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A New Hybrid Approach to Thevenin Equivalent Estimation for Voltage Stability Monitoring

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Abstract—**Voltage instability may be predicted by the Thevenin equivalent based approach using only local measurements taken at the monitored buses. This paper focuses on reliable estimation of Thevenin parameters from measurements, which are crucial in judging voltage stability. Two parameter estimation methods (least squares and Kalman filter) and a proposed new hybrid method are tested on the NPCC 140-bus power system for monitoring real-time margin of the power transferred to a load area against voltage instability.**

Index Terms--**Kalman filter, least squares, Thevenin equivalent, voltage collapse, voltage instability.**

I. INTRODUCTION

Under a substation of a power system, when consumers turn on any kind of equipment, the load connected to the substation increases as a whole. As more and more consumers use electric energy, this can increase the load to the point that it puts stress on the system. A system under stress can undergo voltage collapse if a sudden contingency occurs. This can lead to blackout in the entire system. In addition to active power demand, the load places a demand on reactive power. If the amount of reactive power is too low and no preventative measures are taken, the change in the voltage at the load bus can lead to voltage collapse. Current methods for generating reactive power include supplying by generator through transmission networks, or compensating locally at load buses by shunt capacitors, SVC, STATCOM and synchronous condensers. This corrective action will save the system from voltage instability, but the indication that the system is near voltage collapse is needed in order to signal if this action is appropriate. Traditionally, the corrective action has been to shed load if the voltage drops below a set point, but as demonstrated in [1], this approach is not the most reliable solution. In order to determine the proximity to voltage collapse, it is important to consider maximum power transfer. If the load should increase any further past the maximum power transfer threshold, it will cause the system to become unstable and can lead to voltage collapse. Maximum power transfer can be calculated with the help of the Thevenin equivalent. This paper expands on the works done in [1]-[7], in which the least squares method and the Kalman filter method are used to estimate the Thevenin equivalent of the external system when a load bus or area is monitored against voltage instability. This paper compares these two methods on the NPCC (Northeast Power Coordinating Council) 140-bus power system and accordingly improves the Kalman filter method in [5] by proposing a hybrid method integrating the two methods for increased accuracy in estimating Thevenin parameters.

II. MULTIBUS THEVENIN EQUIVALENT BASED APPROACH

In this paper, the system is set up so that a wide load area is aggregated in one fictitious load [6] using the expressions:

$$\bar{I} = \sum_i (\bar{S}_i / \bar{V}_i)^* \tag{1}$$

$$\bar{S}_L = \sum_i \bar{S}_i \tag{2}$$

$$\bar{V}_L = \bar{S}_L / \bar{I}^* \tag{3}$$

$$\bar{Z}_L = \bar{V}_L / \bar{I} \tag{4}$$

where $\bar{S}_i$, $\bar{V}_i$ are complex power and voltage measured at bus *i*.

The rest of the system is represented by the Thevenin source voltage and Thevenin impedance as shown in Fig. 1.

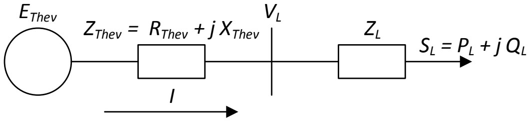

Fig. 1. Thevenin equivalent and local load.

Using the voltage divider equation and (3), the following expression for complex power can be obtained:

$$\bar{S}_L = \bar{Z}_L \frac{\bar{E}_{Thev}}{\bar{Z}_{Thev} + \bar{Z}_L} \left(\frac{\bar{E}_{Thev}}{\bar{Z}_{Thev} + \bar{Z}_L} \right)^* . \tag{5}$$

$$\bar{Z}_{Thev} = Z_{Thev} \cos(\beta) + j Z_{Thev} \sin(\beta), \tag{6}$$

$$\bar{Z}_L = Z_L \cos(\varphi) + j Z_L \sin(\varphi). \tag{7}$$

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Equations (6) and (7) turn the Thevenin impedance and load impedance into their trigonometric forms. These can be substituted back into (5) and the real part of simplified expression is taken to obtain the active power

$$P_L = \frac{E_{Thev}^2 Z_L \cos(\varphi)}{Z_{Thev}^2 + Z_L^2 + 2Z_{Thev}Z_L\cos(\varphi - \beta)}. \qquad (8)$$

In order to find the maximum power, cos(φ) is assumed constant and the derivative of (8) is taken. It is set equal to zero and solved, showing that Z_L is equal to Z_{Thev} at maximum power. By making substitution, $Z_L = Z_{Thev}$, and simplifying, the maximum power equation is obtained

$$P_{Max} = \frac{E_{Thev}^2 \cos(\varphi)}{2Z_{Thev}(\cos(\varphi - \beta) + 1)}. \qquad (9)$$

III. HYBRID METHODS FOR PARAMETER ESTIMATION

The Thevenin source voltage and Thevenin impedance in (9) must be determined through a parameter estimation method. This paper proposes a hybrid method that takes a sliding window from the least squares method and applies it to the Kalman filter method.

By using Kirchhoff's voltage law

$$\bar{V}_L = \bar{E}_{Thev} - \bar{Z}_{Thev}\bar{I}. \qquad (10)$$

Assuming $\bar{E}_{Thev} = E_r + jE_i$, $\bar{V}_L = u + jw$, $\bar{I} = g + jh$, and $\bar{Z}_{Thev} = R_{Thev} + jX_{Thev}$, these quantities can be expanded into their rectangular forms and placed into matrix form as (11), where matrices are labeled as H, x, and z.

$$\underbrace{\begin{bmatrix} 1 & 0 & -g & h \\ 0 & 1 & -h & -g \end{bmatrix}}_{H} \times \underbrace{\begin{bmatrix} E_r \\ E_i \\ R_{Thev} \\ X_{Thev} \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} u \\ w \end{bmatrix}}_{z}. \qquad (11)$$

While g, h, u, and w can be directly measured, this still leaves four unknowns, requiring two or more measurements. For example, with three measurements the expression becomes

$$\begin{bmatrix} 1 & 0 & -g_1 & h_1 \\ 0 & 1 & -h_1 & -g_1 \\ 1 & 0 & -g_2 & h_2 \\ 0 & 1 & -h_2 & -g_2 \\ 1 & 0 & -g_3 & h_3 \\ 0 & 1 & -h_3 & -g_3 \end{bmatrix} \times \begin{bmatrix} E_r \\ E_i \\ R_{Thev} \\ X_{Thev} \end{bmatrix} = \begin{bmatrix} u_1 \\ w_1 \\ u_2 \\ w_2 \\ u_3 \\ w_3 \end{bmatrix}. \qquad (12)$$

The number of measurements that are processed is determined by making a sliding window, i.e. saving a set amount of previous measurements to be calculated as the window shifts through time. This allows new measurements to be considered in the calculation.

To solve for the Thevenin equivalent source voltage and impedance, the method of least squares can be used

$$x = (H^T H)^{-1} H^T z. \qquad (13)$$

Thevenin parameters can be also estimated using the Kalman filter that takes into account the previous measurement and works using two stages, the time update and the measurement update [8].

The time update stage is represented by the equation

$$x_k = Ax_{k-1} + Bv_k. \qquad (15)$$

where A is the state transition matrix which is set to be an identity matrix under the assumption that there is no change in the Thevenin equivalent between time steps and B is the control-input matrix which is set to zero. This stage also updates the state estimate error covariance matrix

$$P_k = AP_{k-1}A^T + Q. \qquad (16)$$

where Q is the process noise covariance matrix.

The measurement update stage takes the measurement H as described in (12) and computes a Kalman gain in order to determine how much correction will be applied to the current measurement

$$K_k = P_k H^T (HP_k H^T + R)^{-1}. \qquad (17)$$

where R is the measurement noise covariance matrix.

The measurement residual is multiplied by the Kalman gain to calculate the update to the current measurement

$$x_k = x_k + K_k(z_k - Hx_k). \qquad (18)$$

Finally, the state estimate error covariance matrix is also updated based on the new values

$$P_k = (I - K_k H)P_k. \qquad (19)$$

where I is the identity matrix.

A hybrid method is proposed integrating the features of the previous two methods. The new method utilizes the sliding window from the least squares method, as exemplified in (12); however, this array of inputs is not used in (13). Instead, it is used as inputs to H and z in (17), (18), and (19), in order to create a sliding window input to the measurement update equations. The effect of using the hybrid method is studied on the artificial 2-bus system identical to the system shown in Fig. 1. All three methods are applied to scenario of a sudden change in Thevenin parameters. The results of Thevenin source voltage estimation are shown in Fig. 2.

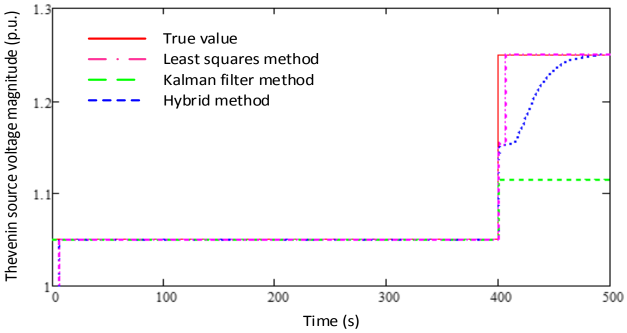


Fig. 2. Thevenin source voltage estimation.

Since the measurements here are from simulation and do not contain any noise, the least squares method performs better than the other two methods. Kalman filter method cannot estimate the value correctly after the sudden parameter change. The hybrid method reacts more slowly than the least squares method but eventually converges to the true value of the parameter. If fed with real measurements with noise, the filtering function with Kalman filter will be needed and the hybrid method will be a compromise of the other two methods.

IV. METHOD TESTING

Three methods are tested on the 140-bus, 48-machine NPCC system as shown in Fig. 3. Voltage collapse is created by load increase in the Connecticut load center (CLC) that is shown in Fig 4, where buses 5, 31 and 35 are three boundary buses. This area is topologically complex since it is supported by both New England and New York areas through three tie lines (6-5, 30-31 and 73-35) from different directions and is perfectly suited for methods testing. Two voltage collapse scenarios are investigated.

- The load continuously increases for 400s. Then generator 21 inside the CLC is tripped and the load increase continues for another 100s until the system collapses.
- The load is changed by 3 sudden steps separated by 100s. Then like the first scenario, generator 21 inside the CLC is tripped and the load increase continues for another 100s until the system collapses.

These scenarios are simulated in Powertech Labs TSAT software. The results are recorded at 30 samples per second (i.e. the typical phasor measurement unit sampling rate).

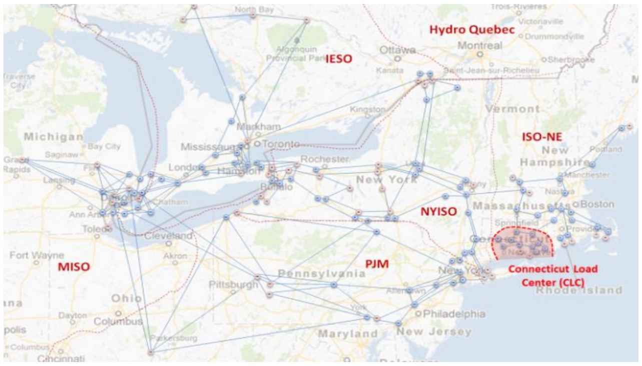


Fig. 3. NPCC system

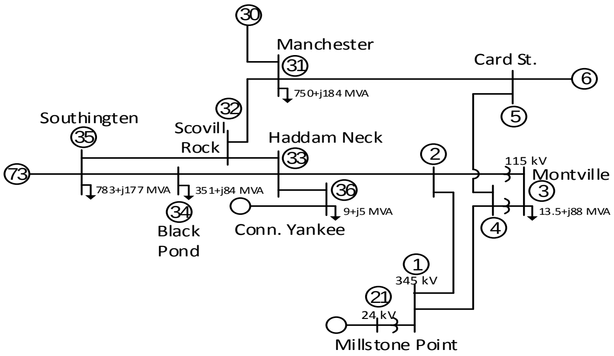
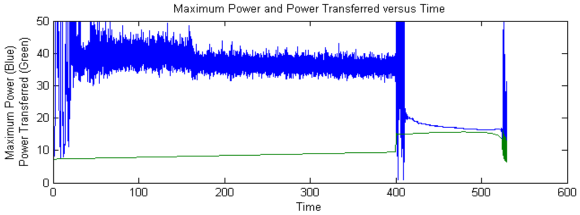


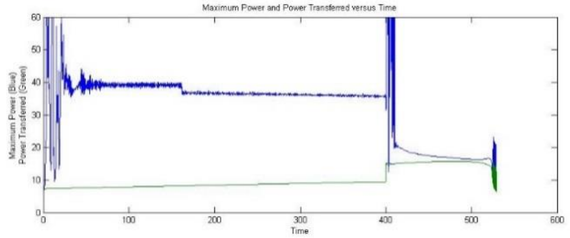
Fig. 4. Connecticut load center

A. Least Squares

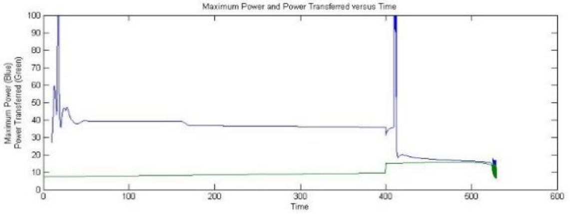
The size of the sliding window is varied and compared in Fig. 5 and Fig. 6 for different scenarios.



(a) 3 measurements

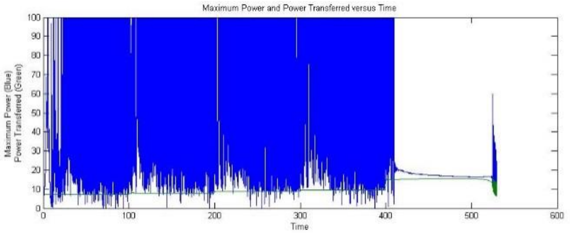


(b) 30 measurements

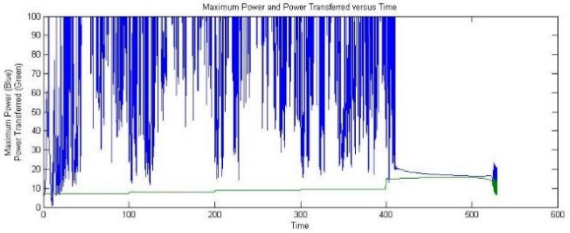


(c) 300 measurements

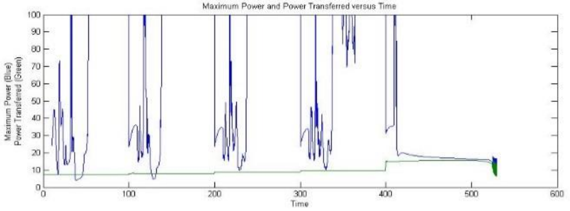
Fig. 5. Maximum power transfer for ramp change in load.



(a) 3 measurements



(b) 30 measurements



(c) 300 measurements

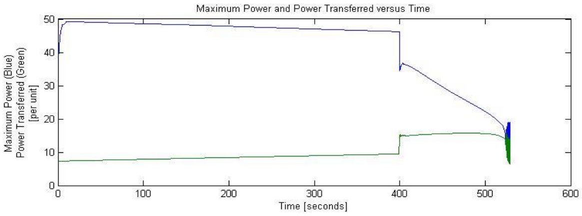
Fig. 6. Maximum power transfer for step change in load.

The method of least squares manages to create a satisfactory maximum power transfer graph with a ramp change in load, as well as providing a desirably small margin between the two parameters near the end of the simulation. It is this small margin that can be used as indication that the system is near voltage collapse.

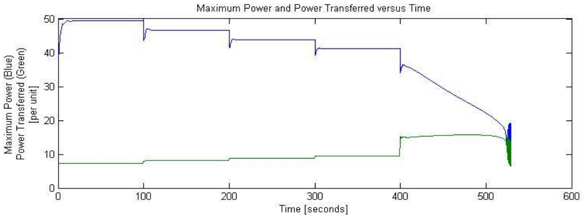
When the step change in load is introduced, the instantaneous changes followed by the periods of no change greatly distort the outcome. Effect of periods with no change can be avoided by suspending estimation during these periods.

B. *Kalman Filter*

A comparison of the two scenarios for the Kalman filter is shown in Fig. 7. This method of estimation is also capable of providing a model to estimate the Thevenin equivalent and is much more flexible towards the step changes in load. However, the margin between the two parameters near the end of the simulation is much larger, causing the indication of voltage collapse to become less reliable.



(a) Ramp change in load



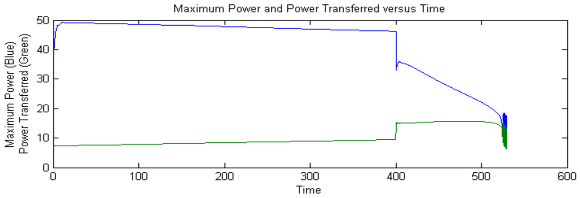
(b) Step change in load

Fig. 7. Maximum power transfer.

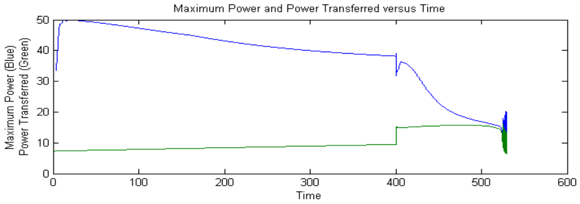
C. *Hybrid Method*

As was shown in the previous section, both methods have areas where they excel, while also having shortcomings in other areas. In fact, each method has a strength where the other lacks, and vice versa. Because of this, a hybrid method has been proposed to combine the best qualities of each method while establishing a compromise for the undesirable parts. In order to accomplish this, the sliding window used in the least squares method was used as the input to the Kalman filter method. The length of this sliding window is varied to show the effect of multiple measurements (Fig. 8 and Fig. 9).

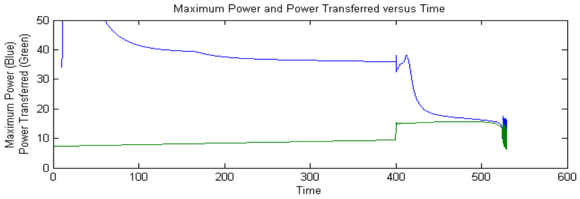
As the number of measurements increase, the maximum power transfer graph begins to shift in appearance from the Kalman filter to the least squares method. With a small number of measurements, the graphs appear the same as the original Kalman filter graphs, but with added measurements, the spikes caused by the sudden changes in load increase as associated with the Least Square method.



(a) 3 measurements

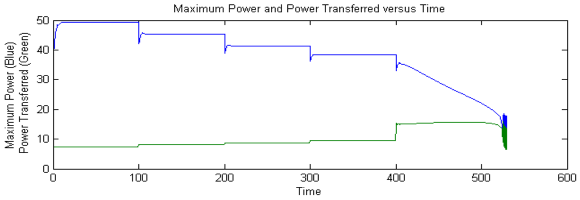


(b) 100 measurements

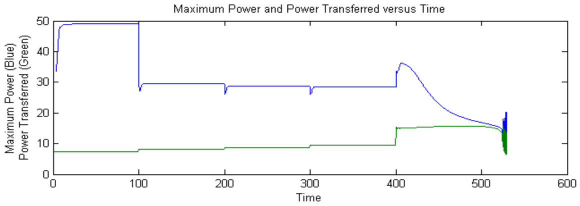


(c) 300 measurements

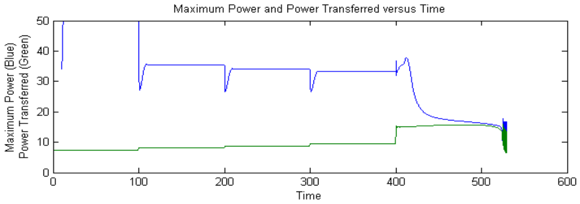
Fig. 8. Maximum power transfer for ramp change in load.



(a) 3 measurements



(b) 100 measurements



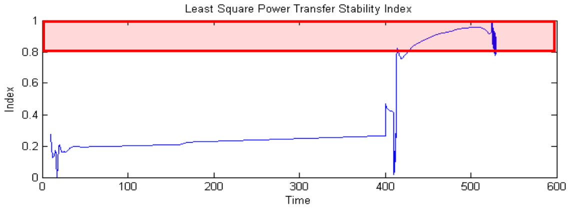
(c) 300 measurements

Fig. 9. Maximum power transfer for step change in load.

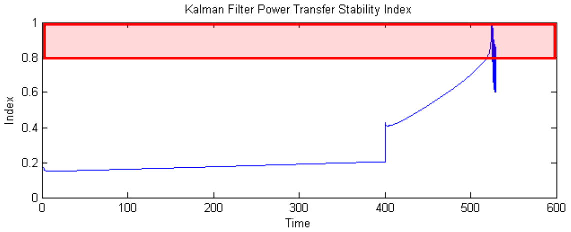
V. APPLICATIONS

These estimation methods can be used directly to show proximity to voltage collapse, or they can be applied as inputs to other indices, such as the power transfer stability index. Reference [9] provides the expression for this index, in which the power transferred is divided by the maximum power, as shown below

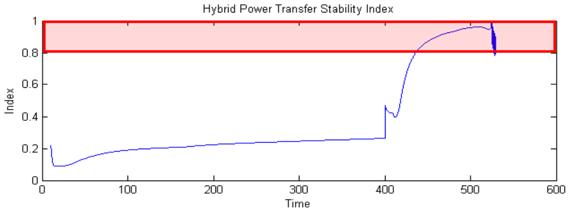
$$PTSI = \frac{P_L}{P_{Max}}. \tag{20}$$



(a) Least squares method

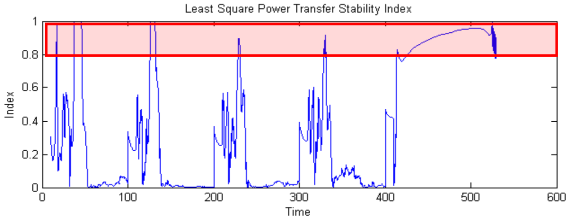


(b) Kalman filter method

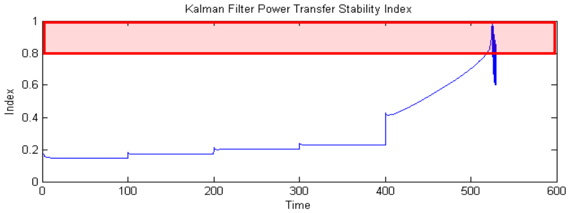


(c) Hybrid method

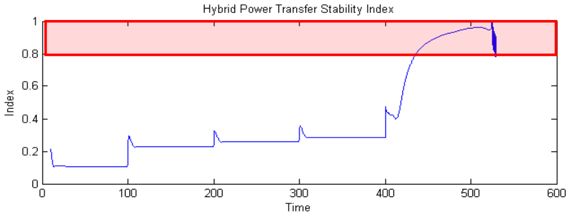
Fig. 10. Power transfer stability index for ramp change in load.



(a) Least squares method



(b) Kalman filter method



(c) Hybrid method

Fig. 11. Power transfer stability index for step change in load.

This is shown in Fig. 10 and Fig. 11 with a 20% safety margin (red area), which very effectively depicts the pros and cons of the least squares and Kalman filter methods, as well as the compromise with the hybrid method.

For the ramp change in load, the least squares method is immediately within the safety margin at the occurrence of the generator trip, which can alert the operator in the earliest time. The Kalman filter method however, does not reach the safety margin until very near voltage collapse. The hybrid method enters the safety margin later than the least squares method, but it still allows for much more time until voltage collapse than the Kalman filter method.

For the step change in load, the sporadic behavior of the least square method can lead to false alarm indications due to the spikes in the graph entering the safety margin. The Kalman filter method is able to handle the step change very well; however, the point where it enters the safety margin still does not allow for much time until voltage collapse. The hybrid method is able to handle the step change without producing false alarms as well as providing a decent time buffer before voltage collapse.

VI. CONCLUSIONS

In order to have the best indication of the system proximity to voltage collapse, a method that provides a small margin near voltage collapse and does not react sporadically towards step changes is needed. Using the least squares method, the model shows that near the point of voltage collapse, the margin is very small, but it does not provide reliable readings when step change is introduced. Using the Kalman filter method, the model is able to handle the step changes very well, but when near voltage collapse, the margin is large enough that it could fail to give indication of voltage collapse. The hybrid method is able to make a compromise between the two methods. It provides a small margin when near voltage collapse as well as being resistant to step changes.

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