Multi-Layer Interaction Graph for Analysis and Mitigation of Cascading Outages

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***Abstract*—This paper proposes a multi-layer interaction graph on cascading outages of power systems as an extension of a single-layer interaction network proposed previously. This multi-layer interaction graph provides a practical framework for prediction of outage propagation and decision making on mitigation actions. It has multiple layers to respectively identify key components and key intra-layer links of components within each layer and key inter-layer links between layers, which contribute the most to outage propagation. Each layer focuses on one of several aspects that are critical for system operators' decision support, such as the number of line outages, the amount of load shedding, and the electrical distance of outage propagation. Besides, the proposed integrated mitigation strategies can limit the propagation of cascading outages by weakening key intra-layer links. All layers are constructed offline from a database of simulated cascades and then used online. A three-layer interaction graph is presented in detail and demonstrated on the Northeastern Power Coordinating Council 48-machine 140-bus system. The key intra- and inter-layer links and key components revealed by the multi-layer interaction graph provide useful insights on the mechanism and mitigation of cascading outages, which cannot be obtained from any single-layer.**

***Index Terms*—Blackout, cascading outage, line outage, load shedding, electrical distance, multi-layer interaction graph.**

I. Introduction

CASCADING outages in power grids may lead to widespread power outages, such as the North America blackout on August 14th, 2003 in North America [2], the Brazil blackout on November 10th, 2009 [3], and the India blackout on July 30th and 31st, 2012 [4]. The simulation, analysis, and mitigation of cascading outages pose a great challenge [5]–[9]. To simulate cascading outages, several models have been proposed, such as CASCADE model [10]–[13], branching process model [14]–[18], OPA model [19]–[21], improved OPA model [22], AC OPA model [23], OPA model with slow process [24], Manchester model [25], [26], and hidden failure model [27], [28], and sandpile model [29]. These models are mainly used in offline studies to discover the mechanisms cascading blackouts, they can only provide very limited information for on-line control or mitigation actions. The commercial TRELSS program [30], [31] can be used to analyze cascading outages by performing detailed simulations that consider protection and operator actions. However, its high computational complexity cannot meet the requirements for online applications. Thus, an advisable approach for online analysis and mitigation of cascading outages would be utilization of a high-level model that is established offline directly from a comprehensive database on historical or simulated cascading events.

Paper [32] proposes an influence model to quantify the influences between the components of a network. Paper [33] quantifies the interactions between transmission lines using a line interaction graph. For fast simulation and analysis of cascading outages, paper [1] constructs an interaction network from a database of cascade outages to capture key components and key links between component outages that play critical roles in outage propagation. The interaction network is useful for understanding the general patterns of outage propagation and has potentials in online applications. An interaction network is also built for an NPCC power system test bed [34].

Due to the computational burdens, it is impractical to online simulate many potential cascading outages and probable mitigation actions. An alternative approach is the combination of offline simulation and prediction and online monitoring and control. Offline analysis is mainly performed on a large number of samples of cascading outages that are either historical outage events or simulated using detailed models. The valuable information extracted from the samples includes the outage components involved, propagation paths of outages, amounts of load shedding, geographic distances between two outages, etc., which if presented together can provide system operators with a comprehensive picture on the propagation patterns of outages and take effective control actions. Paper [1] develops a single-layer interaction network based on the components involved in outages but does not utilize other types of information.

This paper proposes a multi-layer interaction graph as the extension and generalization of the single-layer interaction network in [1]. Different from that interaction network, this multi-layer interaction graph integrates multiple layers that respectively identify the key intra-layer links and components in each layer contributing the most to outage propagation from various perspectives, e.g. the number of line outages, the amount of load shedding and the electrical distance on outage

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propagation. Besides, key inter-layer links that connect components from different layers are also defined to model the transition from one type of consequences to another type. All key intra-layer links and components and key inter-layer links and components together provide comprehensive information on the dominant outage propagation patterns, based on which effective mitigation measures can be further developed.

The multi-layer interaction graph can be obtained offline from a database of historical or simulated samples of cascading outages. Then in the real-time operation environment, with the propagation of component outages, the most critical layer can be identified and the corresponding single-layer control strategy can be designed to weaken the key intra-layer links so as to mitigate cascading outages. Besides, two integrated mitigation strategies involving key intra-layer links in different layers are also proposed and applied to mitigate cascading outages in different stages. Without loss of generality, in this paper we develop an interaction graph with three layers to capture the key intra-layer links and components of each layer in terms of the number of line outages, the amount of load shedding, and the electrical distance of outage propagation. The key inter-layer links and components are also identified. The rest of this paper is organized as follows. Section II introduces the proposed multi-layer interaction graph and the method for identifying key intra- and inter-layer links and components. Also, a method is proposed to determine the minimum number of samples needed by a database for constructing a reliable multi-layer interaction graph. Section III presents strategies to mitigate the propagation of cascading outages and discuss online applications of the proposed multi-layer interaction graph. Section IV demonstrates the multi-layer interaction graph obtained from a simulated database of cascading outages on a Northeastern Power Coordinating Council (NPCC) 48-machine 140-bus system. Finally, conclusions are drawn in Section V.

II. Proposed Multi-Layer Interaction Graph

The schematic diagram of the multi-layer interaction graph is illustrated in Fig. 1, which has three layers respectively in terms of the number of line outages, the amount of load shedding and the electrical distance of outage propagation. In each layer of Fig. 1, the key intra-layer links and components for one power grid are highlighted respectively by thick arrows and lines. Note that the number of key intra-layer links and components can vary based on different selection thresholds, which will be discussed in Section II.C. Fig.1 presents a simple scenario on the propagation of cascading outages within and across layers. As shown in Fig. 2, the path of that scenario involves five components C_1, C_2,..., and C_5 connected by four directed intra-layer links l_1, l_2, l_3, and l_4, which respectively belong to the 1st, 2nd, 3rd and 2nd layers. This entire process of cascading outages can only be observed from the multi-layer interaction graph while any single-layer can only provide partial information for the entire process. The multi-layer interaction graph generated offline provides a big picture on propagation patterns of cascading outages, which is valuable for the system operators to take effective control actions.

In this paper, three control strategies that weaken key intra-layer links located in each layer are studied to mitigate cascading outages respectively from three perspectives, i.e. the number of line outages, the amount of load shedding, and the propagated electrical distance. Besides, integrated mitigation strategies involving key intra-layer links from different layers can be applied.

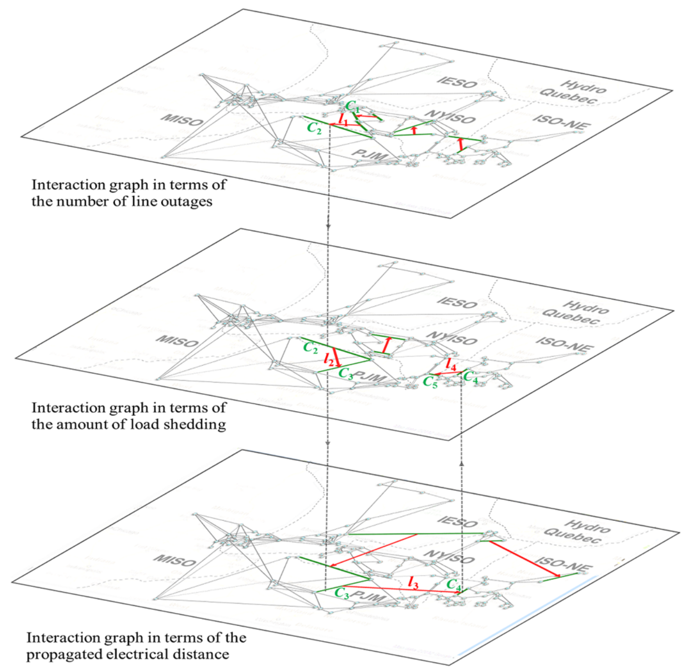

Fig. 1. Illustration on cascade outages propagating in/across multiple layers of an interaction graph about the NPCC power system (thick red arrows and green lines are key intra-layer links and key intra-layer components; broken lines with arrows indicate transitions between layers)

$$C_1 \xrightarrow{l_1} C_2 \xrightarrow{l_2} C_3 \xrightarrow{l_3} C_4 \xrightarrow{l_4} C_5$$

Fig. 2. Path of a sample scenario of cascade outages.

A. Database of Cascades and Links

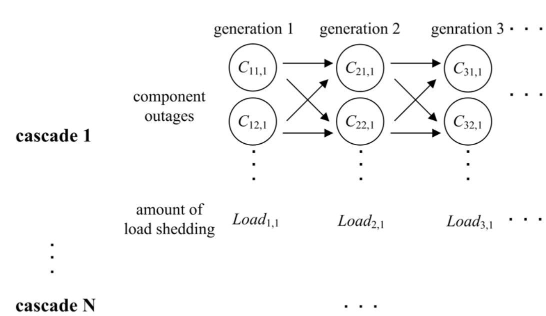

Fig. 3. Schematic diagram of the cascades.

Typically, the transmission lines or transformers can be considered as components. For simplicity, “cascade” in the rest of the paper means one sample of cascading outages, whose outages are clustered into multiple generations by time or sequence of outages. Each cascade starts with initial component outages in generation 1 and continues to generate outages in the following generations until there is no outage or the system collapses as illustrated by Fig. 3. Here $C_{ij,m}$ is the j-th component outage in generation i of the m-th cascade, and $Load_{i,j}$ is the amount of load shedding at the i-th generation of the m-th cascade.

Following the same structure as Fig. 3, a large number of independent cascades triggered by different initial component outages comprise the database of cascades. The component outages in one generation are considered to happen almost at the same time, whose sequence can be ignored. Thus, some causality between any two component outages respectively from two consecutive generations is hypothesized. A component outage $C_{ij,m}$ may potentially be caused by several component outages by going through all of the cascades, among which the one occurring the most times in the database is assumed to be the true cause of $C_{ij,m}$ in paper [1]. Other links connecting the remaining component outages and $C_{ij,m}$ are regarded as the redundant links. Finally, the set of links is obtained and denoted by L_{total}.

B. Link Weights for Different Layers

Different link weights are defined for the three layers:

- The link weight applied in the 1st layer is the empirical probability (denoted by P_l) of the source component outage causing the destination component outage [1].
- The link weight in the 2nd layer is the average amount of load shedding (denoted by W_l) triggered by the link.
- The link weight of the 3rd layer is the electrical distance (denoted by Z_l) in terms of equivalent impedance between the source and destination component outages.

1) Link Weight in Terms of the Number of Line Outages

The link weight is calculated by

$$P_l = \frac{N_{l:i\to j}}{N_i} \tag{1}$$

where $N_{l:i\to j}$ is the number of occurrences for link l and N_i is the number of times of outages of the source component i over the entire database.

2) Link Weight in Terms of the Amount of Load Shedding

In each cascade, the initial outages propagate to subsequent outages in the following generations. For the m-th cascade in the database, the component outages in generation g-1 can produce subsequent outages in generation g resulting in $Load_{g,m}$ of load shedding, which can be assigned to the links connecting the component outages in generation g-1 and g. The average amount of load shedding for link l in the m-th cascade is defined as

$$d_l^m = \frac{Load_{g,m}}{N_{(g-1,g),m}} \tag{2}$$

where $Load_{g,m}$ represents the amount of load shedding at generation g of the m-th cascade and $N_{(g-1,g),m}$ is the total number of links connecting generations g-1 and g. From all of the cascades which involve link l, the average amount of load shedding caused by link l is defined as

$$W_l = \frac{\sum_{m=1}^{N_l} d_l^m}{N_l} \tag{3}$$

If the true causalities between consecutive generations are unknown, for simplicity, we may assume that those $N_{(g-1,g),m}$ links connecting generations g-1 and g have the same amount of load shedding as shown by (2) and hence have uniform weights in one cascade. An alternative method may adopt non-uniform link weights and calculate the link weight by

$$d_{l:i\to j}^m = \frac{Load_{g,m}}{N_{(g-1,g),m}} \frac{F_i}{\sum_{i\in C(i)} F_i} \tag{4}$$

where the source component i outage and destination component j outage are respectively in generations g-1 and g. F_i is the power flow of component i before its outage and $C(i)$ is the set of component outages at generation g-1. An interpretation of (4) is that the links starting from source component i which have large power flows before its outage will be assigned large link weights in that cascade, which is reasonable since the outage of component i may cause power flow redistribution of some heavily loaded lines nearby.

3) Link Weight in Terms of Propagated Electrical Distance

The NERC blackout report [5] shows that outages may propagate to next outages either near or far in the network and become widespread in a late stage of cascading outages. The statistical characterization of how cascading outages typically spread on the network is analyzed in [35] based on standard utility data. In this paper, the links' spatial spreading is quantified by the electrical distance defined by [36]-[37], which originally quantifies the distance between two buses in power systems. Here, we adapt it to quantify the electrical distance between two components (i.e. lines) as

$$\begin{aligned} &Z_{i(i_s,i_d)\to j(j_s,j_d)} = \min\{Z_{i_s j_s}^{equ}, Z_{i_s j_d}^{equ}, Z_{i_d j_s}^{equ}, Z_{i_d j_d}^{equ}\} \\ &Z_{\alpha,\beta}^{equ} = Z_{\alpha\alpha} - 2Z_{\alpha\beta} + Z_{\beta\beta} \quad \alpha \in \{i_s, i_d\}, \beta \in \{j_s, j_d\} \end{aligned} \tag{5}$$

where $Z_{i(i_s,i_d)\to j(j_s,j_d)}$ is the electrical distance for the link $i\to j$ with source component i and destination component j, $Z_{i_s j_s}^{equ}$, $Z_{i_s j_d}^{equ}$, $Z_{i_d j_s}^{equ}$, and $Z_{i_d j_d}^{equ}$ are four equivalent impedances which each start from a bus of the source component and end at a bus of the destination component as calculated by the above formula. $Z_{\alpha\beta}$ is the α-th row β-th column entry of the network impedance matrix, and $Z_{\alpha\alpha}$ and $Z_{\beta\beta}$ are the α-th and β-th diagonal elements, respectively.

C. Construction of Multi-Layer Interaction Graph

The proposed multi-layer interaction graph has three layers. Each layer is comprised of key intra-layer links and components contributing the most to outage propagation from one specific perspective, i.e. the number of line outages, the amount of load shedding and the propagated electrical distance.

For the 1st layer, a directed acyclic subgraph starting from the link $i \to j$ can be extracted from L_{total} as shown in Fig. 4, and this subgraph (denoted by $C(i \to j)$) is unique and comprised of all the components influenced by link $i \to j$.

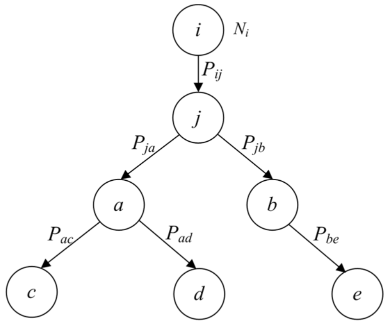

Fig. 4. Subgraph influenced by a link in the 1st layer.

To identify the key links, an index called the Cascading Outage Contribution (COC) is proposed to quantify the contribution of a link to propagation of outages in terms of the number of line outages. Given N_i times of component i outage, the expectation of the number of component j outages is

$$E_j = N_i P_{ij} \tag{6}$$

where P_{ij} is the empirical probability that the outage of component i causes the outage of component j. For any component $c \in C(i \to j), c \neq j$, the expectation of the number of outages given the times of its source component outage is

$$E_c = E_{c_s} P_{c_s j} \tag{7}$$

where c_s is the source outage of component c. Then define

$$COC^{I}_{i \to j} = \sum_{c \in C(i \to j)} E_c \tag{8}$$

which quantifies the total expected value of component outages that propagate through link $i \to j$. It can characterize the extent of the outage propagation in terms of the number of line outages. The larger $COC^{I}_{i \to j}$, the more critical the link. The set of key links can be obtained from those having the largest weights, e.g.

$$L^{I}_{key} = \left\{ l \mid COC^{I}_{l} \geq \lambda_I COC^{I}_{l\max} \right\} \tag{9}$$

where $COC^{I}_{l\max}$ is the largest value of COC^{I} for all links in the database and $1 > \lambda_I \geq 0$ is the threshold.

To quantify the contribution of component i to the propagation of outages, define the out-strength index (OS) as

$$OS^{I}_{i} = \sum_{l \in \zeta(i)} COC^{I}_{l} \tag{10}$$

where $\varsigma(i)$ is the set of links starting from component i. The out-strength of a component quantifies how much a component influences the others components. The components with large OS^{I} can cause great consequences and thus play crucial roles in the propagation of outages. The set of key components is obtained by selecting those components having the largest OS^{I}'s as

$$C^{I}_{key} = \left\{ i \middle| OS^{I}_{i} \geq \beta_I OS^{I}_{i\max} \right\} \tag{11}$$

where $OS^{I}_{i\max}$ is the largest OS^{I} for all components in the database and $1 > \beta_I \geq 0$ is the threshold.

Note that if an isolated key component exists in one layer that is not involved in any key link, it should be removed from C^{I}_{key} since it is not very useful to analyze the propagation of outages. The same handling is applied to C^{II}_{key} and C^{III}_{key} defined later.

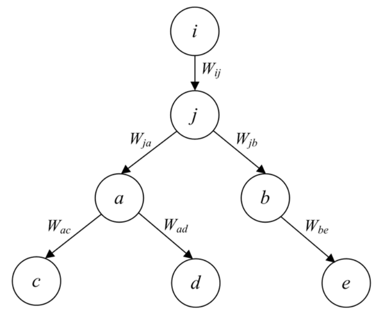

Fig. 5. Subgraph influenced by a link in the 2nd layer.

Using directed acyclic subgraph shown in Fig. 5, the key links in the 2nd layer can be identified by means of the COC of link $i \to j$ defined in (12) to quantify the consequence with outage propagation in terms of the amount of load shedding, where W_l is the average amount of load shedding triggered by link l.

$$COC^{II}_{i \to j} = \sum_{l \in C(i \to j)} W_l \tag{12}$$

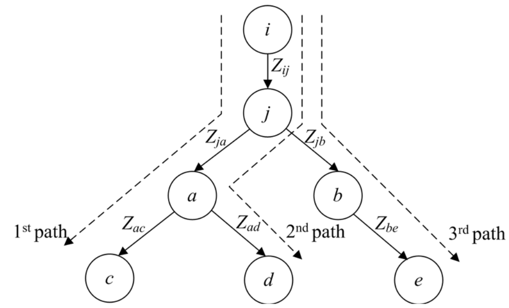

Fig. 6. Subgraph influenced by a link in the 3rd layer.

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Similarly, using the subgraph in Fig. 6, the key links in the 3rd layer are identified by the COC of link $i \to j$ defined in (13), where D^{path}_{t} is the total electrical distance for the t-th path starting from link $i \to j$, N_{path} is the number of paths starting from component j, G_t is the number of links along the t-th path, and Z_k is the electrical distance for the k-th link along the t-th path. $COC^{III}_{i \to j}$ can quantify the average distance of outage propagation starting from link $i \to j$.

$$COC^{III}_{i \to j} = \frac{\sum_{t=1}^{N_{path}} D^{path}_{t}}{N_{path}}, \quad \text{where } D^{path}_{t} = \sum_{k=1}^{G_t} Z_k \tag{13}$$

The key links and components in the 2nd and 3rd layers can be identified by a method similar to (9)-(11). Their sets of key links and key components are denoted by L_{key}^{II}, C_{key}^{II}, L_{key}^{III}, and C_{key}^{III}, respectively. Note that the key links and key components within a single layer are called key intra-layer links and components in order to distinguish them with the following inter-layer links and inter-layer components.

In real-world power systems, for example, consider how the transmission line outages spread in the August 10 1996 Western interconnection blackout in [5]. Occurrences of early outages did not cause obviously severe consequences on the system but did cause some transitions underneath to increase vulnerability of the system. In this paper, inter-layer links are also proposed in order to understand the transitioning of the outage propagation across different layers.

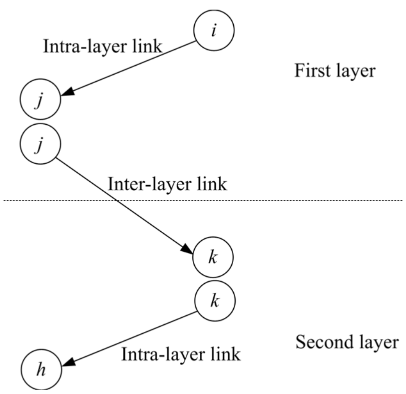

Fig. 7. An inter-layer link between the 1st and 2nd layers.

Fig. 7 illustrates an inter-layer link connecting the 1st and 2nd layers, where link $i \rightarrow j$ is an intra-layer link in the 1st layer and link $k \rightarrow h$ is an intra-layer link of the 2nd layer. Link $j \rightarrow k$ is an inter-layer link starting with the destination component outage of the 1st link and ending at the source component outage of the 2nd link. The total number of candidate inter-layer links between any two layers equals the product of the number of their key intra-layer links. The following two steps can identify the key inter-layer links on the most frequent inter-layer transitions and the key inter-layer components involved in those transitions: 1) for any two layers s and d, select the top γ % links from $L_{total} - L_{key}^{s} \cup L_{key}^{d}$ as the key inter-layer links; 2) the source components of those key inter-layer links are defined as key inter-layer components.

D. Number of Cascades Needed for a Database

More cascades tend to contain more information about the property of cascading outages. As a result, the number of identified links will increase and the COCs and the rankings of links can change. However, the rankings of the top-T_L key intra-layer links will stay unchanged after the number of cascades exceeds a threshold M^{min}. M^{min} can be determined by a procedure similar to the methods in [1] and [18]: gradually increasing the number of cascades, recording the set of top-T_L key intra-layer links, and finding the smallest number of cascades beyond which the top key intra-layer links do not change. Specifically, for the multi-layer interaction graph, the following steps are taken to determine M^{min} that make every layer have stable top-T_L key intra-layer links. Because the set of inter-layer links depend on the components of the key intra-layer links, the resulting set of inter-layer links are also stable.

Let M_i, i=1, 2,…, T_M be a series of numbers gradually increased by ΔM as candidate numbers of cascades to be included into the database. $S_{T_L}(M_i)$ is the set of top-T_L key intra-layer links from M_i cascades. Define

$$R(M_i) = | S_{T_L}(M_{i-1}) \cap S_{T_L}(M_i) | / T_L \tag{14}$$

which gives the ratio between the number of common key intra-layer links from two consecutive numbers of cascades and T_L. Then we calculate the standard deviation σ_i of $R(M_i)$ around M_i as (15). Then M^{min} is determined as the smallest M_i with σ_i less than a given tolerance τ .

$$\sigma_i \triangleq \sigma([R(M_{i-2}), \quad R(M_{i-1}), \quad R(M_i)]), \ i \geq 2 \tag{15}$$

The number of cascades needed is the maximum value among the M^{min}s for the three layers.

III. Mitigation of Cascading Outages

A. Mitigation Strategies

The propagation of cascading outages could be mitigated by weakening key intra-layer links [1]. When the source component of a key intra-layer link is tripped and causes the overloading of its destination component, that destination component will be tripped at a reduced probability to simulate intentional relay blocking as mitigation of outage propagation. In this way, cascades can be generated by simulations with that mitigation strategy.

In this paper, the following six mitigation strategies that weaken a number (denoted by K, e.g. 20-100) of key intra-layer links from different perspectives are compared:

- *Strategy-LO*: Weaken top-K key links in the 1st layer about the number of line outages.
- *Strategy-LS*: Weaken top-K key links in the 2nd layer about the amount of load shedding.
- *Strategy-ED*: Weaken top-K key links in the 3rd layer about the propagated electrical distance.
- *Strategy-3L*: Weaken top-K/3 key links in every layer.
- *Strategy-LOLS*: Weaken top-K/2 key links respectively in the 1st and 2nd layers at different stages of cascades.
- *Strategy-R*: Weaken randomly selected K links for comparison purposes.

Strategy-LO, *Strategy-LS,* and *Strategy-ED* are single-layer mitigation strategies. *Strategy-3L* is an integrated mitigation strategy in which the key intra-layer links are from the three layers. *Strategy-LOLS* is an integrated mitigation strategy in which the key intra-layer links are from the 1st and 2nd layers. The key intra-layer links from the two layers are weakened in different stages of cascading outages. The key intra-layer links in the 1st layer are weakened in the early stage (generation 1 to 2) of cascading outages, and the key intra-layer links from the 2nd layer are weakened in the later stage (generation 2 to 3) of cascading outages.

B. Validation of Mitigation Strategies

To validate a mitigation strategy, simulation of cascading outages is performed with each of the above strategies to generate a new database of cascades. Then, compared to the original database without mitigation, each mitigation strategy is evaluated by the reduced proportion of the average number of line outages, the average amount of load shedding, or the average electrical distance of outage propagation for each cascade. More specifically, the propagated electrical distance for one cascade is

$$ED_{cascade} = \sum_{g=1}^{N_{gen}-1} \max\{ED_{g,g+1}\} \tag{16}$$

where $\max\{ED_{g,g+1}\}$ is the maximal electrical distance between generation g and g+1 by going through all of links connecting these two generations, and N_{gen} is the number of generations for one cascade.

In general, a mitigation strategy that weakens the key intra-layer links from one layer can significantly mitigate the problem concerned by that layer compared to the random intra-layer link weakening and the key intra-layer link weakening for a different layer. It is anticipated that *Strategy-LO* can reduce the number of line outages much more effectively than *Strategy-R*, *Strategy-LS,* and *Strategy-ED*. Thus, the multi-layer interaction graph provides a way to choose the most effective strategy mitigating the most critical type of problems in cascading outages.

C. Online Decision Supports Using the Interaction Graph

For a utility-scale power system having tens of thousands of buses, a single simulation run in time domain is very time consuming using detailed and accurate models on all power system components. Thus, it is more reasonable to utilize the proposed interaction graph offline constructed based on a database of historical or offline simulated cascades. For simulated cascades, the models to be used in offline simulation can be detailed sufficiently to consider protective actions and system dynamics over a wide range from transient dynamics to mid-term or long-term dynamics to ensure the credibility of the database. Thus, online simulation can be avoided and the real-time application of this interaction graph for real-time prediction will be fast and practical.

In the real-time operation environment, if initial outages have occurred and tend to spread to a wide area, system operators needs to make an immediate decision to evaluate and mitigate the outages. The multi-layer interaction graph can provide operators with decision supports. If an outage just happens on any key intra-layer or inter-layer component, the key intra-layer links involving that component actually predicts how outage may propagate and what type of consequences may happen, e.g. line outages, load shedding and propagation over a distance. An inter-layer link involving that component predicts a transition between different types of consequences. Any control action weakening those key intra-layer or inter-layer links involving that component can contribute to mitigation of cascading outages. As simple examples, the mitigation strategies introduced above only consider intentional relay blocking so that the optimal power flow module with the DC OPA model (which emulates control actions from the control center compared to line tripping as local protection) is performed more often. In practice, system-wide mitigation actions should be taken to effective weaken key intra-layer or inter-layer links.

IV. Case Studies

A database of 10000 independent cascades is produced by simulations in MATLAB using the improved DC OPA model in [22] on an NPCC 140-bus system shown in Fig. 8. The total time cost is about 13800 seconds on a desktop PC with Intel Core i7-3770K 3.40GHz and 4GB RAM. Line flow limits are predetermined to ensure the "N-1" reliability criterion, i.e. no overloaded line after any single line outage. The initial outages for simulations are selected from random "N-k" (k≥2) contingencies in the NYISO area. The load variation at each load bus is assumed to follow the uniform distribution in [0.95, 1.05]. The probability of the line tripping depends on its loading according to Table I from [38].

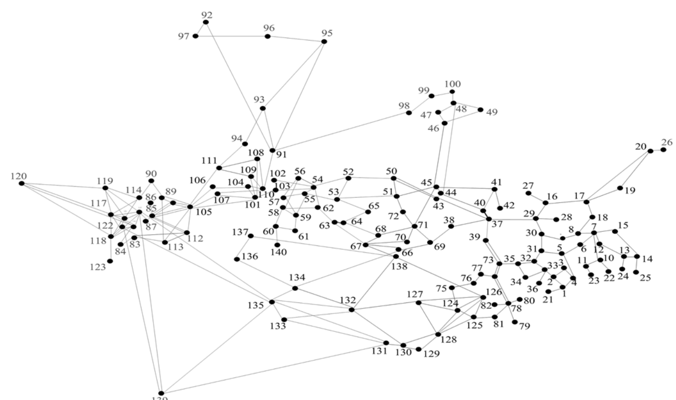

Fig. 8. NPCC 140-bus system.

TABLE I
Probability of Tripping with Line Loading [38]

Line Loading (% of Limit)	100	110	120	130	140	150
Probability of Tripping	0.10	0.30	0.60	0.80	0.95	1.00

A. A Multi-Layer Interaction Graph

A multi-layer interaction graph with three layers is visualized in Figs. 9-11. Each layer contains top-100 key intra-layer links and top-20 key intra-layer components. All components (i.e. transmission lines) of the NPCC system are represented by gray nodes (each located at the middle of the line), key intra-layer components are highlighted as green dots and key intra-layer links are represented by red arrows.

Note that the multi-layer interaction graph may vary with changes on the system topology and load level. If the changes are insignificant, e.g. slight load variations and line disconnections in a local area, most of key intra-layer links and components in other areas are still valid and important predictors on how outages may propagate. Thus, only some key intra-layer links and components in that local area need to be updated using the database having new cascades included. However, if the changes are significant, e.g. system-wide load variations, the set of key intra-layer links and component in each layer can vary significantly. Subsection V-C compares the interaction graphs respectively from the original database and a new database of cascades having load variances everywhere in the system, which indicate reduced overlapping in key intra-layer links.

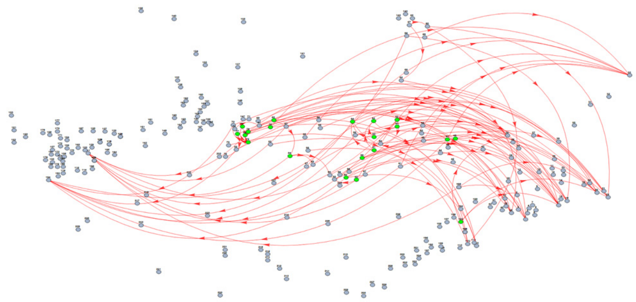

Fig. 9. Top-100 key intra-layer links and top-20 key intra-layer components in terms of the number of line outages.

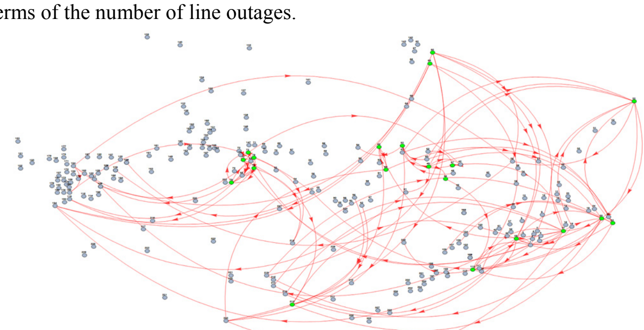

Fig. 10. Top-100 key intra-layer links and top-20 key intra-layer components in terms of the amount of load shedding (excluding 2 isolated components).

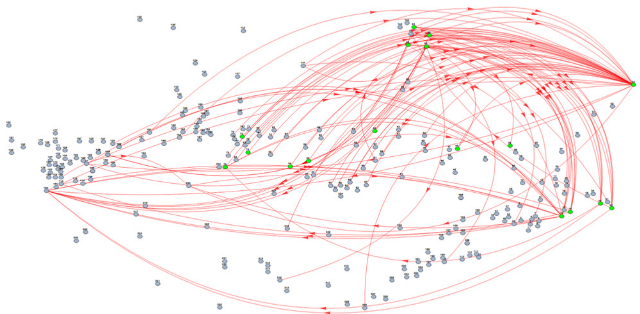

Fig. 11. Top-100 key intra-layer links and top-20 key intra-layer components in terms of the propagated electrical distance (excluding 3 isolated components).

TABLE II
NUMBERS OF OVERLAPPED LINKS BETWEEN DIFFERENT LAYERS

Number of Key Intra-layer Links in Each Layer	Number of Overlapped Links between Two layers		
	1st and 2nd Layers	1st and 3rd Layers	2nd and 3rd Layers
20	0	0	0
50	1	4	0
100	4	7	7

The overlapped links between different layers are very few as listed in Table II, indicating the independency of the three types of key intra-layer links.

The numbers of occurrences for the top-100 key intra-layer links in different layers are given by Fig. 12. The key intra-layer links in the 1st layer has much more occurrences than the key intra-layer links in the 2nd layer. It indicates that the links with large occurrences have high probabilities to be identified as the key intra-layer links of the 1st layer, which is reasonable from the definitions (6)-(8). Those links with a large number of occurrences have high probabilities to be identified as the key intra-layer links in the 1st layer. However, these links may not be identified as the key intra-layer links in terms of the amount of load shedding. For the key intra-layer links in the 2nd layer, although they occur much less often than those of the 1st and 3rd layers, they contribute the most to the amount of load shedding. For the key intra-layer links in the 3rd layer, the number of occurrences is distributed more dispersedly.

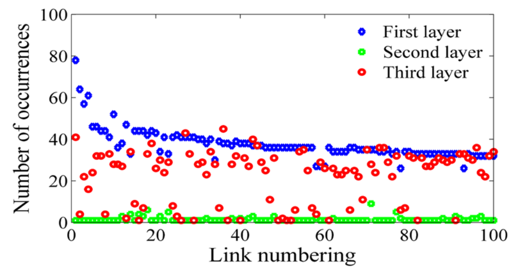

Fig. 12. Numbers of occurrences for key intra-layer links in different layers.

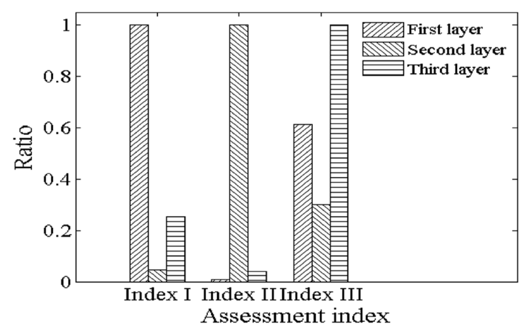

Fig. 13. Comparing assessment indices on top-100 key intra-layer links in three layers (Index I - average number of line outages; Index II - average amount of load shedding; Index III- average propagated electrical distance).

For a further comparison, the average number of line outages, average amount of load shedding and average propagated electrical distance are calculated as Indices I, II and III, respectively for the top-100 key links from different layers and then are normalized to [0, 1] as shown in Fig. 13, which confirms the distinct focuses of three layers; i.e. the index matching the focus of the layer has the highest value.

B. *Key Inter-Layer Links and Components*

Cascading outages may propagate within a single layer of the interaction graph or may cross to a different layer directly or through an inter-layer link. It is important for the system operators to monitor the inter-layer transitioning since it indicates the consequences of outages becoming less monotype. Here the numbers of inter-layer links between different layers for different numbers of key intra-layer links are shown in Fig. 14. The number of inter-layer links increases a lot with the increase of the number of key intra-layer links.

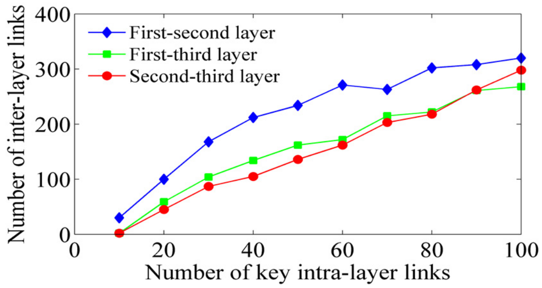

Fig. 14. Inter-layer links vs. key intra-layer links.

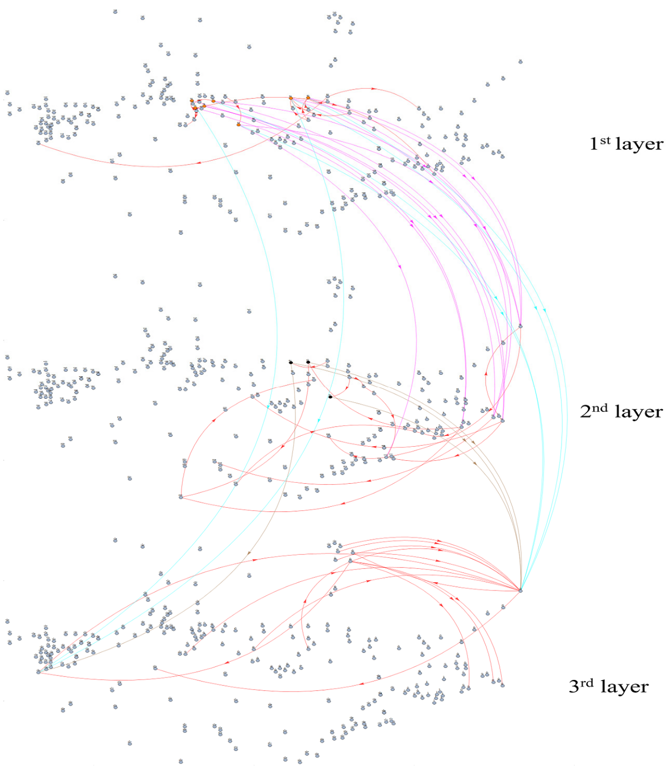

Fig. 15. 3-layer interaction graph (key inter-layer links between the 1st and 2nd layers, between the 1st and 3rd layers, and between the 2nd and 3rd layers are magenta, brown and cyan arrows, respectively; intra-layer links in each layer are red arrows; key inter-layer components involved in the key inter-layer links between the 1st and 2nd layers, between the 1st and 3rd layers, and between the 2nd and 3rd layers are orange, pink and black dots).

Fig. 15 shows only the top-15 key intra-layer links in each layer and the key inter-layer links between different layers. The number of inter-layer links between the 1st and 2nd layers, the 1st and 3rd layers, and the 2nd and 3rd layers are 65, 22, and 16, respectively. The sets of key inter-layer links between different layers can be selected from the top γ=50% of candidate inter-layer links. The number of key inter-layers between the 1st and 2nd layers, the 1st and 3rd layers, and the 2nd and 3rd layers are determined as 12, 6, and 4, respectively. The numbers of key inter-layer components involved in the key inter-layer links between the 1st and 2nd layers, the 1st and 3rd layers, and the 2nd and 3rd layers are 7, 5, and 3, respectively, as highlighted in Fig. 15.

Note that the set of key inter-layers links depends on the selection of the key intra-layer links. An inter-layer link could become an intra-layer link in another case and vice versa. From the number of inter-layer links and key inter-layer links between different layers, we can find that the correlation between the 1st and 2nd layers is tighter than any other two layers. Many inter-layer links connect the key intra-layer links in the 1st layer to those in the 2nd layer. This observation can help to propose an integrated mitigation strategy by combing the key intra-layer links from the two layers and weaken them in different stages of cascading outages.

C. Multi-Layer Interaction Graph for Increased System Load

Variations in system load will definitely affect the multi-layer interaction graph. As discussed above, load variations in a local area may only influence the key links and components of that area and neighboring areas. However, if the entire system has load variations, the multi-layer interaction graph may need to be significantly updated. In this case study, all system loads are scaled up by 10% uniformly. The new database having the same number of cascades as the original database is generated for the new system load level. Figs .16-18 show three layers with key intra-layer links and components.

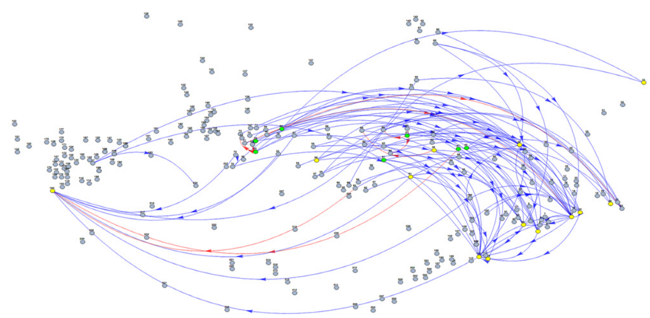

Fig. 16. Top-100 key intra-layer links and top-20 key intra-layer components in terms of the number of line outages at 110% system load (red/blue arrows are overlapped/different links and green/yellow dots are overlapped/different components compared to 100% system load).

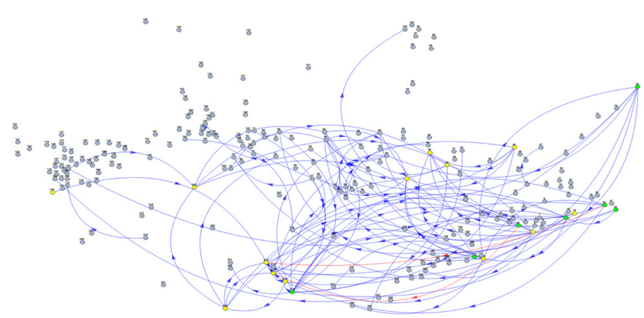

Fig. 17. Top-100 key intra-layer links and top-20 key intra-layer components in terms of the amount of load shedding with 110% system load.

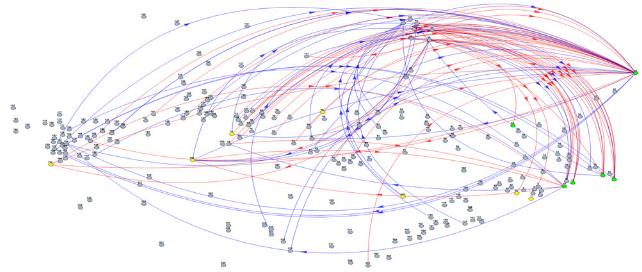

Fig. 18. Top-100 key intra-layer links and top-20 key intra-layer components in terms of the propagated electrical distance with 110% system load (excluding 7 isolated components).

The identified key intra-layer links in each layer with 110% and 100% load levels of the system are compared. The numbers of overlapped links by comparing different numbers of key intra-layer links is shown in Fig. 19. From a sensitivity study on the number of overlapped links, the key intra-layer links in the 1st and 2nd layers are more sensitive to the load variation of the system than those in the 3rd layer.

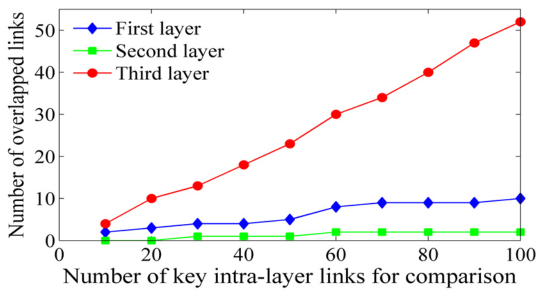

Fig. 19. Number of overlapped links vs. number of key intra-layer links.

D. Distribution of Key Intra-Layer Links among Generations

A key intra-layer link in a specific layer may connect two consecutive generations in either early or later stages of a cascade. For each of the three layers, the distribution of the top-100 key intra-layer links in transitions between generations is shown in Fig. 20. The maximum number of generations involved in the links is 6.

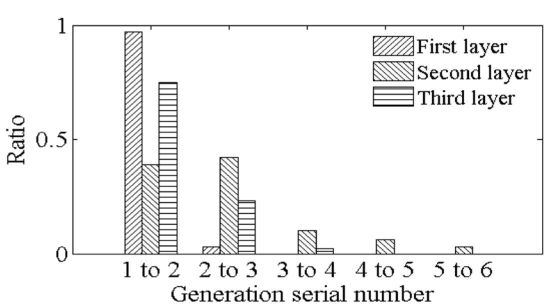

Fig. 20. Distribution of key intra-layer links in transitions of generations.

The majority of the key intra-layer links of the 1st layer connect generations 1 and 2, and also the majority of the key intra-layer links in the 3rd layer connect generations 1 and 2. These observations suggest that reducing line outages to limit the spreading of outages should have a high priority in the early stage of cascading outages, and the system operators should pay more attention to the 1st and 3rd layers of the interaction graph. The key intra-layer links in terms of load shedding are more dispersed and mainly connect generations 1, 2, and 3. The increased percentage of key links from generations 2 to 3 indicates that load shedding is the main problem in later stages. It is reasonable to have more load shedding in later stages of a cascading outage.

These observations enlighten us to propose the *Strategy-LOLS* mentioned in Section III-A, i.e. an integrated mitigation strategy to weaken the key intra-layer links in the 1st layer in the early stage (generations 1 to 2) of cascading outages and key intra-layer links in the 2nd layer in the later stage of cascading outage (generations 2 to 3). In fact, the majority of cascades in the database contain 3 generations and the average number of generations is 3.14.

E. Validation of Mitigation Strategies

When the source component of a key intra-layer link fails, the destination component may become overloaded and be tripped by protective relays. Each mitigation strategy considered here reduces the probability of the relay tripping to 10%. The number (i.e. K) of intra-layer links to be weakened is set up as 20 for all the mitigation strategies. For *Strategy-3L,* the numbers of key intra-layer links in the 1st, 2nd, and 3rd layers are set up as 7, 7, and 6, respectively. The case for *Strategy-R* is simulated for 20 times. These mitigation strategies are compared in terms of three assessment indices as shown in Table III.

TABLE III
COMPARISON OF MITIGATION STRATEGIES BY ASSESSMENT INDICES

Mitigation Strategies	Average Number of Line Outages	Average Amount of Load Shedding	Average Propagated Electrical Distance
Strategy-LO	-24.36%	-3.29%	-6.44%
Strategy-LS	+3.67%	-69.21%	+4.52%
Strategy-ED	-8.15%	-4.73%	-18.87%
Strategy-3L	-26.54%	-65.31%	-15.65%
Strategy-LOLS	-25.78%	-56.46%	-9.11%
Strategy-R	-3.23 to +4.21%	-6.59 to +7.94%	-1.66 to +2.79%

Strategy-LO can significantly reduce the average number of line outages by 24.36%. *Strategy-ED* can reduce the average number of line outage but not significantly because the 1st and 3rd layers show relevant properties from Fig. 12 and Fig. 13. *Strategy-LS* and *Strategy-R* may even increase the number.

Similarly, *Strategy-LS* significantly reduces the average amount of load shedding by 69.21%. However, *Strategy-ED* and *Strategy-LO* reduce it slightly and *Strategy-R* even increases it for some cascades.

The average propagated electrical distance can be reduced by 18.87% by *Strategy-ED*. *Strategy-LO* only reduces it slightly and *Strategy-LS and Strategy-R* may even increase the propagation.

The results validate that a specific problem, e.g. number of line outages, amount of load shedding, and propagated electrical distance, can be effectively mitigated by a strategy that weakens key intra-layer links from a matched layer.

Strategy-R may even increase the average number of line outages, load shedding and average electrical distance, which indicates the ineffectiveness of the random mitigation strategy.

Strategy-LS increases the average number of line outages and the average propagated electrical distance. This is because the mitigation strategy is not to stop the propagation of outages but change the direction of propagation. Weakening key links of a wrong type may even bring negative impacts.

Strategy-3L reduces the average number of line outages, the average amount of load shedding, and the average propagated electrical distance by 26.54%, 65.31%, and 15.65%. It is obvious since the key intra-layer links are combined from the three layers.

Strategy-LOLS is applied to different stages of cascading outages with the key intra-layer links from the two layers. A link will be weakened if it is between generations 1 and 2 and belongs to the top-10 key links of the 1st layer, or if it is between generations 2 and 3 and belongs to the top-10 key links of the 2nd layer. It reduces the average number of line outages and average amount of load shedding by 25.78% and 56.46%, respectively. This validates that the key links in the 1st layer are highly relevant to early stages of outages and key links in the 2nd layer are more involved in late stages of outages.

In general, each key link is mitigated for more times under *Strategy-3L* than that under *Strategy-LOLS*. That is because a link may appear in different stages of outages varying from different cascades. *Strategy-3L* will weaken a key link wherever it appears. However, *Strategy-LOLS* only weaken a key link when it appears in an anticipated stage of outages.

It is suggested that the system operators can monitor the propagation of outages based on the 1st and 3rd layers of the interaction graph during the early stage of outages. With the spread of cascading outages, more attentions should be paid to the 2nd layer. Then the strategy of weakening key intra-layer links of the corresponding layer can be applied, which is shown by *Strategy-LOLS* as one example. Incomplete information from a single layer interaction graph could mislead the system operators to take inappropriate control actions and may enlarge the propagation of outages undesignedly. The multi-layer interaction graph can provide comprehensive information helpful for online monitoring and mitigation of cascading outages.

F. Determining the Number of Cascades

As an example, the number of cascades for identifying top-100 key intra-layer links in the 2nd layer is presented here. The number of cascades in other layers can be determined by the same approach.

Set T_M=100, M_1=100, ΔM =100, and τ=0.01. Fig. 21 shows how $R(M_i)$ and its standard deviation σ_i change with the number of cascades. They become flat when cascades reach a specific number M^{min}, which is around 3600, meaning that the top-100 key links in the 2nd layer can be identified using 3600 cascades. The number of cascades needed for a database can be determined as 5200, which is the maximum value among all M^{min}'s for three layers. The number of cascades used in this paper, i.e. 10000, is large enough.

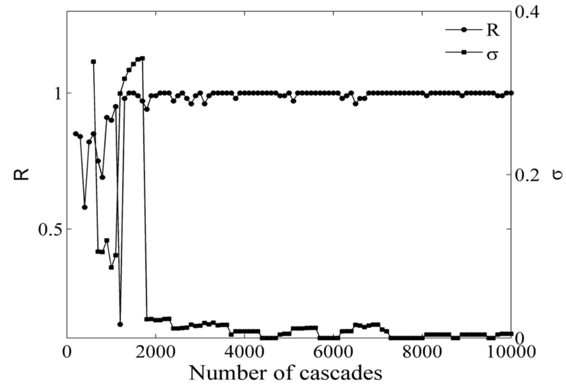

Fig. 21. R and σ for different numbers of cascades.

V. Conclusions

This paper proposes a multi-layer interaction graph on cascading outages of power systems as an extension of the single layer interaction network proposed in [1]. It intends to assist the system operators in predicting propagation of outages and making decisions on mitigation actions. The multi-layer interaction graph can be obtained offline from a database of simulated or historical cascades and then applied online. The graph comprises multiple layers respectively depicting key components and key links that contribute the most to outage propagation from different perspectives, i.e. the number of line outages, the amount of load shedding and the electrical distance of outage propagation. Two types of key links, i.e. key intra-layer link and key inter-layer link and their corresponding key components are proposed and defined. They together provide comprehensive information for the monitoring and mitigation of cascading outages. Meanwhile, key intra-layer link based mitigation strategies corresponding to each layer and two integrated mitigation strategies are also proposed and validated on the NPCC 140-bus system.

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