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Measurement-based Voltage Stability Assessment for Load Areas Addressing n-1 Contingencies

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Abstract: This paper proposes an online measurement-based method for n-1 voltage stability assessment on a load area by means of an equivalent system of N+1 buses introduced by ref. [1] to represent N boundary buses of the load area and the external system. This method utilizes real-time synchro-phasor measurements on N boundary buses and online state estimation on the load area to analyze n-1 sensitivities for equivalent transfer admittances and power flows by means of a Modified Line Outage Distribution Factor (MLODF) in order to estimate the voltage stability margin of power transfer to each boundary bus under an anticipated n-1 condition. The method is demonstrated on the Northeast Power Coordinating Council 48-machine, 140-bus power system.

Index Terms: Voltage Collapse, Voltage Stability, Measurement-based Voltage Stability Assessment, n-1 Contingencies, Load Center.

1. Introduction

Prediction and mitigation of voltage instability is one of major concerns in today's power system operations [2]. Especially for a load center area supported by remote generation through tie lines, voltage instability may originate from a boundary bus and then spread out to wide-area or even system-wide voltage instability. The root cause of voltage instability is often loss of a key system component such as a tie line or a local generator in the load area. Thus, online Voltage Stability Assessment (VSA) for both the current condition and any foreseen n-1 condition is a necessary task in power system operations.

Among existing VSA techniques in literature, real-time Measurement-based VSA (MBVSA) as a data-driven approach has unique advantages compared to traditional model-based VSA. Many researchers have made significant contribution to the model-based VSA [3-7]. The drawbacks of model-based VSA include the dependency of accurate simulation models and a convergent power-flow solution as the basis for assessment. MBVSA methods typically use synchro-phasor measurements, e.g. PMU (phasor measurement unit) data, collected at the load buses or the boundary buses of a load area, and then online identify an equivalent of the system for estimating voltage stability margins on those buses. For instance, widely-used Thevenin equivalent based methods represent the system outside a load bus or a

load area by a Thevenin equivalent circuit [8-21]. Because voltage stability margin depends on the gap between the magnitudes of the Thevenin impedance and the load impedance, these methods use real-time measurements to estimate the Thevenin impedance and also calculate the load impedance so that voltage stability margin can be calculated directly without conducting simulation or powerflow analysis on the original system. The Thevenin equivalent based methods work well on a local load bus or the transmission corridor of a load pocket area, but may not be accurate for a load area supported by multiple tie lines [22]. In order to apply the Thevenin equivalent to a meshed power network, Ref. [16] decouples the network into many "decoupled single-port circuits" that each connects a load bus with an equivalent generator so as to estimate voltage stability margin at each load bus. However, this method ideally needs to deploy PMUs at all generators and all load buses (or at least the interested load buses for practical applications) to obtain an accurate network model.

Ref. [1] has extended the Thevenin equivalent, which is a 1+1 bus equivalent, to an equivalent with N+1 buses (called "the N+1 equivalent" in this paper) that is able to represent N tie lines and to calculate the voltage stability margin individually on each boundary bus.

Most of existing MBVSA methods only provide real-time voltage stability margin information for the current system condition but is unable to look ahead to provide voltage stability margin for foreseen critical n-1 conditions as model-based VSA methods do. Although model-based VSA methods provide margin information for n-1 conditions, they need to run contingency analysis by either time-domain simulation or powerflow analysis, which could be time consuming for a large system.

This paper will enhance the MBVSA method proposed in [1] to provide real-time voltage stability margin also for n-1 conditions using only measurement data. First, n-1 sensitivity analysis is conducted on the powerflow model of the system for insights on how the N+1 equivalent may vary under n-1 conditions. With the model and tie line flow under n-1 contingency updated, the n-1 voltage stability margins and the criticality of the post-contingency scenarios can be obtained using N+1 equivalent MBVSA.

To determine post-contingency quantities in performing contingency analysis, the sensitivity and Distribution Factor (DF) methods are widely utilized. Sauer found Line Outage Distribution Factor (LODF) using admittance values in [23] to determine the real power line flows. Then in [24], an improved technique based on phasor measurements was proposed. In [25], Ilic and Phadke used the decoupled method to estimate the distribution factors to determine reactive power line flows. In [26], Singh and Srivastava have obtained

the P-δ and Q-V relationship based on the load flow Jacobian matrix. In this paper, a Modified LODF has been proposed to provide the real power sensitivity of the tie-line flows.

The rest of the paper is organized as follows. Section 2 briefly introduces the *N*+1 equivalent and the corresponding MBVSA algorithms for the load areas. Section 3 presents two sensitivity analyses respectively on equivalent parameters and on tie-line power flows, by which margin information for an anticipated n-1 condition can be predicted. Section 4 validates the proposed approach using the NPCC system. Finally, conclusions are drawn in Section 5.

2. Measurement-Based VSA Using the *N*+1 Equivalent

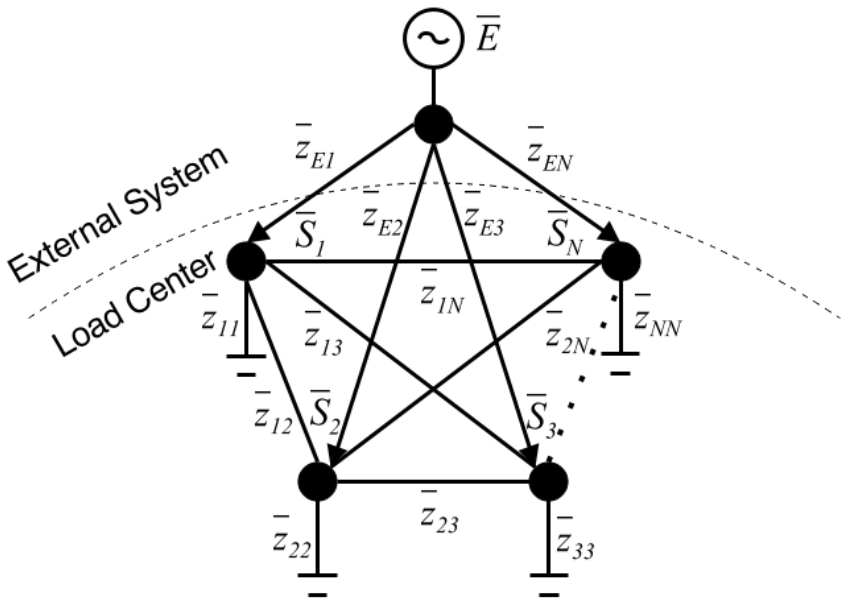

Fig. 1. *N+1 equivalent*

For multiple tie lines feeding a load area, their power transfer limits may be met at different time instants if the coupling relationship among their boundary buses is weak. The *N*+1 equivalent proposed in [1] retains the coupling relationship among boundary buses of a load area as in Fig. 1. The external system is represented by a single voltage source with phasor $\overline{E}$ connected by *N* branches with impedances $\bar{z}_{E1} \sim \bar{z}_{EN}$, and the load area is modelled by *N* equivalent loads with impedances $\bar{z}_{11} \sim \bar{z}_{NN}$ on boundary buses with connections of impedances $\bar{z}_{ij}$'s between them. The *N*+1 equivalent in [1] is extended from the widely-used Thevenin equivalent which represents the external system as a single equivalent voltage

source. The effectiveness of the Theveinin equivalent has been verified by a large number of papers and case studies. As long as the external system is strong and has no angular stability concern, it is usually valid to represent the external system by an adaptive equivalent voltage source to simplify voltage stability analysis on the load area. The reason is that a critical voltage source, e.g., a generator or generation area close to the load area, often dominates the voltage stability of the load area. Even if that voltage source may shift from one area to another with the change of operating conditions, real-time estimation of its equivalent parameters guarantees the equivalent voltage source always represents the most critical voltage source. As an extension from the Thevenin equivalent, the N+1 equivalent does not lose any information on the external system since its parameter estimation on the external voltage source is basically inherited and extended from the Thevenin equivalent; moreover, application of an N+1 equivalent enables more detailed voltage stability monitoring on the boundary of the load area.

Let $\bar{S}_i = P_i + jQ_i$ denote the complex power fed to boundary bus i and let $\bar{V}_i$ denote the bus voltage phasor. Using synchronized measurements on $\bar{S}_i$ and $\bar{V}_i$, all parameters of the equivalent can be identified on-line (e.g. every 0.1~1s) using the latest measurements of a sliding time window containing K measurement points $\bar{S}_i(k) = P_i(k) + \mathrm{j}Q_i(k)$ and $\bar{V}_i(k) = V_i(k)\angle\theta_i(k)$ (k=1, …, K) [1].

As all parameters are continuously updated by solving two optimization problems, each power transfer limit can be calculated analytically as an explicit function of all parameters of the N+1 equivalent, which has been derived in [1]. Then, assume the power factor over the time window to be constant for each load, active power transfer P_i is a function of all parameters of the equivalent and its maximum $P_{i,j}^{\mathrm{Max}}$ can be solved by letting its partial derivative with respect to the change of y_{jj} at bus j be zero, i.e.,

$$\frac{\partial P_i(y_{11}, \cdots, y_{NN})}{\partial y_{jj}} = 0 \qquad i, j=1, \ldots, N \tag{1}$$

Ref. [1] proved that the analytical solution of $P_{i,j}^{\mathrm{Max}}$ can be obtained by solving a quadratic equation. Thus, in real time, the active power limit of tie line i with respect to the change of the load at bus j can directly be calculated and compared with the actual active power for system operators' situational awareness or to trigger a preventive control action once their gap (i.e. voltage stability margin for the present condition) becomes insufficient.

3. Sensitivity Analyses for n-1 conditions

Even if the voltage stability margins on all tie lines are sufficient for the present condition, voltage instability may still be a threat under a foreseen n-1 contingency in the load area. This section will present the proposed method for n-1 voltage stability assessment based on n-1 sensitivity analyses for both the N+1 equivalent and real power tie-line flows. More specifically, the first sensitivity analysis is conducted on admittances of the equivalent system under n-1 contingencies inside the load area. The purpose is to predict the values of those admittances after a foreseen n-1 contingency in order to calculate power transfer limits for each tie line under the contingency using (1). Then, the second sensitivity analysis is conducted using a proposed Modified Line Outage Distribution Factor (MLODF) to predict the tie-line flow after that n-1 contingency. Thus, the power transfer margin under that n-1 contingency is determined by the predicted post-contingency transfer limit and power transfer.

3.1. n-1 Sensitivity Analysis on the Equivalent

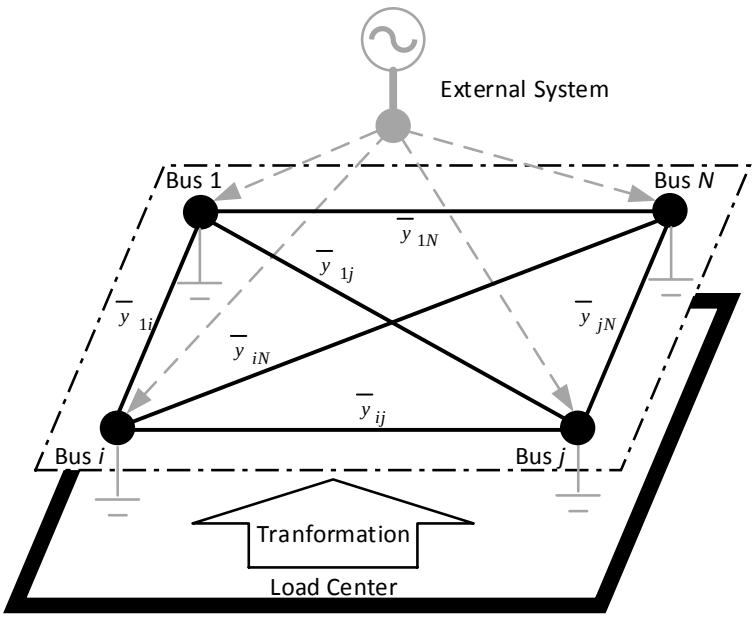

Fig. 2. *System transformation*

The N+1 equivalent can be derived from a reduction on the admittance matrix of the load area as follows. Firstly, replace the external system of the load area with N hypothetical voltage sources located at the N boundary buses, respectively. Assume that the load area has N boundary buses and R internal buses, whose admittance matrix $\mathbf{Y}_{bus}$ is $(R+N)\times(R+N)$. As shown in Fig. 2, reduce this matrix to the N boundary buses by eliminating elements

representing R internal buses. Thus, an N-bus complete graph is formed, which has a load admittance $\overline{y}_{ii}$ at each boundary bus and a transfer admittance $\overline{y}_{ij}$ between any two boundary buses.

In the following process of network reduction, all loads in the load area are considered as impedance loads and all generators in the load area will be considered as negative loads. Those loads and generators are admittances added to the diagonal elements of the matrix.

The nodal equation $\mathbf{I}_{bus} = \mathbf{Y}_{bus}\mathbf{V}_{bus}$ for the load area is:

$$\begin{bmatrix} \mathbf{I}_N \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{NN} & \mathbf{Y}_{RN} \\ \mathbf{Y}_{NR} & \mathbf{Y}_{RR} \end{bmatrix} \begin{bmatrix} \mathbf{V}_N \\ \mathbf{V}_R \end{bmatrix} \tag{2}$$

where $\mathbf{I}_N$ represents the vector of the currents injected to N boundary buses of the load area through N tie lines, and $\mathbf{Y}_{NN}$, $\mathbf{Y}_{RR}$, and $\mathbf{Y}_{RN}$ are respectively the admittance matrices about N hypothetical voltage sources $\overline{E}_1 \sim \overline{E}_N$, the internal load buses, and their connections.

Expand equation (2) to obtain,

$$\begin{cases} \mathbf{I}_N = \mathbf{Y}_{NN}\mathbf{V}_N + \mathbf{Y}_{RN}\mathbf{V}_R \\ 0 = \mathbf{Y}_{NR}\mathbf{V}_N + \mathbf{Y}_{RR}\mathbf{V}_R \end{cases} \tag{3}$$

from which vector $\mathbf{V}_R$ can be eliminated to find

$$\mathbf{I}_N = (\mathbf{Y}_{NN} - \mathbf{Y}_{NR}\mathbf{Y}_{RR}^{-1}\mathbf{Y}_{RN})\ \mathbf{V}_N \tag{4}$$

Therefore, the admittance matrix on the reduced N-bus system for the load area is

$$\mathbf{Y} = \mathbf{Y}_{NN} - \mathbf{Y}_{NR}\mathbf{Y}_{RR}^{-1}\mathbf{Y}_{RN} \tag{5}$$

Equation (5) actually provides a transformation from the original system to the equivalent system. To assess the sensitivity on $\mathbf{Y}$ under a line i-j admittance change, three possible locations of the line are considered. The resulting changes of $\mathbf{Y}$, denoted by $\Delta\mathbf{Y}$, are shown in Table 1.

Table 1 Sensitivity of the equivalent transfer admittances

Location of line i-j	$\Delta\mathbf{Y}$
$i, j \in N$	$\mathbf{Y}'_{NN} - \mathbf{Y}_{NN}$
$i, j \in R$	$\mathbf{Y}_{NR}(\mathbf{Y}'^{-1}_{RR} - \mathbf{Y}_{RR})\mathbf{Y}_{RN}$
$i \in N, j \in R$ or $i \in R, j \in N$	$\mathbf{Y}'_{NR}\mathbf{Y}'^{-1}_{RR}\mathbf{Y}'_{RN} - \mathbf{Y}_{NR}\mathbf{Y}_{RR}^{-1}\mathbf{Y}_{RN}$

Thus, the equivalent transfer admittances between any two boundary buses for the load area can be obtained from the estimated admittances matrix $\mathbf{Y}$ and its change $\Delta\mathbf{Y}$. As it is shown in Table 1, when the outaged line connects two boundary buses, there is $\Delta\mathbf{Y} = \mathbf{Y}_{NN}^{'} - \mathbf{Y}_{NN}$ since $\mathbf{Y}_{NN}$ is the only admittance changed in the 1st case. If there is no direct connection between the two ends of the outaged line and the boundary buses, we have $\Delta\mathbf{Y} = \mathbf{Y}_{NR}(\mathbf{Y}_{RR}^{'-1} - \mathbf{Y}_{RR})\mathbf{Y}_{RN}$. For the rest of the cases, the following relationship exists, $\Delta\mathbf{Y} = \mathbf{Y}_{NR}^{'}\mathbf{Y}_{RR}^{'-1}\mathbf{Y}_{RN}^{'} - \mathbf{Y}_{NR}\mathbf{Y}_{RR}^{-1}\mathbf{Y}_{RN}$. Thus, a sensitivity analysis for any n-1 contingency in the load area can be conducted based on the transfer admittances retrieved from $\mathbf{Y} + \Delta\mathbf{Y}$.

In order to obtain the limit under n-1 contingency, some equations in Ref. [1] need to be modified. Firstly, the voltage on boundary buses used for n-1 contingency sensitivity becomes

$$\mathbf{V} = \overline{E}\,\frac{adj(\mathbf{Y} + \Delta\mathbf{Y} + \mathbf{Y}_{\mathbf{E}}^{\mathbf{D}})}{\det(\mathbf{Y} + \Delta\mathbf{Y} + \mathbf{Y}_{\mathbf{E}}^{\mathbf{D}})}\mathbf{Y}_{\mathbf{E}} \tag{6}$$

For simplicity, let $\alpha_i^{'}$ denote the i-th element of $adj(\mathbf{Y} + \Delta\mathbf{Y} + \mathbf{Y}_{\mathbf{E}}^{\mathbf{D}})$ and let $\gamma^{'}$ = $\det(\mathbf{Y} + \Delta\mathbf{Y} + \mathbf{Y}_{\mathbf{E}}^{\mathbf{D}})$. Thus,

$$\overline{V}_i = \overline{E}\,\alpha_i^{'} / \gamma^{'} \tag{7}$$

According to the derivation in ref. [1], there is

$$P_i^{'}(\overline{y}_{11}, \cdots, \overline{y}_{NN}) = \left|\overline{E}\right|^2 \mathrm{Re}\left[\left(\overline{y}_{Ei}^{*} - \overline{y}_{Ei}^{*}\,\frac{\alpha_i^{'*}}{\gamma^{'*}}\right)\frac{\alpha_i^{'}}{\gamma^{'}}\right] \tag{8}$$

Based on the sensitivity analysis and the updated power injection expression, equation (1) can be used to calculate the power transfer limits for each tie line under n-1 contingency.

3.2. n-1 Sensitivity Analysis on Tie Line Flows

A Line Outage Distribution Factor (LODF) can be used to determine the impact of a line outage on other line flows without explicitly solving post-contingency power flows [27]. In this paper, a Modified Line Outage Distribution Factor (MLODF) is proposed to estimate the post-contingency real power transfers on tie lines with moderate computational burdens.

For a typical load center powered by the external system through multiple tie lines, the schematic Fig. 3 explains how the n-1 sensitivity analysis is conducted on tie-line flow S_{ij} when line *l-m* is outaged.

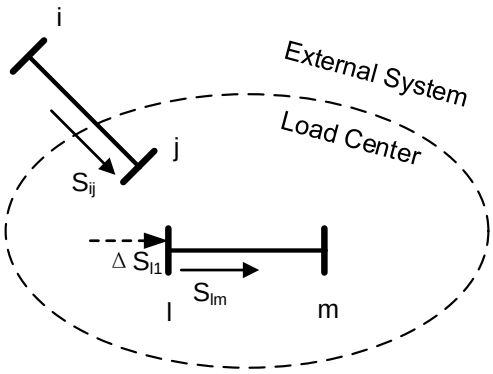

Fig. 3. *MLODF for load center*

According to the LODF defined in [27], the load center in Fig. 3 has a LODF

$$\tau_{ij,lm} = \frac{\Delta \bar{S}_{ij}}{\bar{S}_{lm}} = \frac{\rho'_{ij,l} - \rho_{ij,l}}{\rho_{lm,l}} \tag{9}$$

where $\Delta \bar{S}_{ij}$ is the power transfer change on the tie line from bus *i* to bus *j*, $\bar{S}_{lm}$ represents the power transfer of the outaged line. Variables $\rho'_{ij,l}$, $\rho_{ij,l}$ and $\rho_{lm,l}$ are the Power Transfer Distribution Factors (PTDFs), representing the sensitivity of line loading with respect to bus demands. Here $\rho_{ij,l}$ and $\rho'_{ij,l}$ are the PTDFs before and after the line *l*-*m* outage, respectively. A PTDF $\rho_{ij,k}$ relating the power transfer $\bar{S}_{ij}$ on the line from bus *i* to bus *j* with respect to the power injection at bus *k* $\bar{S}_k$ is denoted as

$$\rho_{ij,k} = \frac{\Delta \bar{S}_{ij}}{\Delta \bar{S}_k} \approx \frac{\partial \bar{S}_{ij}}{\partial \bar{S}_k} = \frac{\partial \left[(\bar{V}_i - \bar{V}_j) / \bar{z}_{ij} \right]^*}{\partial (\bar{I}_k^*)} \tag{10}$$

where, $\bar{z}_{ij}$ represents the transmission line impedance between bus *i* and bus *j*. Variables $\bar{V}_i$, $\bar{V}_j$ and $\bar{V}_k$ are the voltages on bus *i*, *j* and *k*, respectively. Current $\bar{I}_k$ represents the current injection into bus *k*. In the derivation of a formula to practically calculate PTDFs, Ref. [27] assumes all the buses have nearly unity bus voltages and hence considers the PTDFs as a function of only impedances of the system. However, for the voltage stability assessment, that approach may cause large errors since the voltages on the boundary buses as well as internal buses of the load area may not be close to 1 p.u., especially when the system is approaching the collapse point. In this paper, a modified formula (11) for calculating a PTDF is proposed, which retains voltages of all buses involved:

$$\rho_{ij,k} = \frac{\bar{V}_j}{\bar{V}_k} \frac{\left(\frac{\partial \bar{V}_i}{\partial \bar{I}_k} - \frac{\partial \bar{V}_j}{\partial \bar{I}_k} \right)^*}{\bar{z}_{ij}^*} = \frac{\bar{V}_j}{\bar{V}_k} \frac{(\bar{Z}_{ik} - \bar{Z}_{jk})^*}{\bar{z}_{ij}^*} \tag{11}$$

where, $\overline{Z}_{ik}$ and $\overline{Z}_{jk}$ are the element at the *i*-th row and *k*-th column, and the element at the *j*-th row and *k*-th column of the impedance matrix $\mathbf{Z}_{bus} = (\mathbf{Y}_{bus})^{-1}$, respectively.

Then, an LODF $\tau_{ij,lm}$ is calculated using a modified formula

$$\tau_{ij,lm} = \frac{\rho'_{ij,l} - \rho_{ij,l}}{\rho_{lm,l}} = \frac{\overline{V}_j}{\overline{V}_m} \frac{z^*_{lm}(Z'_{il} - Z'_{jl})^* - z^*_{lm}(Z_{il} - Z_{jl})^*}{z^*_{ij}(Z_{ll} - Z_{lm})^*} \tag{12}$$

where notations having a prime are based on the post-contingency impedance matrix $\mathbf{Z}'_{bus}$ of the system. With $\overline{V}_j$ and $\overline{V}_m$ obtained from either the state estimator or PMUs, the change on a tie-line flow can be calculated accurately. The result of (12) is named a MLODF (Modified LODF) in the rest of the paper.

In the proposed MBVSA method for n-1 conditions, the changes of all tie-line power transfers will be estimated by (10) and (12), which are much faster than power flow calculation and meet the requirements for online applications.

Remark: Ref. [21] also employs model-based sensitivity analysis to estimate post-contingency voltage and current phasors, and combines that analysis with a Thevenin equivalent based MBVSA method. Two steps are performed in order to calculate sensitivity indices in [21]: 1) calculate the Jacobian matrix on the system's power flow condition utilizing synchronized phasor measurements at all buses; 2) calculate sensitivity indices by the inverse matrix of the Jacobian matrix. However, when the Jacobian matrix becomes singular at the voltage collapse point, that approach has difficulty in calculating the sensitivity indices. Compared to the method in [21], the method proposed in this paper extends the *N*+1 buses equivalent system to analyze n-1 sensitivities of the system. Since this method only predicts post-contingency voltage stability for the tie lines of the load center, it needs much fewer measurements and has fewer computational burdens than the method in [21]. In addition, this method does not need to calculate the Jacobian matrix and its inverse matrix.

3.3. Implementation of the Proposed Method for n-1 Voltage Stability Assessment in a Hybrid Framework

The proposed method complements the traditional MBVSA techniques for the present condition and it improves operators' situational awareness on potential voltage instability under a foreseen n-1 condition in a vulnerable load area without conducting time-consuming n-1 contingency analysis or simulation. The voltage stability margin information for n-1

conditions is obtained based on two sensitivity analyses respectively on the N+1 equivalent and the tie-line flows as introduced in sections 3.1 and 3.2. Although the margin information may not be 100% accurate, this method is appropriate for an online preventive control scheme to trigger a remedial action when voltage instability under any credible n-1 contingency is predicted. A conservative threshold can be selected for the margin of each tie line considering the error due to sensitivity analysis. Another application of this method is an online contingency pre-screening tool to identify a critical set of n-1 contingencies whose occurrences may lead to negative or insufficient voltage stability margin. Then online contingency analysis or simulation can be conducted focusing only on those critical n-1 contingencies identified by this measurement-based method. In such a case, a hybrid scheme integrating this MBVSA method and a simulation-based VSA tool is proposed for online n-1 voltage stability assessment on a load area. The scheme conducts the following steps:

1) Obtain the most recent State Estimator (SE) solution (typically available every 1 to 3 minutes) to conduct two sensitivity analyses as introduced above;
2) Continuously (e.g. every 0.1 to 1 second) calculate the voltage stability margins for foreseen n-1 contingencies in the load area using PMU data collected at boundary buses of the load area;
3) If the n-1 margin given by the proposed measurement-based method on any tie line is found below a pre-set threshold (e.g. 10%), perform simulation-based VSA by power flow calculation (or dynamic simulation if time allows) on the corresponding n-1 contingency;
4) If the simulation-based VSA verifies the voltage instability following that n-1 contingency, the system does not meet n-1 security criteria any more, so a preventive control action should be taken to recover n-1 security of the system;
5) If the measurement-based margin drops to another even lower threshold (e.g. 5%) before simulation-based VSA finishes its computation, a preventive action needs to be taken immediately;
6) Whenever a new online SE solution is available (e.g. every 1 to 3 minutes), perform n-1 contingency pre-screening based on the solution and use this result to update the less accurate result from the sensitivity analysis on tie line flows.

Fig. 4 illustrates such a hybrid scheme for online VSA.

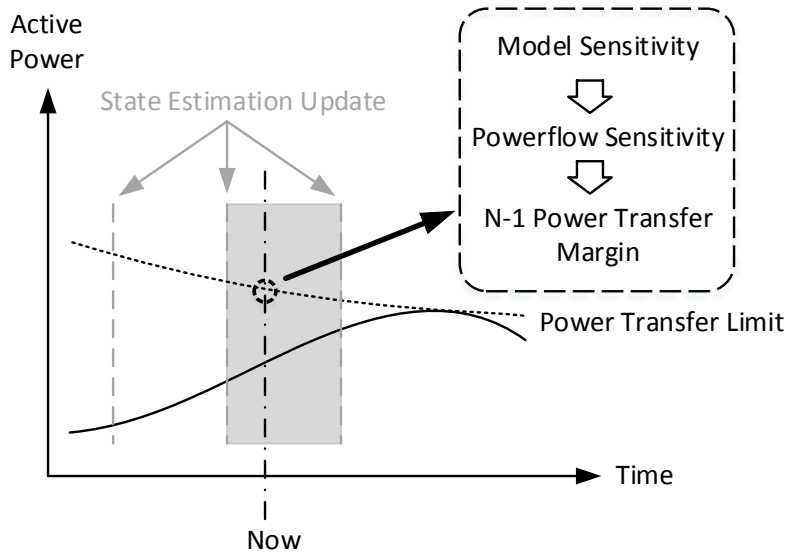

Fig. 4. Implementation of the hybrid VSA scheme for n-1 conditions

4. Case Studies

The Northeast Power Coordinating Council (NPCC) 48-machine, 140-bus system model is utilized to demonstrate the proposed MBVSA method for n-1 conditions on the Connecticut Load Center (CLC) area, which is supported by three tie lines, i.e. 73-35, 30-31 and 6-5 as shown in Fig. 5 [28]. The well-known IEEE 39-bus standard system is actually reduced from the NPCC 140-bus system used in this paper. Compared with the NPCC system, the IEEE 39-bus system retains the topology of the ISO-NE region completely and reduces the rest of the system including the transmission lines from the ISO-NE region to the NYISO region to an equivalent generator as bus 39. Powertech's TSAT is used to create simulated PMU measurements about the CLC area. Simulation results on the voltages at boundary buses 35, 31 and 5 and the complex powers of the three tie lines are recorded at a frequency of 30 Hz according to a typical PMU sampling rate. The raw data are preprocessed by an averaging filter over 15 samples to be downgraded to 2 Hz. The processed data are then fed to the proposed method to estimate the external and load-area parameters and to calculate transfer limits for foreseen n-1 contingencies. The parameter estimation is performed every 0.5 s on data of the latest 5-second time window. Two optimization models formulated in [1] are adopted and solved by the Sequential Quadratic Programming (SQP) method to estimate the parameters of the equivalent system. In practice, the optimization over the time window can effectively reduce the impact from errors or missing data with real PMU measurements.

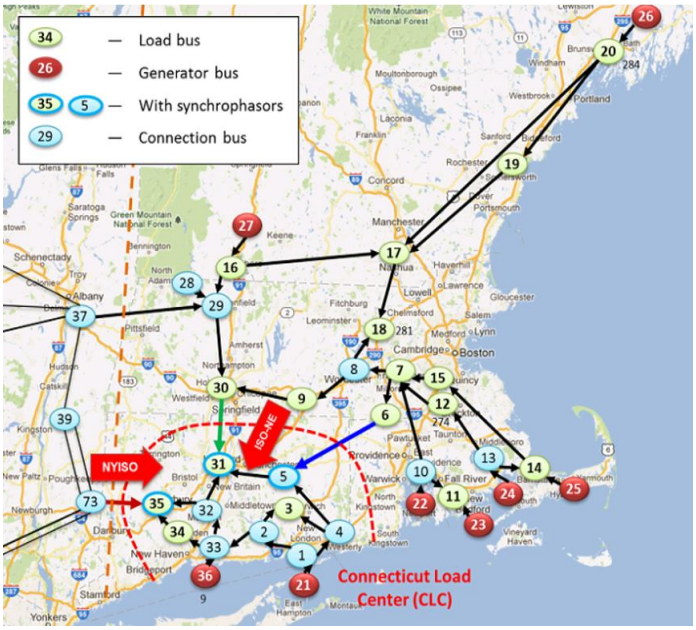

Fig. 5. *Map of CLC area*

The following n-1 scenario is selected to demonstrate the proposed method: all loads of the load area are uniformly increased by a total of 1.53 MW per second from its original load of 1906.5 MW with constant load power factors. At t=200 s, the transmission line between buss 4 and 5 is tripped, which pushes the system to be closer to the voltage stability limit. After another 200 s load increase, voltage collapse happens around t=400 s as shown in Fig. 6 on three boundary-bus voltages. Fig. 7 indicates the P-V (power-voltage) curves drawn using the measurements at three boundary buses. In the NPCC system, the load is not represented by the constant power load model. With the ZIP load model, the system may pass the “nose” point of a P-V curve and even operate on the lower part of that curve. The discontinuities of the P-V curves are caused by the n-1 contingency.

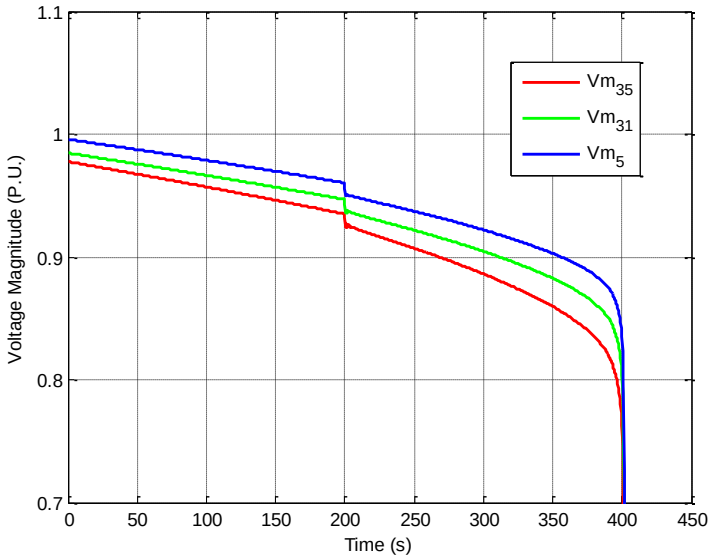

Fig. 6. *Voltage magnitudes at CLC boundary buses*

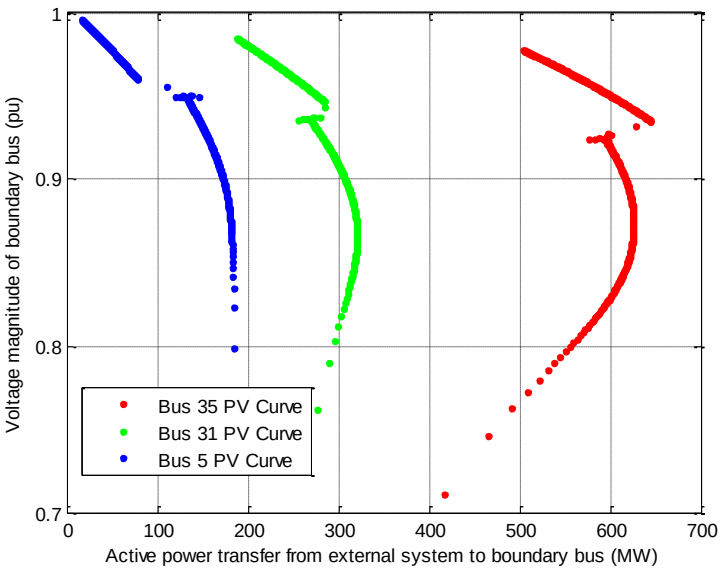

Fig. 7. *PV curves*

4.1. n-1 Sensitivity Analysis on the Equivalent

The load area is represented by a 3-bus complete graph having three transfer admittances connecting three boundary buses and three loads respectively at three boundary buses. Fig. 8 and Fig. 9 compare the changes of the conductances and susceptances of three transfer admittances due to this n-1 contingency. Their values are calculated by the approach in Section 3.1. These two figures clearly show that the transfer admittances will remain constant unless a topological change happens in the load area.

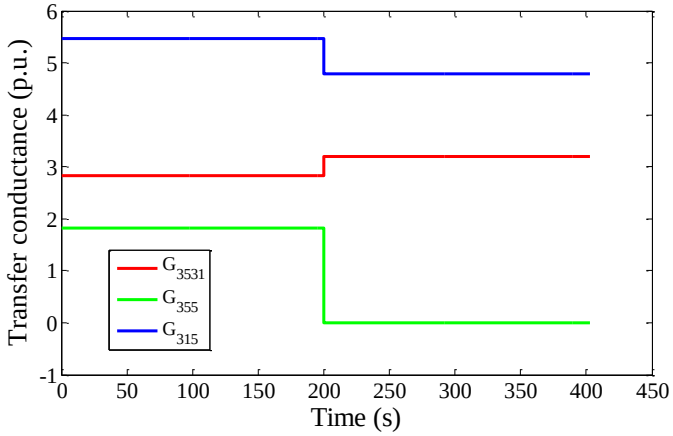

Fig. 8. *Transfer conductance comparison*

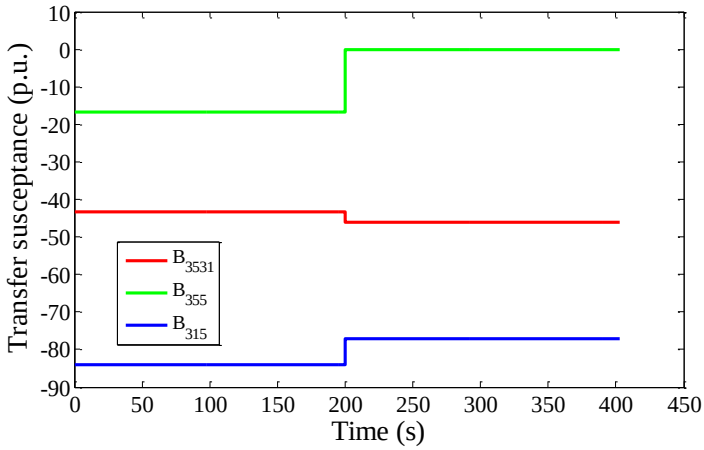

Fig. 9. *Transfer susceptance comparison*

4.2. n-1 sensitivity analysis on tie line flows

The MLODF calculated by formula (12) is utilized to predict tie-line power flows for the post-contingency condition. For the NPCC 48-machine, 140-bus system model, there are 12 n-1 line outage contingencies in the load area. All 12 contingencies are tested on a desktop computer with Intel Xeon E5620 2.4GHz CPU and 48GB RAM. The PSAT software by Powertech Labs needs more than 5 seconds to finish the n-1 power flow calculation for 12 cases using Newton-Raphson (NR) method. However, the proposed measurement-based method only takes 0.124 seconds to significantly save the computational time. For the most critical tie line 73-35, Table 2 lists the errors of line flows estimated by MLODF method compared to the actual line flows calculated by the Newton-Raphson (NR) method. All errors are below 3%, showing a good accuracy with MLODF method.

Table 2 Errors in line flows estimated by the MLODF

Contingency	Error	Contingency	Error
Line 4-5 trip	0.56%	Line 33-34 trip	-1.92%
Line 5-31 trip	-2.59%	Line 1-2 trip	-2.51%
Line 1-4 trip	-0.90%	Line 34-35 trip	-0.52%
Line 32-33 trip	-2.03%	Line 32-35 trip	2.09%
Line 2-33trip	2.07%	Line 3-4 trip	-0.17%
Line 31-32 trip	2.19%	Line 2-3 trip	-0.04%

4.3. Estimation of n-1 Voltage Stability Margin

Fig. 10 gives the results from the proposed MBVSA method for n-1 conditions. Each tie line has three n-1 limits respectively on three potential directions of load increase (i.e. near buses 35, 31 and 5). Before the trip of line 4-5, the method keeps computing totally nine power transfer limits under the n-1 condition corresponding to that line tripping, and all tie lines have sufficient margins to their limits. As shown in Fig. 10, voltage stability limits for that anticipated n-1 condition (line 4-5 trip) are the blue dashed lines starting at t=200s. When that line tripping actually happens, the MBVSA for the present condition can immediately estimate the real-time stability limits of the three tie lines. From the figure, the active powers of the three tie lines become very close to their limits. Compared with the real-time limits, all nine n-1 limits predicted before t=200s are fairly accurate.

After the line tripping happens, P_{35} of tie line 73-35 is the most critical tie line as it has the smallest margin. As loads keep increasing, P_{35} reaches the limit $P_{35,5}^{\mathrm{Max}}$ at t=307 s. From Fig. 10 (b) and (c), the other two lines keep positive margins until the final voltage collapse happens around t=400 s.

The aforementioned monitoring for each anticipated n-1 condition offers an effective way for contingency pre-screening so that more detailed power flow analysis or time-domain simulation is only needed for very few critical contingencies. The n-1 limit and margin information on individual tie line from this proposed measurement-based method will tell the operators in real time which contingency will be critical for further, more detailed analysis. With this information, a huge number of non-critical simulations will be skipped and only the most critical ones will be analyzed online. Therefore, system operators will have enough time to run the simulation and find the optimal remedial action before system collapse.

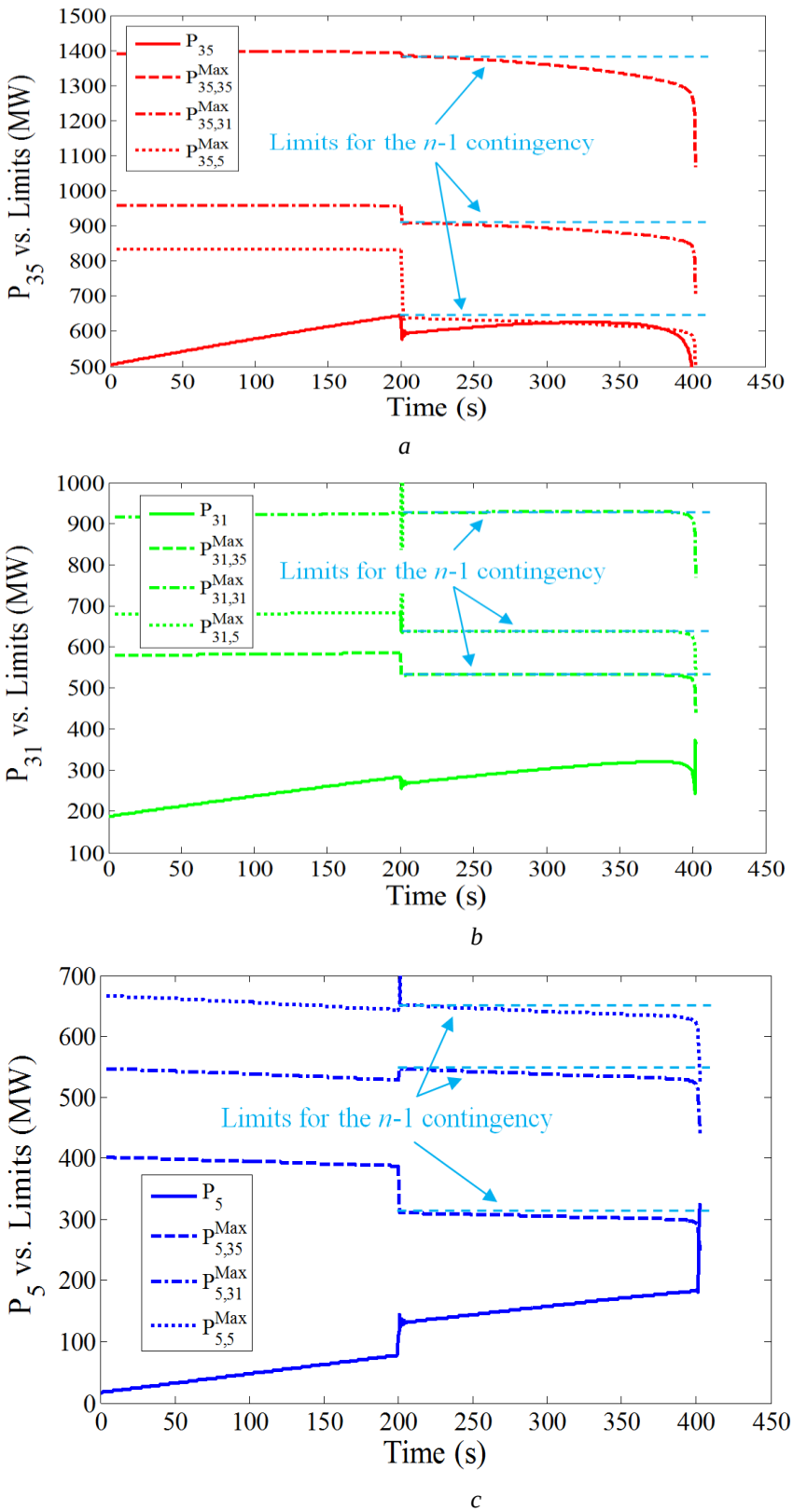

Fig. 10. *Transfer limits of each tie line under n-1 line trip contingency*
a P35 vs. its limits
b P31 vs. its limits
c P5 vs. its limits

4.4. n-1 Contingency Sensitivity Analysis Study with Preventive Control

In the framework of a hybrid scheme, preventive control against the worst line tripping is demonstrated in this section on the 140-bus NPCC system. This scenario has load of the area increased by 1.53 MW per second; the line 4-5 is tripped at *t*=200 s; after additional 200 s load increase, voltage collapse happens around *t*=400 s. The simulation-based VSA module is performed on the power flow condition obtained before the actual line tripping for all n-1 contingencies in the load area. Table 3 lists the smallest tie-line power transfer margins under all n-1 contingencies from the simulation-based VSA. From the result, the worst contingency is identified to be line 4-5 tripping. In the following, we will demonstrate how the proposed MBVSA method can predict that most critical contingency and take a timely remedial action to prevent the system from voltage collapse.

Table 3 Margins of all n-1 line trip contingency at 200s

Contingency	Margin(MW)	Contingency	Margin(MW)
Line 4-5 trip	33	Line 33-34 trip	67
Line 5-31 trip	54	Line 1-2 trip	67
Line 1-4 trip	57	Line 34-35 trip	72
Line 32-33 trip	60	Line 32-35 trip	75
Line 2-33trip	62	Line 3-4 trip	76
Line 31-32 trip	63	Line 2-3 trip	78

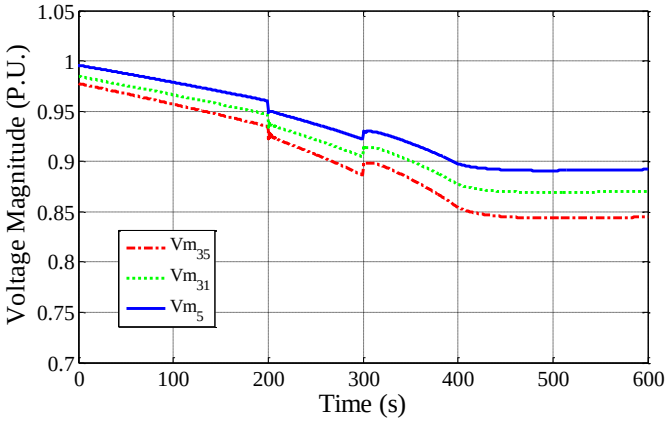

Fig. 11. *Voltages of three boundary buses*

The scenario is demonstrated in three stages corresponding to the three operating conditions as shown in Tables 4.

Table 4 Stages of the simulated instability scenario

Stage Number	Description
1	No Contingency
2	Line 4-5 tripped
3	Remedial action triggered

In Stage 1, there is no contingency and the system is operating securely under n-1 criteria. Note that the n-1 limit for the worst contingency (i.e. line 4-5 trip) is provided to the operator by the measurement-based module when the n-1 margin is lower than a pre-designed threshold. In addition, the limit for the current operating condition is also calculated by the measurement-based module. At this stage, the value of MBVSA for the operator is to monitor voltage stability limits for the current condition (i.e. n-0 condition), and take actions if margin is insufficient.

In Stage 2, upon the line 4-5 tripping, the system operates under a contingency. Immediately after the line tripping, the limits from MBVSA change, informing the operator happening of an event. Note that the MBVSA at this stage is especially valuable immediately after the event and before the simulation-based module finishes its computation. The MBVSA tells the operator whether there is still sufficient margin for the present operating condition and whether preventive control is needed. Once the simulation-based module finishes its calculation of accurate n-1 limits, its results will update the predicted n-1 limits from the MBVSA.

In Stage 2, if the simulation-based module does not finish its computation on more accurate n-1 limits before a zero or very small margin is predicted by the MBVSA, the MBVSA will trigger a preventive control action automatically to prevent voltage collapse. If the simulation-based module finishes its computations for predicting the voltage instability and for the determination of a remedial action, the action will be taken and the system will enter Stage 3. Note that even if the results from the simulation-based module are available, the MBVSA module is still important because it provides situational awareness for the operator on the criticality of the system condition in real time especially when there is not enough time to perform any additional simulation.

In Stage 3, the effect of a remedial action is simulated as shown in Fig. 12. The remedial action switches in additional reactive power sources at bus 33 at t=300 s. It can be observed that voltage collapse is prevented, the system is no longer under emergency

condition and the operators can take additional actions to bring the system back to a secure operating condition.

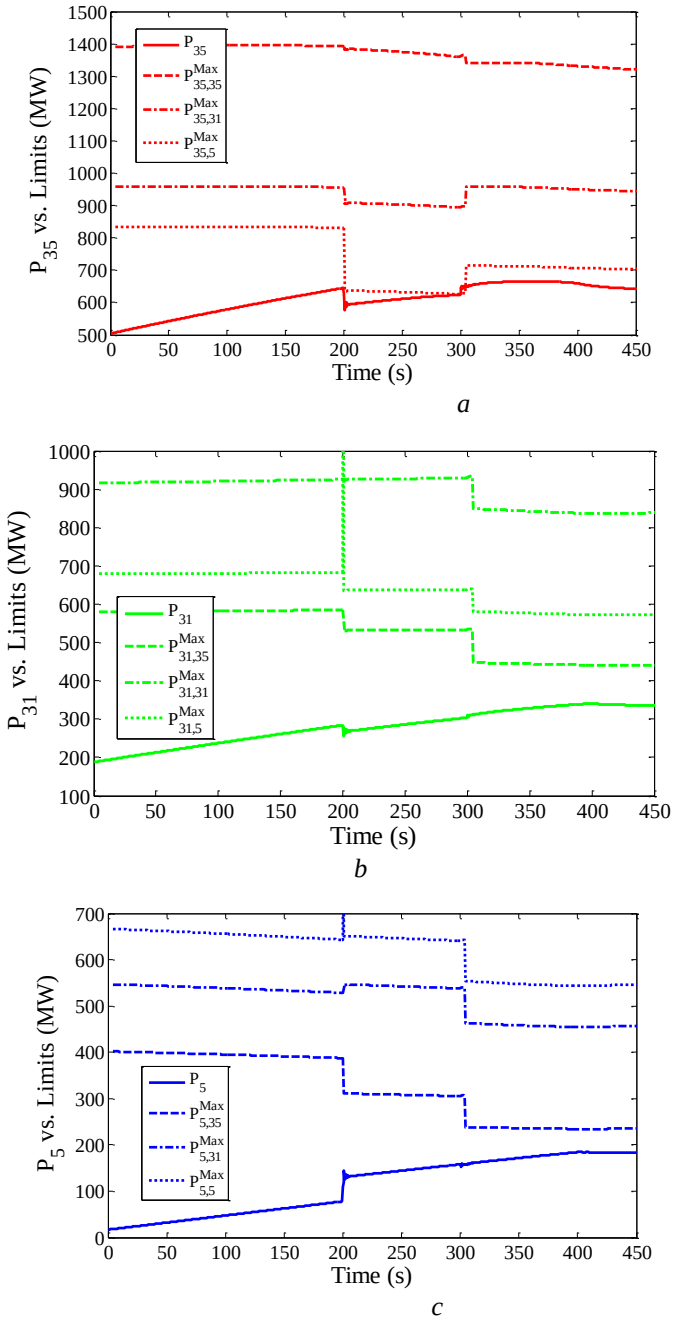

Fig. 12. *Scenario demonstration*
a P35 vs. its limits
b P31 vs. its limits
c P5 vs. its limits

5. Conclusion

Two n-1 sensitivity analyses have been proposed for the parameters of the *N*+1 equivalent and for tie-line flows of the load area in order to provide voltage stability margin information using MBVSA for n-1 conditions. The proposed approach has been demonstrated

on the NPCC 140-bus system for prediction of voltage instability under anticipated n-1 contingencies and for preventive control against voltage collapse. How to implement this approach in a hybrid VSA framework together with simulation-based VSA is also presented.

6. Acknowledgment

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7. References

[1] F. Hu, K. Sun, A. D. Rosso, E. Farantatos, and N. Bhatt, "Measurement-Based Real-Time Voltage Stability Monitoring for Load Areas," *IEEE Trans. Power Systems*, vol. 31, No. 4, pp. 2787-2798, July 2016.

[2] T. Van Cutsem, and C. Vournas, *Voltage Stability of Electric Power Systems*. Norwell, MA: Kluwer, 1998.

[3] V Ajjarapu, and C. Christy, "The Continuation Power Flow: A Tool for Steady State Voltage Stability Analysis," *IEEE Transactions on Power Systems*, vol. 7, no. 1, pp. 416-423, Feb. 1992.

[4] H. D. Chiang, A. J. Flueck, K. S. Shah, and N. Balu, "CPFLOW: A Practical Tool for Tracing Power System Steady-state Stationary Behavior due to Load and Generation Variations," *IEEE Trans. Power Syst.*, vol. 10, no. 2, pp. 623–634, May 1995.

[5] N. Flatabo, R. Ognedal, and T. Carlsen, "Voltage Stability Condition in a Power Transmission System Calculated by Sensitivity Methods," *IEEE Transactions on Power Systems*, vol. 5, no, 4, pp. 1286-1293, Nov. 1990.

[6] S. Greene, I. Dobson, and F. Alvarado, "Sensitivity of the Loading Margin to Voltage Collapse with Respect to Arbitrary Parameters," *IEEE Transactions on Power Systems*, vol. 12, no. 1, pp. 262–272, Feb. 1997.

[7] T. Van Cutsem, L. Wehenkel, et al., "Decision Tree Approaches to Voltage Security Assessment," *IEE Proceedings C Generation, Transmission and Distribution*, vol. 140, no. 3, pp. 189-198, 1993.

[8] K. Vu, M. M. Begovic, D. Novosel et al., "Use of Local Measurements to Estimate Voltage-stability Margin," *IEEE Trans. Power Systems*, vol. 14, no. 3, pp. 1029-1035, 1999.

[9] I. Smon, G. Verbic, and F. Gubina, "Local Voltage-stability Index Using Tellegen's Theorem," *IEEE Trans. Power Systems*, vol. 21, no. 3, pp. 1267-1275, 2006.

[10] B. Milosevic, and M. Begovic, "Voltage-stability Protection and Control Using a Wide-area Network of Phasor Measurements," *IEEE Trans. Power Systems*, vol. 18, no. 1, pp. 121-127, 2003.

[11] S. Corsi, and G. N. Taranto, "A Real-time Voltage Instability Identification Algorithm Based on Local Phasor Measurements," *IEEE Trans. Power Systems*, vol. 23, no. 3, pp. 1271-1279, 2008.

[12] C. D. Vournas, and N. G. Sakellaridis, "Tracking Maximum Loadability Conditions in Power Systems," *in Proc. 2007 Bulk Power System Dynamics and Control-VII*, Charleston, SC, Aug. 2007.

[13] M. Glavic, and T. Van Cutsem, "Wide-area Detection of Voltage Instability from Synchronized Phasor Measurements. Part I: Principle," *IEEE Trans. Power Systems*, vol. 24, no. 3, pp. 1408-1416, 2009.

[14] S. M. Abdelkader, and D. J. Morrow, "Online Thévenin Equivalent Determination Considering System Side Changes and Measurement Errors," *IEEE Trans. Power Systems*, vol. PP, no. 99, pp. 1-10, 2014.

[15] W. Li, Y. Wang, and T. Chen, "Investigation on the Thevenin Equivalent Parameters for Online Estimation of Maximum Power Transfer Limits," *IET Generation, Transmission & Distribution*, vol. 4, no. 10, pp. 1180-1187, 2010.

[16] Y. Wang, I.R. Pordanjani, et al., "Voltage Stability Monitoring Based on the Concept of Coupled Single-port Circuit," *IEEE Trans. Power Systems*, vol. 26, no. 4, pp. 2154-2163, 2011.
[17] J.-H. Liu, and C.-C. Chu, "Wide-area Measurement-based Voltage Stability Indicators by Modified Coupled Single-port Models," *IEEE Trans. Power Systems*, vol. 29, no. 2, pp. 756-764, 2014.
[18] W. Xu, I. R. Pordanjani, et al., "A Network Decoupling Transform for Phasor Data Based Voltage Stability Analysis and Monitoring," *IEEE Trans. Smart Grid*, vol. 3, no. 1, pp. 261–270, Mar.2012.
[19] I. R. Pordanjani, et al, "Identification of Critical Components for Voltage Stability Assessment Using Channel Components Transform," *IEEE Trans. Smart Grid*, vol. 4, no. 2, pp. 1122-1132, 2013.
[20] K. Sun, F. Hu, and N. Bhatt, "A New Approach for Real-time Voltage Stability Monitoring Using PMUs," *2014 IEEE ISGT - Asia*, Kuala Lumpur, 20-23 May 2014.
[21] H. Yuan, and F. Li, "Hybrid Voltage Stability Assessment (VSA) for n-1 Contingency," *Electric Power Systems Research*, vol. 122, pp. 65–75, May. 2015.
[22] F. Hu, K. Sun, et al, "An Adaptive Three-bus Power System Equivalent for Estimating Voltage Stability Margin from Synchronized Phasor Measurements," *IEEE PES General Meeting*, 2014.
[23] P. Sauer, "On the Formulation of Power Distribution Factors for Linear Load Flow Methods," *IEEE Transactions on Power Apparatus and Systems*, vols. PAS-100, no. 2, 1981.
[24] Y. C. Chen, A. D. Dominguez-Garcia, and P. Sauer, "Online Estimation of Power System Distribution Factors-A Sparse Representation Approach," in North American Power Symposium, Manhattan, KS, USA, 2013.
[25] M. Ilic-Spong, and A. Phadke, "Redistribution of Reactive Power Flow in Contingency Studies," *IEEE Transactions on Power Systems*, vols. PWRS-1, no. 3, 1986.
[26] S. N. Singh, and S. C. Srivastava, "Improved Voltage and Reactive Power Distribution Factors for Outage Studies," *IEEE Transactions on Power System*, vol. 12, no. 3, 1997.
[27] G. T. Heydt, *Computer Analysis Methods for Power Systems*, MacMillan Pub. Co., 1986.
[28] J. H. Chow, R. Galarza, P. Accari, et al., "Inertial and Slow Coherency Aggregation Algorithms for Power System Dynamic Model Reduction," *IEEE Trans. Power Systems*, vol. 10, no. 2, pp. 680-685, 1995