Measurement-Based Voltage Stability Assessment Considering Generator VAR Limits

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Abstract—This paper proposes a measurement-based voltage stability assessment method considering VAR limits of generators. Traditional measurement-based methods for monitoring a load bus or a load area often simplify the external system as a simple constant electromotive force (e.m.f.) behind a Thevenin impedance, which may give inaccurate voltage stability margin when some external generator reaches its VAR limit. In this paper, a novel method is proposed to consider the VAR limits of generators by deriving a relationship between the variations of their e.m.f. and the load, which can be identified and tracked in real time. Thus, a more accurate voltage stability margin is derived by an analytical expression under variations of both generator terminal voltages and loads. The proposed method is compared with a typical Thevenin method and demonstrated on a two-area system and the IEEE 39-bus power system.

Index Terms—Voltage stability assessment, Thevenin equivalent, generator capability, maximum power transfer, VAR limit.

I. INTRODUCTION

Due to fierce competitions in power markets, electric power grids are often operated close to their security limits, which significantly increases the risk of instabilities, e.g. voltage instability and angle instability [1], [2]. Voltage instability becomes one of the major concerns for system operators as it may propagate and induce cascading failures. Thus, it is of great importance to implement online Voltage Stability Assessment (VSA) to help system operators determine the voltage stability margin.

Traditionally, the VSA is performed with model-based approaches, such as Continuation Power Flow (CPF) or time-domain simulations [3]-[5]. Recently, the holomorphic embedding method is proposed to evaluate the voltage stability margin [6]-[9]. Several issues prevent the model-based approach from on-line applications, e.g., inaccurate models, possible divergence of state estimation and heavy computational burdens [1].

With wide deployment of the GPS-synchronized Phasor Measurement Units (PMUs), there are growing interests in the Measurement-Based VSA (MBVSA). One major category of MBVSA methods is based on Thevenin’s theorem. Except for the load bus of interest, the rest of the system is reduced to its Thevenin Equivalent (TE) including a voltage source and an equivalent impedance, which are both estimated by local measurements [10]-[13]. A comparative study regarding four approaches to estimate the TE is provided in [14]. The authors in [15] derive a new TE circuit to consider the ZIP load model. Ref. [16]-[20] leverage the concepts of coupled single-port circuits or the channel component transformation to investigate the voltage stability issue for a load area. In [21], all the boundary buses within the load areas are merged together into a fictitious load bus, and then the TE is applied to this fictitious bus to evaluate the voltage stability margin for the load area. This method requires all boundary buses to be equipped with PMUs. Then, authors in [22] and [23] generalize the TE method to an $N+1$ bus equivalent system, where $N$ is the number of boundary buses. Furthermore, ref. [24] extends the $N+1$ bus equivalent system to address potential $N-1$ contingencies in real time.

The existing TE-based approaches may perform well in predicting some of the Saddle Node Bifurcation (SNB) cases, but they are generally unable to predict the Limit Induced Bifurcation (LIB) due to the over-simplification of the external system. In practice, the LIB occurs often when generators or other voltage regulating devices reach their VAR limits while load continues to increase. Generators’ reactive power capabilities play a crucial role in maintaining system voltage stability, as they can cause the system to lose its structural stability within a very short period, leading to voltage collapse [25], [26]. An MBVSA method capable of predicting both SNB and LIB is in great need.

Authors in [27] propose an alternative generator equivalent model considering VAR limits. The generators are modeled by time-varying internal voltages and impedances, whose parameters are estimated using the real-time measurements. Then, the $L$-index in [28] is extended on the model to give the voltage stability margin. Compared with the existing work, the proposed method in this paper can capture the relationship between the voltage and load when some generator hits its VAR limit. In addition, an analytical expression is derived to predict the voltage stability margin quickly and accurately.

This paper investigates the impacts of LIB in MBVSA and proposes a novel approach to incorporate generator VAR limits. Compared with existing approaches, the proposed one
can provide system operators with more accurate stability margins and early warning of voltage instability by being able to handle both SNB and LIB.

The remainder of this paper is organized as follows. Section II presents the proposed solution and discusses its online implementation. Section III demonstrates the proposed approach on the two-area system and compares its performance against the existing TE-based method. Further experimental results are presented in Section IV, and conclusions are drawn in Section V.

II. PROPOSED APPROACH

A. Improved Generator Model for VSA

Power system voltage stability margin and its trajectory can vary significantly when one or more generators hit the VAR limits, as the incremental behavior(s) of that/those generator(s) will change dramatically. However, this effect is not considered in most of the existing TE-based approaches, which model the generator as a constant voltage source behind a Thevenin impedance. Considerable error in the estimation of system voltage stability margin has been observed using this type of simplified models.

The basic idea of the proposed method is to derive an approximate relationship on how the magnitude of the voltage source decreases with the increase of load after one or more generators reach VAR limits. Then, the voltage stability margin can be predicted in real time by evaluating an explicit expression considering VAR limits, so that the prediction of margin can be predicted in real time by evaluating an explicit expression considering VAR limits, so that the prediction of margin can be more accurate. This method is used to prevent the mid-term or long-term voltage instability.

A decline in generator terminal voltage can be observed once the system enters LIB. To model and predict the variation of equivalent voltage source representing the external system, relation between the terminal voltage of this equivalent source and the load impedance can be derived by substituting the constant voltage source with a single-equivalent source and the load impedance can be derived by

\[ E_i = \frac{E'_i}{R_s + jX_s} \]

After one or more generators reach their VAR limits, the voltage stability margin can be more accurate. Thus, the terminal voltage can be determined by

\[ E_i = \frac{jX_s a d f d (Z_T + Z_L)}{R_s + jX_s + Z_T + Z_L} \]

Eliminate the load reactance \( jX_i \) as \( T_L \)

\[ E_i \approx \frac{E'_i}{R_s + jX_s} \]

where armature resistance \( R_s \) is typically very small and can be neglected. Equation (5) can be decomposed into the real and imaginary part, expressed in (6) and (7) respectively.

\[ E_i \approx \frac{E'_i}{R_s + jX_s} \]

When the load keeps increasing, the equivalent load resistance \( R_L \) keeps decreasing. The terminal voltage \( E_i \) keeps constant until the field current, i.e. \( i_{fd} \), of a generator hits its maximum limit. After that, the terminal voltage \( |E_i| \) decreases. The proposed model basically can predict the trend of \( E_i \) to give a more accurate margin after a generator hits its VAR limit.

The single-generator model hits its VAR limit, the effective internal voltage \( |E_i| = X_{ad} i_{ad} \) reaches its maximum. Rewriting (8) to obtain

\[ X_{ad}^2 i_{ad} (R_f + R_L) - E_{ad} X_{ad}^2 i_{ad} = - |E_i|^2 2E_{ad} X_{ad}^2 i_{ad} (R_f + R_L) \]

and set the error term as

\[ e = |E_i|^2 2E_{ad} X_{ad}^2 i_{ad} \]

Therefore, the load resistance \( R_L \) can be determined by

\[ R_L = \frac{E_{ad} X_{ad}^2 i_{ad}}{X_{ad}^2 i_{ad} + e} \]

Neglecting small error term \( e \), the equation (11) becomes

\[ R_L \approx \frac{X_{ad}^2 i_{ad} E_{ad}}{X_{ad}^2 i_{ad} E_{ad}} \]

Eq. (12) reveals an approximate linear relationship between the load resistance \( R_L \) and real part of generator terminal voltage \( E_{ad} \). The error of this linear approximation is induced by the error term \( e \) in Eq. (11), which will be monitored later in the case studies.

In (12), the generator terminal voltage \( E_i \) is typically controlled by the Automatic Voltage Regulator (AVR) by adjusting the field current \( i_{fd} \). \( E_i \) can be maintained at its reference value, until \( i_{fd} \) reaches its limit. With the linear approximation, coefficients \( X_{ad}/(X_{ad} i_{ad}) \) and \( R_L \) in (12) can be estimated using \( E_{ad} \) and \( R_L \) measurements in a pre-defined time window. Therefore, the variation of generator terminal voltage
under heavy loading condition can be predicted, which can be more accurate than constant terminal voltage.

B. Network Transformation and Impact Factor

To generalize the single-generator model proposed in Section II-A, the multi-generator model shown in Fig. 2 can be regarded as an equivalent model representing many generators in a “source” area. Among these generators, the critical ones are the ones with higher impact factors than the others, which hold dominant components in the aggregated voltage source. With PMUs installed at these critical generators, the visibility of the system is largely improved, and the performance of generator over-excitation detection is further enhanced. Thus, the impact of critical generators to a power system can be assessed promptly utilizing the PMUs installed at these critical generators. To accurately address the impact factor and to create the equivalent coupled single-port model, the following network transformation is utilized.

In general, buses in an \( M \)-load \( N \)-generator power system can be categorized into three groups, namely, the load bus group \( L \), the generator bus group \( T \), and the bus group \( Z \) with zero current injection. Using the single-port decoupled method \[18\], power flow equations can be collectively written as

\[
\begin{bmatrix}
-I_L \\
0 \\
I_T
\end{bmatrix} =
\begin{bmatrix}
Y_{LL} & Y_{LZ} & Y_{LT} & Y_{TV} & Y_{VZ} & Y_{VT} & V_L \\
Y_{ZL} & Y_{ZZ} & Y_{ZT} & Y_{VZ} & Y_{VT} & V_Z \\
Y_{TL} & Y_{TL} & Y_{TT} & Y_{ET} & V_T
\end{bmatrix}
\begin{bmatrix}
I_{L} \\
I_{T}
\end{bmatrix}
\tag{13}
\]

where \(-I_L\) and \(I_T\) are vectors representing current injections at bus group \( L \) and \( T \), respectively, and \( V_L, V_Z\) and \( V_T\) represent the voltage phasors of the three groups. The system admittance matrix is rearranged accordingly as shown in (13).

The following transformation equation is proposed as

\[
E_{eq} = V_L + Z_{eq}I_L
\tag{14}
\]

where the equivalent voltage source \( E_{eq} \) and impedance \( Z_{eq} \) of the single-port decoupled network are

\[
Z_{eq} = (Y_{LL} - Y_{LZ}Y_{ZL}^{-1})^{-1}
\tag{15}
\]

\[
E_{eq} = Z_{eq}(Y_{ZZ}Y_{ZT}^{-1}Y_{TT} - Y_{VT})E_T
\tag{16}
\]

Therefore, an impact factor matrix \( K \) can be defined as

\[
K = Z_{eq}(Y_{ZZ}Y_{ZT}^{-1}Y_{TT} - Y_{VT})
\tag{17}
\]

In the following discussion, matrix \( K \) will be utilized to identify critical generators as well as to provide the ranking of generators in terms of their impacts in voltage stability monitoring and analysis.

Contingencies induced by switching actions can also be considered in the proposed method. In real-time operation, if there is an \( N \)-1 contingency, the admittance matrix in (13) will be updated, so a new voltage stability margin can be calculated using the updated matrix.

Fig. 2 shows a reduced power system model considering VAR limits of multiple generators. For example, a load \( Z_{l1} \) is connected to \( N \) generators through multiple equivalent transmission lines with impedances, \( Z_{T11} \), \( Z_{TN1} \), which are defined in the 1st column of matrix \( Z_{eq} \). Fig. 3 shows the single-port equivalent model considering VAR limits after the network transformation in (15)-(17).
\[
H = \begin{bmatrix}
V_{r,1} & V_{i,1} & -p_t & -q_t \\
V_{r,3} & -V_{r,3} & -q_t & p_t \\
\vdots & \vdots & \vdots & \vdots \\
V_{r,n} & V_{i,n} & -p_n & -q_n \\
V_{r,n} & -V_{r,n} & -q_n & p_n
\end{bmatrix}
\]

(21)

\[
Z' = [V_{r,i} + V_{i,i}] 
\begin{bmatrix}
0 & V_{r,1} & \cdots & V_{r,n}
\end{bmatrix} \begin{bmatrix}
0 & V_{i,1} & \cdots & V_{i,n}
\end{bmatrix}
\]

(22)

where \( \bar{E}_t = E_{ir} + jE_{iR}, \) \( Z_T = R_T + jX_T, \) and matrices \( H \) and \( Z \) are calculated based on PMU measurements collected at \( n \) time instants within the specified time window. Variables \( V_{r,k} \) and \( V_{i,k} \) are real and imaginary parts of voltage at the load bus at the \( k \)th time step, respectively. Variables \( p_t \) and \( q_t \) are active and reactive power injections at the load bus at the \( k \)th time step. As mentioned before, this method is used to prevent the mid-term or long-term voltage instability, so \( \bar{E}_t \) and \( \bar{Z}_t \) are assumed to be constant in the time windows of parameter identification.

With at least two sets of Thevenin equivalent parameters obtained, least squares approach can be used to identify coefficients \( a \) and \( b \) as shown below to model the linear relationship between \( E_{ir} \) and \( R_L \):

\[
E_{ir} = X_{ad}i_{ad}R_L + X_{ad}i_{ad}R_T' + \epsilon' = aR_L + b + \epsilon'
\]

(23)

where \( \epsilon' \) is the derived error term, i.e. \( \epsilon' = (a(R_T + R_L)X_{ad}i_{ad})_t \), in which \( \epsilon \) is the error term in Eq. (10). \( \epsilon' \) will be monitored later in the case studies.

C. Voltage Stability Limit Calculation

With generators’ reactive power capabilities considered, more accurate voltage stability limits can be evaluated with prediction of \( E_{ir} \), which is discussed in this subsection. As mentioned in the previous subsection, the real part of each terminal voltage \( E_{ir} \) varies linearly with the \( j \)th load resistance \( R_L \), and the equivalent voltage source \( E_{eq} \) for this load is the linear combination of all terminal voltages \( E_{ij} \) (\( j = 1-N \)), according to (19). The following derivation proves that the real and imaginary parts of equivalent voltage source \( E_{eq} \) of single-port equivalent model also have the linear relationship with the load resistance \( R_L \), expressed as

\[
E_{eqr} = \text{Re}\left(\sum_{j=1}^{N} K_{ij}E_{ij}\right) = \sum_{j=1}^{N} \text{Re}(K_{ij})(a_jR_L + b_j) + \sum_{j=1}^{N} \text{Im}(K_{ij})E_{ij} = a'R_L + b'
\]

\[
E_{eqi} = \text{Im}\left(\sum_{j=1}^{N} K_{ij}E_{ij}\right) = \sum_{j=1}^{N} \text{Im}(K_{ij})(a_jR_L + b_j) + \sum_{j=1}^{N} \text{Re}(K_{ij})E_{ij} = a''R_L + b''
\]

(24)

where \( a_j, b_j \) are the linear coefficient of the \( j \)th generator and \( a' \), \( b' \), \( a'' \) and \( b'' \) are parameters, such that

\[
a' = \sum_{j=1}^{N} a_j \text{Re}(\bar{K}_{ij}), \quad b' = \sum_{j=1}^{N} b_j \text{Re}(\bar{K}_{ij}) - \text{Im}(\bar{K}_{ij})E_{ij}
\]

\[
a'' = \sum_{j=1}^{N} a_j \text{Im}(\bar{K}_{ij}), \quad b'' = \sum_{j=1}^{N} b_j \text{Im}(\bar{K}_{ij}) + \text{Re}(\bar{K}_{ij})E_{ij}
\]

(25)

For the single-port model, shown in Fig. 3, assuming the impedance of a critical load is represented as \( \bar{Z}_L \), its power consumption can be calculated as

\[
\bar{S}_L = \bar{V}_L \bar{T}_L = P_L + jQ_L
\]

(26)

\[
P_L = \frac{R_L |E_{eq}|^2}{(R_{eq}^2 + X_{eq}^2)(X_{eq}^2 + X_L^2)} = \frac{(a'R_L + b')^2 + (a''R_L + b'')^2}{(R_{eq} + R_L)^2 + (X_{eq} + X_L)^2}
\]

(27)

\[
Q_L = \frac{X_L |E_{eq}|^2}{(R_{eq} + R_L)^2 + (X_{eq} + X_L)^2} = \frac{(a'R_L + b')^2 + (a''R_L + b'')^2}{(R_{eq} + R_L)^2 + (X_{eq} + X_L)^2}
\]

(28)

where \( R_L \) and \( X_L \) are resistance and reactance of the load.

A linear or quadratic function can be utilized to represent the load impedance characteristic. To simplify the mathematical derivation, an assumption can be made that the load reactance is proportional to its resistance as:

\[
\gamma = \frac{X_L}{R_L}
\]

(29)

Thus, by taking derivative of (27) with respect to load resistance, the ‘nose’ point can be calculated.

\[
\frac{dP_L}{dR_L} = 0
\]

(30)

Solve (30) to obtain the following equations:

\[
(cR_L + d)f(R_L) = 0
\]

(31)

where parameters

\[
c = 2(a'^2 + a''^2)
\]

(32)

\[
d = 2(a'b' + a''b'')
\]

(33)

and

\[
f(R_L) = c(\gamma^2 + 1)R_L^2 + 4c(R_L + \gamma X_L) - d(\gamma^2 + 1)R_L^2 + 3c(R_L^2 + X_L^2)R_L + d(R_L^2 + X_L^2)
\]

(34)

Obviously, \(-d/c\) is one of the roots of the 4th order polynomial equation regarding \( R_L \) in (31). According to (23), both \( c \) and \( d \) are positive real numbers. Therefore, this root is practically infeasible and can be neglected as \( R_L \) cannot be negative.

The roots exist in the following cubic equation.

\[
c(\gamma^2 + 1)R_L^3 + 4c(R_L + \gamma X_L) - d(\gamma^2 + 1)R_L^2 + 3c(R_L^2 + X_L^2)R_L + d(R_L^2 + X_L^2) = 0
\]

(35)

Since all coefficients in this cubic equation are real, there exists at least one real root. All remaining roots of this cubic equation can be found algebraically, as shown in (36). The maximum power that can be supplied to the load is shown in (41).
need to be collected in order to evaluate active and reactive power consumptions which ultimately are used for the voltage stability margin calculation. This stability margin will be provided to operators and the simulation tools for remedial action scheme (RAS). When the real-time stability margin is smaller than a pre-defined threshold, the commercial software for model-based VSA will be called to conduct power flow calculations or dynamic simulations for multiple contingencies. Additionally, the RAS actions will also be validated.

It is worthwhile to mention that $P_{max}$ is calculated by Eq. (41) in an analytical manner, which is fast to be evaluated in real time. Meanwhile, $P_{max}$ can be calculated with higher accuracy when some generators hit the VAR limits, because it considers the additional information of the linear approximation between the variations of voltage sources and the increase of load.

This MBVSA method is able to predict voltage stability limit, i.e. $P_{max}$, considering both SNB and LIB. When one or more generators hit their VAR limits, their over excitation limit (OEL) alarm signals will be transmitted to the EMS system in the control center. If there is no OEL alarm activated, i.e. SNB, the voltage stability margin will be calculated by the method in [23] without considering VAR limits. If some OEL alarms are activated in generators, i.e. LIB, the margin will adapt to the proposed method. It can give a more accurate stability margin with considering the trend of $E_i$ for those generators working under VAR limits.

Contingencies induced by switching actions can also be considered in the proposed method. If there is an $N$-1 contingency, the admittance matrix in (13) will be updated in real time, so a new voltage stability margin can be calculated using the updated matrix.

### III. Demonstration on the Two-Area System

The two-area system given by P. Kundur [29] is used to demonstrate the proposed approach. Fig. 5 shows a one-line diagram of this two-area system, a circuit model of which is built and simulated using MATLAB. In this circuit model, all the generators are represented as voltage sources, where the voltage magnitude of generator G4 is declading to emulate the scenario when VAR limit of generator G4 is reached. The load at Bus 9 is increased from 1767 MW to 2665 MW, with voltage collapse observed at the end of the simulation.

![Fig. 5. The two-area system.](image)

Fig. 6(a) shows the PV curve of Bus 9, from which it can be seen that the “nose” point is at 0.72 p.u. when load at Bus 9 reaches 2665 MW. Fig. 6(b) shows the impact factors of all four generators, they are calculated using the system model.
admittance matrix. Basically, generator G4 at Bus 4 has the largest impact factor which is greater than the summation of the impact factors of the rest three, and therefore is considered as the critical generator.

Assume PMUs are deployed at all critical generators and Load 9 in the system. They are monitoring the voltage phasors of the generators and Load 9, which are utilized to calculate the coefficients \(a\) and \(b\) in (23). This equation is solved by least squares approach showing in (20)-(22). With such a linear relation estimated, the equivalent voltage source \(E_{eq}\) can be obtained using (19). Finally, the voltage stability can be calculated according Section II-C.

Fig. 6. (a) PV curve of bus 9 and (b) Impact factor of the generators.

Fig. 7. \(P_{\text{max}}\) comparison.

Fig. 7 shows the active power transfer (black line) and its corresponding limits calculated using both classical TE-based method [1] (blue line) and the proposed method (red line). As the figure shows, the proposed method always provides more accurate result for the active power limit calculation, since \(P_{\text{MaxNew}}\) estimated by the new method is closer to the actual \(P_{\text{Max}} = 2665\text{MW}\), which is identified by time domain simulation, and more flat than \(P_{\text{MaxOld}}\) which is calculated by the TE based method. The classical TE-based method gives a wrong positive margin when the load reaches its ‘nose’ point with zero margin, while the proposed approach can identify it more accurately.

Comparison of the active power transfer margins is plotted in Fig. 8. These margins are calculated as the differences between the calculated (actual) maximum power transfer limit and the actual power transfer divided by the corresponding maximum power transfer limit. At \(t = 0\) s, errors in the transfer margin calculation are 3.45% and 8.07% using the proposed approach and the classical TE approach, respectively. As load increases, estimation error associated with the proposed method drops significantly and gets close to zero after 200s. However, the error resulting from classical TE-based method is always significantly higher. Therefore, the effectiveness and advantage of the proposed approach have been demonstrated by the two-area system.

IV. CASE STUDIES ON THE IEEE 39-BUS SYSTEM

The proposed method is further tested on the IEEE New England 39-bus system. As highlighted in Fig. 9, the system has a Connecticut Load Center (CLC) area supported by power from three tie lines, i.e., Line 9-8, 3-4, and 15-14. Line 9-8 is from the NYISO region and the other two are from the northern ISO-NE region.

Fig. 8. Comparison of \(P\) margins.

Fig. 9. Map of IEEE 39 system.

\(\text{DSATool}^{\text{TM}}\) is used to simulate the voltage instability scenario. In the simulation, all loads are modeled as constant PQ loads. Simulation results on the voltages and the complex
powers at Bus 8 are recorded at 30Hz. The proposed method is performed every 1/30s using data collected from the latest 1s time window.

![Fig. 10. PV curve of Bus 8.](image)

To simulate the voltage collapse in the CLC area, load at Bus 8 is continuously increased by a total of 576 MW from its original loading level of 522 MW with consistent load power factor. Fig. 10 shows the PV curve at Bus 8. The voltage collapse happens around $t = 173s$ as shown in Fig. 11.

The generator terminal voltage magnitudes recorded throughout this simulation are plotted in Fig. 11. The black curve $V_{mE}$ represents voltage at the aggregated bus for Load 8. Three stages with different slopes can be observed on $V_{mE}$, which are in $[0s, 60s]$, $[60s, 150s]$ and $[150s, 173s]$, respectively. The stage shift at 60s is caused by the over-excitation of generator G31, and the stage change at 150s is caused by the over-excitation of generator G32. The voltages at Bus 31 and Bus 32 start dropping significantly after 60s and 150s, respectively.

![Fig. 11. Voltage magnitudes of generator buses and aggregated bus.](image)

According to (17), generator impact factors are calculated and shown in Fig. 12. Generators G31, G32 and G39 have the highest impact factors. As shown in the geographic map (Fig. 9), these three generators are the nearest ones to Load 8. Therefore, both the electrical distance and geographic information tell the same story for the critical generators.

With the impact factors calculated, the equivalent voltage source can be estimated according to (19). In practice, the equivalent voltage source is updated continually in real time using the PMU measurements from all the generator buses. Additionally, Load 9 is monitored by PMU, which helps identifying the coefficients in (23). At the end, the voltage stability limit can be solved in real time by (36)-(41).

To further verify the aggregation results, the correlation coefficient is also used to measure the linear dependence between the voltage profiles at the aggregated bus and the corresponding original ones. If there are $N$ measurements in each voltage profile, the Pearson correlation coefficient is evaluated as

$$
\rho(V_{mi}, V_{mE}) = \frac{\text{cov}(V_{mi}, V_{mE})}{\sigma_{Vmi} \sigma_{VmE}} = \frac{1}{N-1} \sum_{j=1}^{N} \frac{(V_{mi}(j) - \mu_{Vmi}) (V_{mE}(j) - \mu_{VmE})}{\sigma_{Vmi} \sigma_{VmE}}
$$

where $\mu_{Vmi}$ and $\sigma_{Vmi}$ are the mean and standard deviation of $V_{mi}$ ($i=30-39$), respectively, and $\mu_{VmE}$ and $\sigma_{VmE}$ are the mean and standard deviation of voltage $V_{mE}$ at the aggregated bus. As shown in Fig. 13, the correlation coefficients for each generator terminal voltage magnitude and the equivalent voltage magnitude are calculated, among which the top three are the ones between $V_{m31}$, $V_{m32}$ and $V_{m35}$.

By comparing the actual point of operation to the desired limit, the regulator determines when it is appropriate to adjust the generator field current in order to stay within the desired operating conditions. Fig. 14 shows the field currents for all ten generators in the system. The three solid curves reaching their limits sequentially are the field currents of generators G31, G32 and G35. The field current of generator G31 firstly reaches the limit at 60s, then field current of generator G35 hits its limit at 130s, and finally field current of generator G32 reaches its limit at 150s. The remaining generators do not hit their field current limits, and therefore their field currents keep increasing until the voltage collapse happens.
When field current of a generator hits its limit, reactive power output of that generator constricts. Further, if the load continues to increase, reactive power supports from these generators will be significantly reduced, and their terminal voltage will decline. The system may enter voltage instability status even with a small disturbance. As shown in Fig. 15, generators G31, G32 and G35 reach their field current limits, and their reactive power outputs drop significantly since then.

Fig. 16 shows the results for the active power limit calculation using TE-based method and the proposed method with generator VAR limits considered. The active power limit calculation has been conducted for all measurements, which means that it was calculated 5201 times. The total calculation costs 19.2 seconds, so every calculation takes 0.0037 second. However, it is not necessary to calculate it for all time instant, the configuration can be adjusted according to computation resource and the performance requirement. The active power of Load 8 increases from 522 MW and reaches its limit at 170s, causing voltage collapse. For Stage 1, no generator reaches its over-excitation limit, so the limit predicted by the new method is closed to the TE-based method. When the system enters Stage 2 and Stage 3, the field currents of the generators G31, G32 and G35 approach their limits, and accuracy of the proposed approach become significantly higher than the classical TE approach. The existing approach still gives positive margin when the system collapses at 173s. The active power limit calculated by the proposed approach is reached at the system collapse point.

In Fig. 17(a), an abrupt decline in the $P$ margin occurs at
60s as the field current of generator G31 reaches its limit, as shown in Fig. 17(b)-(d). Immediate after that the system enters Stage 2, and the transition reflects the impact from generator G31 with the highest impact factor.

Fig. 18. P margin and field currents.

In Fig. 18, system enters Stage 3 which is triggered by the field current of generator G32 being reached. Similarly, the field current of generator G35 also reaches its limit at 130s, but the impact was limited due to its small impact factor which is less than 0.05. With the quantifiable impact factor, the critical generators are identified correctly and promptly.

Fig. 19 shows improvements in the active power transfer margin calculation using the proposed method. Since no generator hits its \( Q \) limit before 60s, the proposed approach and classical TE method give similar margin. After the system enters Stage 2 at 60s, the margin was improved by less than 10% by the proposed method. When the system enters the final stage, as all three generators hit their \( Q \) limits, which makes the \( Q \) limits a critical factor for the stability margin evaluation. As shown in the figure, the margin calculation improved by the proposed method reaches up to 43%.

Fig. 20 shows the error of the equivalent voltage identified by this method. This error is mainly induced by the linear regression of Eq. (23). It can be observed that, in the 1st and 2nd stages, the error is less than 0.05 p.u. and in all the simulation period except for the critical time of voltage collapse, the error is less than 0.1 p.u.

For industrial applications, PMU measurement errors may exist. In order to analyze the impact of PMU measurement errors on the proposed MBVSA method, two additional study cases with white Gaussian noise on PMU measurements are implemented. Signal to Noise Ratio (SNR) is the metric to quantify the level of noise.

Case A: The SNR is 45dB, which is a reasonable value quantified from comprehensive statistical studies from wide-scale PMUs in different voltage levels [30].

Case B: The SNR is 10dB, which is given for evaluating a worse PMU measurement environment.

Fig. 21 compares the voltage stability margins of Case A and Case B. It can be noticed that the proposed method has
Fig. 22 compares the margin improvement under different levels of noise. It can be observed that the PMU measurement errors on the VSA results can be neglected, if the measurement noise is limited.

V. CONCLUSION

A measurement-based voltage stability assessment method considering generator VAR limit has been proposed. The proposed approach is capable of predicting both Saddle Node Bifurcation and Limit Induced Bifurcation. The proposed approach gives more accurate system power transfer limits in terms of voltage stability as compared to the existing TE-based method. The proposed method is demonstrated on the two-area system and further validated using the IEEE 39-bus system.

REFERENCES

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