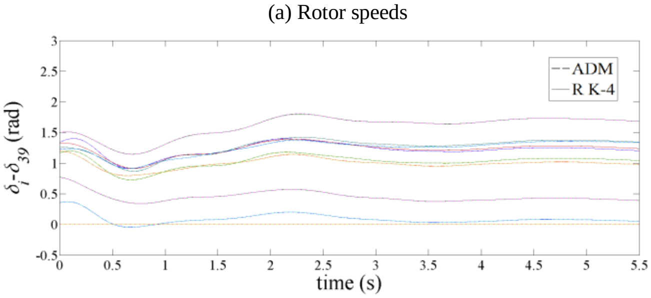
Fig. 11. Comparison of the simulation results of rotor speeds given by the R-K 4 and the 2-term SAS using a fixed time window of 0.02 s.

When an SAS is evaluated over a fixed time window T for power system simulation, the last SAS terms, i.e. divergence indicator I_D's, of all state variables can distinguish numerical instability from power system instability: if the simulated system trajectory becomes unstable while all I_D's are still small, e.g. much less than the predefined $I_{D,\max}$, it is very likely to be power system instability; if some I_D also increases drastically to approach or exceed $I_{D,\max}$ when the system trajectory appears to be unstable, numerical instability may happen. Thus, a smaller T should be used to re-evaluate the SAS for verification of numerical instability. For example, if T is increased to 0.02 s $\approx 10\times T_{\max}$, the simulation results diverge with numerical instability introduced on purpose as shown in Fig. 11, where the results from the R-K 4 method are still stable. That numerical instability can be detected by $I_D > I_{D,\max}$ for many windows. From the results of Fig.9 to Fig. 11, as T increases from 0.001 s to 0.01 s and then to 0.02 s, the largest I_D of all states variables increases from 0.0023 p.u. to 0.0279 p.u. (i.e. 12.1 times) and then to 0.1051 p.u. (i.e. 45.7 times), which indicates the occurrence of numerical instability. I_D can be utilized to avoid numerical instability by changing the time window adaptively. The detailed method will be proposed in the next sub-section.

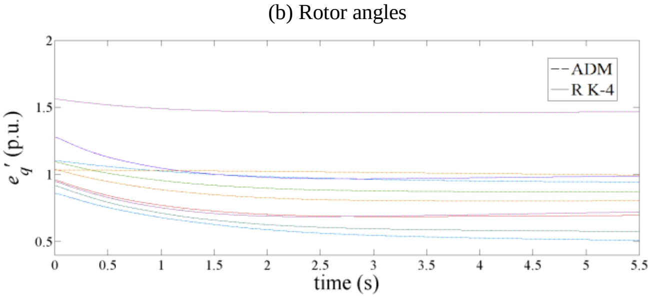
The M-ADM is also tested on the system having each generator represented by the 6th-order DE model in (9), (11) and (12) containing the exciter and governor. The parameters of exciters and governors are set up as T_{Ak} = 0.02 s, K_{Ak} = 5, T_{gk} = 0.5 s, R_k = 0.01 for all machines. A 2-term SAS is derived for each of the six state variables, and the time window is selected to be 0.001 s within the estimated $T_{\max}$. Under the same contingency on line 3-4, The R-K 4 simulation indicates the frequency oscillation is better damped than that without a governor. Fig. 12 compares the results from the M-ADM (dash lines) and R-K 4 (solid lines) for each state variable, which match well.



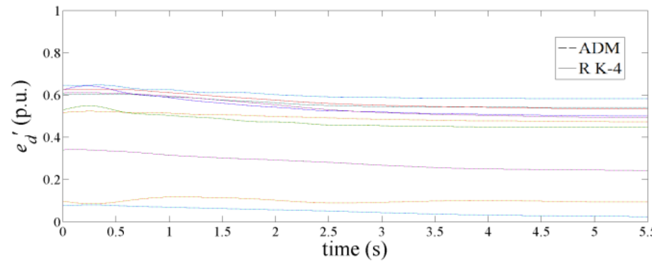
(a) Rotor speeds



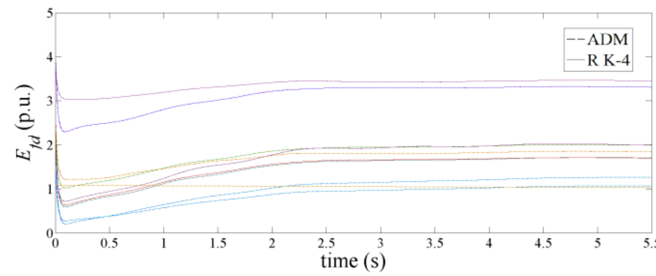
(b) Rotor angles



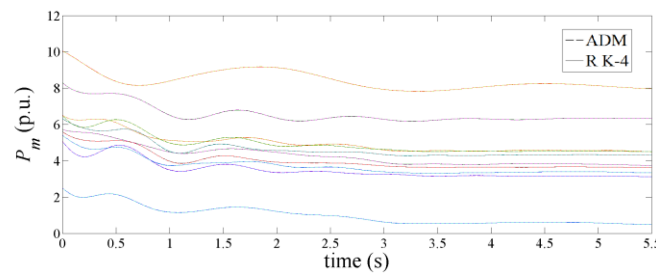
(c) q-axis transient voltages



(d) d-axis transient voltages



(e) Field voltages



(f) Governor outputs

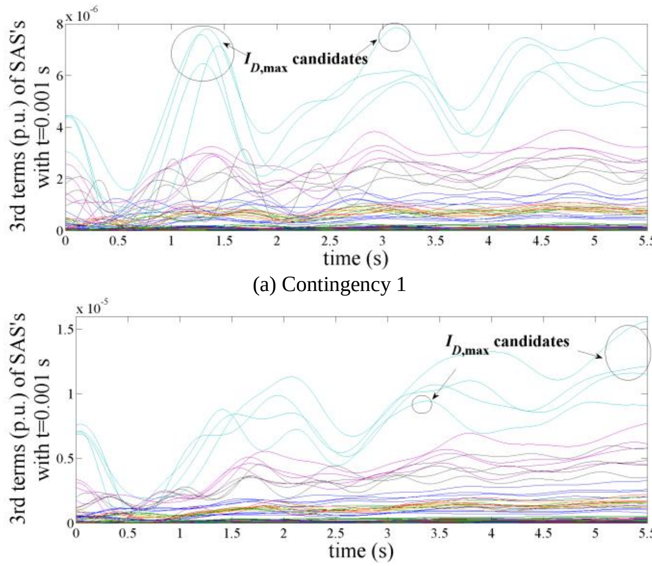
Fig. 12. Comparison of the simulations using the 6th-order generator model by the R-K 4 and the 2-term SAS using a fixed time window of 0.001 s.

B. *Adaptive Time Window*

The first step is to use a list of contingencies to determine an $I_{D,\max}$ that can guarantee the accuracy of an SAS and avoid numerical instability in simulation by the M-ADM. For the illustration purpose, the above contingency on line 3-4 and a second contingency adding a three-phase fault lasting 0.08 s on line 15-16 at bus 15 are considered. Consider the 3rd SAS term of each state variable (except the rotor angle) in per unit as an I_D. Fig. 13 plots the I_D's for all those state variables of 10 generators, where 3 random variations are added and the resulting trajectories are also plotted in the same figure. The effective $I_{D,\max}$ for two contingencies are found both associated with $|e'_{d5,2}|$, which are 6.5×10^{-6} and 9.4×10^{-6} (p.u.), respectively. Fig. 14 gives the result from a 3-term SAS evaluated over an adaptive time window, which is identical to the R-K 4 result.

Fig. 15 plots how the length of the time window changes with time during a 5.5-s simulation for three cases: 1) the 2-term SAS with an initial T=0.001 s, 2) the same SAS with an initial T=0.01s, and 3) the 3-term SAS with an initial T=0.001 s. The comparison of the cases 1) and 2) in Fig. 15 verifies that, if an adaptive time window is used, the accuracy of simulation is independent of the choice of the initial time window since the T of the case 2) adaptively decreases below 0.002 s soon after simulation starts. For the cases 1) and 2), the largest T reaches 0.0022 s. A main advantage of using an adaptive time window is that the total number of windows for evaluation is effectively reduced. The M-ADM using a fixed 0.001 s window evaluates 5500 windows to finish 5.5-s simulation while the case 1) using an adaptive time window only takes 4500 windows (i.e. 4500/5500=81.8%) to finish the same simulation period. For the case 3), the reduction of time windows is even more significant. As shown in Fig. 15, the largest T reaches 0.005 s, which is more than twice of the largest T for the 2-term SAS. Also, the total number of windows drops to 2000 (i.e. 2000/5500=36.4%). Thus, a conclusion is that using an adaptive time window enables the M-ADM to better exploit the advantage with a higher order SAS in terms of the reduction of the window number.

In the future development of a practical M-ADM based power system simulation tool, the optimal size of the time window and the proper number of SAS terms should be decided in a more adaptive way based on the information of the simulated power system to minimize the user intervention. It is not the focus of this proof-of-concept paper but will be addressed in the future work.



(a) Contingency 1

(b) Contingency 2

Fig. 13. Estimation of $I_{D,max}$.

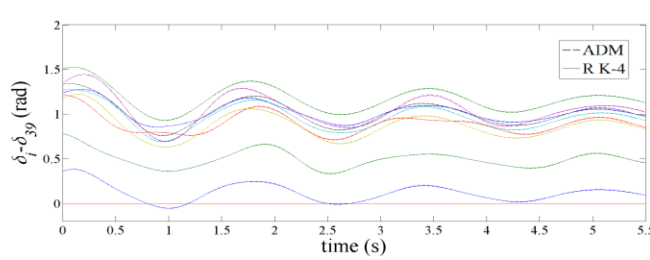


Fig. 14. Comparison of rotor angles given by the R-K 4 and the 3-term SAS using an adaptive time window initiated from 0.001 s.

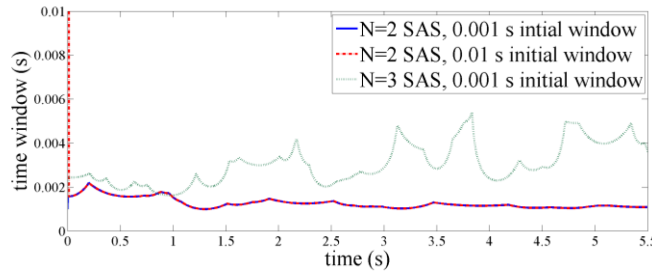


Fig. 15. Adaptive changing of time window length.

C. Time Performance

To demonstrate the time performance of the proposed SAS-based approach, the following three cases are tested:

- ***Case-A**:* only symbolizing time t and initial state variables, i.e. for one specific simulation.
- ***Case-B**:* beside *Case-A*, also symbolizing the reduced admittance matrix **Y** about 10 generator EMFs, i.e. for simulating different faults under one specific loading condition. Magnitudes and angles of elements of the reduced admittance matrix are symbolized separately to generate two symmetric symbolic 10×10 matrices.
- ***Case-C**:* beside *Case-B*, also symbolizing generators' mechanical powers to make the SAS be also good for simulating various loading conditions.

Here, the load at each bus is represented by a constant impedance load model and is embedded in the reduced admittance matrix **Y**. In the online stage, for a given power-flow condition with all loads known, load impedances will first be calculated, and then with the knowledge of the post-fault network topology, all elements of **Y** can be calculated in order to evaluate the SAS.

The offline stage is implemented in MAPLE and the online stage is performed in MATLAB. For 4th-order and 6th-order generator models, the numbers of CU's comprising the 3-term SAS's of each state variable are given in Tables III and IV, respectively, for three cases.

TABLE III
THE NUMBER OF CUS FOR THE 4TH-ORDER MODEL SYSTEM

State Variable	Case-A	Case-B	Case-C
ω_k	4,269	11,430	11,430
δ_k	150	150	150
e'_{qk}	225	301	301
e'_{dk}	223	299	299

TABLE IV
THE NUMBER OF CUS FOR THE 6TH-ORDER MODEL SYSTEM

State Variable	Case-A	Case-B	Case-C
ω_k	4,272	11,434	11,434
δ_k	150	150	150
e'_{qk}	227	303	303
e'_{dk}	223	299	299
E_{fd}	2,644	5,234	5,234
P_m	153	155	155

For *Case-A*, it only takes less than 3 μs to evaluate one CU. If all such CU's are evaluated simultaneously on parallel processors, it takes about 3 μs to evaluate one SAS for each time window plus the time costs for communication in parallel computing. Because summating the values of all CU's for a state variable is essentially the addition of constants, it is extremely fast. The additions for different state variables can also be performed in parallel. Thus, the final time for summating all CUs equals the time for the most complex SAS expression, often on a rotor speed, which only takes 7 μs. Therefore, the ideal total time cost for evaluations of state variables of one generator is 3+7=10 μs per time window *T*. If evaluations for various generators are also done simultaneously on an unlimited number of parallel processors, that time is also the time cost τ for SAS evaluation over each time window *T*. The R-K 4 method takes 0.37 s to finish a 5.5-s simulation with all generators represented by the 4th-order model on one computer processor. (It takes 0.48 s if all generators are represented by the 6th-order model.) Given the fact that a 3-term SAS only needs 2000 adaptive time windows for a 5.5-s simulation, it can be concluded that the online stage ideally only takes 0.00001×2000=0.02 s to finish simulation on parallel processors, which is about 18 times faster than the time cost of the R-K 4. Ratio T/τ =5.5/0.02=275, i.e. the number of times faster than wall-clock time. For *Case-B* and *Case-C*, T/τ =137.5 as given by Table IV, which indicates how many times the simulation can be faster than the wall-clock time.

By comparing Tables III and IV, it can be easily noticed that even after the exciter and governor models are added, the state variables that have the most CUs are still rotor speeds. Meanwhile, the number of CUs of each rotor speed's SAS only increases very slightly (by 3 for Case-A and 4 for Case-B and Case-C.) when the generator model changes from the 4th-order to the 6th-order. Basically, adding those details or controllers to each generator does not influence the online performance of the proposed approach.

The time performance of the offline stage is not as critical as the online stage, so it is evaluated in a sequential computing manner. Tables V and VI summarize the time performances of both offline and online stages for two systems respectively using the 4th and 6th order generator models under the assumption of an ideal parallel computing capability.

TABLE V
TIME PERFORMANCE ON THE 4TH-ORDER MODEL SYSTEM

	Case-A	*Case-B*	*Case-C*
Offline time cost (s)	198.05	682.18	711.17
Online time cost (s)	0.02	0.04	0.04
Ratio T/τ	275.0	137.5	137.5

TABLE VI
TIME PERFORMANCE ON THE 6TH-ORDER MODEL SYSTEM

	Case-A	*Case-B*	*Case-C*
Offline time cost (s)	6215.51	13472.91	16339.71
Online time cost (s)	0.02	0.04	0.04
Ratio T/τ	275.0	137.5	137.5

Considering that the number of parallel processors cannot be infinity in practice, we also studied how the time performance changes with the number of available processors. As theoretical estimates, ideal parallelism among all available processors is assumed. Thus, all processors are assumed to take equal computational burdens. The results are listed in Table VII for *Case-A* using 3-term SASs. From the table, when the number of processors drops to 100, the simulation time increases to 0.3 s, which is close to 0.37 s of the R-K 4. If the number of parallel processors is further decreased, the simulation using the M-ADM becomes slower than the R-K 4.

TABLE VII
INFLUENCE OF PARALLEL CAPABILITY ON TIME PERFORMANCE

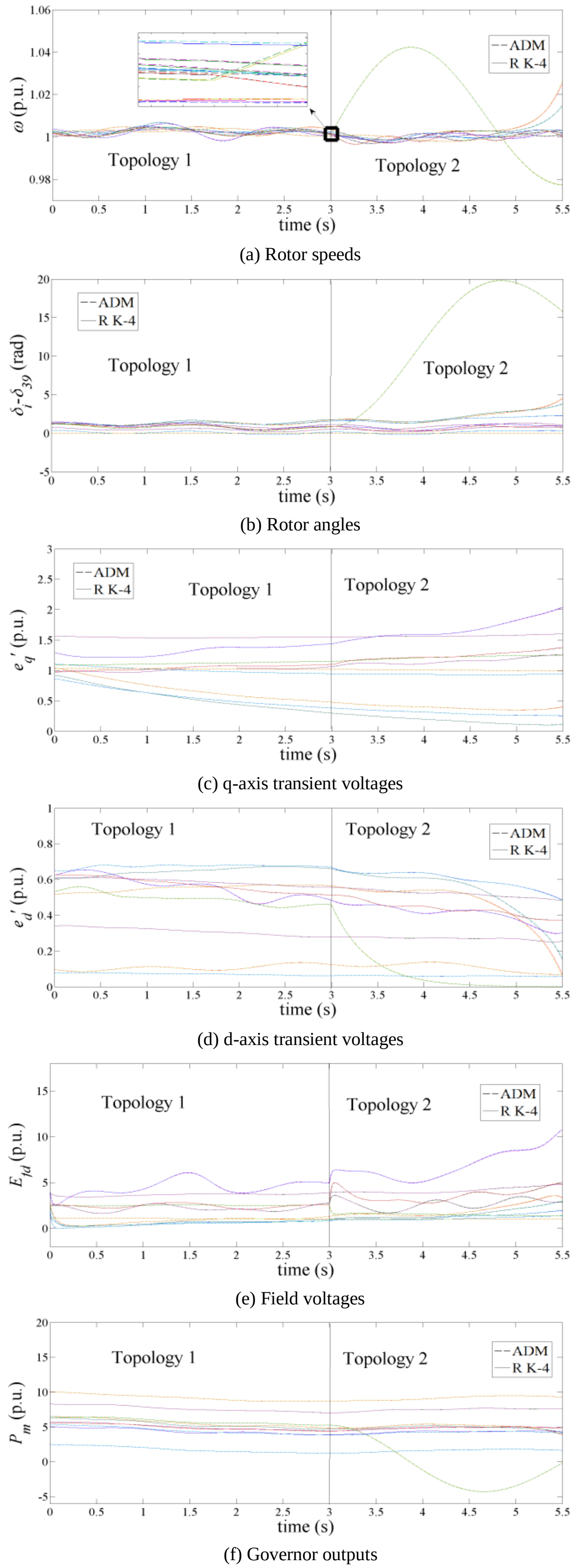
Number of Parallel Processors	Time Cost of Each Time Window (s)	Time Cost for a 5.5-s simulation (s)
∞	1.0×10^{-5}	2.0×10^{-2}
1000	1.4×10^{-5}	2.8×10^{-2}
100	1.5×10^{-4}	3.0×10^{-1}
10	1.5×10^{-3}	3.0

When a long list of contingencies need to be simulated, parallel processors may simulate multiple contingencies simultaneously, so power system simulation using the proposed SAS-based approach will be parallelized also at the contingency level besides the aforementioned CU level. Thus, a more sophisticated hierarchy for parallel implementation of the proposed SAS-based approach should be designed and will be addressed in the future work.

D. Simulation of a contingency with multiple disturbances

The proposed SAS-based approach can be used to simulate a contingency containing multiple disturbances, e.g. "n-1-1" and even "n-k" contingencies, which involve one or more disturbances during the simulation period. The same SAS can be used for the entire simulation period as long as all parameters that may change during the simulation period are defined as symbolic variables like an SAS from *Case-B* or *Case-C*.

In the following, we demonstrate how to use the SASs of *Case-B* to perform an "*n*-1-1" simulation involving a topological change of the system during the simulation period. The 6th-order generator models are adopted. The initial contingency is still the same as that in Fig. 9-Fig. 12 except that at *t*=3 s, the line 22-35 is opened, making the system have a different topology in the remaining 2.5 s. The SAS's derived for *Case-B* treat all elements of reduced **Y** matrix as symbolic variables. Therefore, at *t*=3 s, the time when topology changes, new values of the elements in the reduced **Y** matrix should be plugged into the SASs. The simulation results are shown in Fig. 16. Generator 35 loses its stability. The online time cost is 0.04 s with ideal parallelism on enough processors.



(a) Rotor speeds

(b) Rotor angles

(c) q-axis transient voltages

(d) d-axis transient voltages

(e) Field voltages

(f) Governor outputs

Fig. 16. Comparison of the simulation results with a topology change at t=3 s given by the R-K 4 and a 3-term SAS using an adaptive time window.

V. Conclusions

This paper has proposed a new approach for transient stability simulation, which is based on the SAS of power system DEs derived by the ADM. A two-stage implementation scheme, i.e. offline SAS solving and online SAS evaluation, was presented to minimize the online computational burden, and was validated using the IEEE 39-bus power system. Moreover, an approach using an adaptive time window for SAS evaluation is proposed to further reduce the time cost in the online stage. At the end, the performance of the proposed approach is tested and analyzed in a parallel computing framework. This new approach has potentials to be faster than a traditional R-K 4 based approach if parallel processors reach to a certain number.

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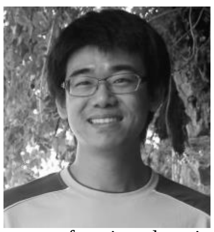
BIOGRAPHIES



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