a. Express the machine impedance in per unit based on the machine ratings.

b. Using the results of part (a), find the per-unit impedance based on a new \( S_{\text{base}} = 50 \text{ MVA} \) and \( V_{\text{base}} = 34.5 \text{ kV} \).

**Solution**

\[
Z_{\text{base}} = (13.8)^2 / 30 = 6.35 \Omega \\
Z_s = R_s + jX_s = 2 + j10 = 10.2 /78.7^\circ \Omega \\
Z_{su} = Z_s / Z_{\text{base}} = (2 + j10) / 6.35 = 0.315 + j1.575 \\
= 1.606 /78.7^\circ \text{ pu } \Omega
\]

\[
Z_{su,\text{new}} = 1.606 /78.7^\circ (50/30)(13.8/34.5)^2 \\
= 0.428 /78.7^\circ = 0.084 + j0.420 \text{ pu } \Omega
\]

**References**


**Problems**

3.1 Given the complex numbers \( A_1 = 5 /30^\circ \) and \( A_2 = -3 + j4 \), calculate the following, giving the answers in both rectangular and polar forms.

a. \( A_1 + A_2 \)  
   d. \( (A_2)^2 \)

b. \( A_1A_2 \)  
   e. \( A_1[1 + (A_2)^2] \)

c. \( A_1/(A_2)^* \)

3.2 For a given electrical circuit, the expressions for voltage and current as functions of time are given as follows:

\[
v(t) = 283 \sin(\omega t) \text{ V} \\
i(t) = 35 \cos(\omega t + 25^\circ) \text{ A}
\]

a. Find the rms values of the voltage and current.

3.3 The electrical network shown in Fig. 3.13 has a voltage source \( V = 100 /0^\circ \), and the values of the impedances are as follows:

\[
Z_1 = 8 - j6 \Omega, \quad Z_2 = 3 - j4 \Omega, \quad Z_3 = 5 + j5 \Omega
\]

Determine (a) the real power absorbed by each impedance and (b) the reactive power taken by each impedance.

![Figure 3.13](image)

3.4 A single-phase source supplies a load consisting of a resistor \( R = 20 \Omega \) and a capacitive reactance \( X_C = 10 \Omega \), which are connected in parallel. The instantaneous voltage of the source is given by

\[
v(t) = 120 \cos(\omega t + 45^\circ) \text{ V}
\]

Find the following:

a. Phasor voltage \( V \) of the source
b. Phasor current \( I \) supplied by the source
c. Instantaneous current \( i(t) \) supplied by the source

3.5 Repeat Problem 3.4 if the resistor and capacitor are connected in series.

3.6 For the parallel \( R - C \) load of Problem 3.4, determine

a. Instantaneous power absorbed by the resistor
b. Instantaneous power absorbed by the capacitor
c. Real power absorbed by the resistor
d. Reactive power absorbed by the capacitor
e. Power factor of the combined load

3.7 Repeat Problem 3.6 if the resistor and capacitor are connected in series.

3.8 Consider a single-phase load with an applied voltage \( v(t) \) and load current \( i(t) \) specified as follows:

\[
v(t) = 220 \cos(\omega t + 20^\circ) \text{ V} \\
i(t) = 40 \cos(\omega t - 30^\circ) \text{ A}
\]
60 CHAPTER 3 BASIC AC CIRCUIT CONCEPTS

3.9 A series circuit has an impedance of $25 \angle 53.1^\circ$ and is connected to a single-phase 220-V source.

a. Find the resistance and reactance of the load.

b. Find the real and reactive power absorbed by the load.

c. Find the power factor of the circuit, and state whether lagging or leading.

d. Calculate the reactance in ohms of capacitors to be connected in parallel with the load in order to improve the power factor to 0.9 lagging.

3.10 A single-phase source has a terminal voltage $V = 120 \angle -15^\circ$. It supplies a current of $15 / 45^\circ$ to an electrical load.

a. Find the complex power supplied by the source.

b. Determine the real power, the resistance of the source is delivering or absorbing.

c. Determine the reactive power, and state whether the source is delivering or absorbing.

3.11 Two ideal voltage sources are connected to each other through a feeder with impedance $Z = 1.5 + j 6 \, \Omega$ as shown in Fig. 3.14. Let $E_1 = 120 \angle 0^\circ$ V and $E_2 = 110 \angle 45^\circ$ V.

a. Determine the real power of each machine, and state whether the machine is supplying or absorbing real power.

b. Determine the reactive power of each machine, and state whether each machine is delivering or receiving reactive power.

c. Determine the real and reactive power of the impedance, and state whether supplied or consumed.

3.12 Repeat Problem 3.11 if the feeder impedance between the voltage sources of Fig. 3.14 is $Z = 1.5 - j 6 \, \Omega$.

3.13 A single-phase electrical load draws 10 MW at 0.6 power factor lagging.

a. Find the real and reactive power absorbed by the load.

b. Draw the power triangle.

c. Determine the kVAR of a capacitor to be connected across the load to raise the power factor to 0.95.

3.14 A capacitor is connected across the series impedance of Problem 3.9. This capacitor supplies 1000 VARs.

a. Find the real and reactive power supplied by the source.

b. Find the resultant power factor.

3.15 An industrial plant consists of several induction motors. The plant absorbs 300 kW at 0.6 PPF lagging from the substation bus.

a. Compute the required kVAR rating of the capacitor connected across the load to raise the power factor to 0.9 lagging.

b. A 200-hp, 90% efficiency, synchronous motor is operated from the same bus at rated conditions and 0.8 power factor lagging. Calculate the resulting power factor.

3.16 A 230-V source supplies two loads in parallel. One draws 5 kVA at a lagging power factor of 0.80, and the other draws 3 kW at a lagging power factor of 0.90. Find the source current.

3.17 A 440-V, 30-hp, three-phase motor operates at full load, 88% efficiency, and 65% power factor lagging.

a. Find the current drawn by the motor.

b. Find the real and reactive power absorbed by the motor.

3.18 A three-phase, 50-kVA, 600-V, 60-Hz generator operates at rated terminal voltage and supplies a line current of 48 A per phase at a 0.8 lagging power factor to a balanced three-phase load. Determine the real, reactive, and apparent power.

3.19 A 345-kV, three-phase transmission line delivers 500 MVA, 0.866 power factor lagging, to a three-phase load connected to its receiving-end terminals. Assume that the load is $\Delta$ connected and the voltage at the receiving end is 345 kV.

a. Find the complex load impedance per phase.

b. Calculate the line and phase currents.

c. Find the real and reactive power per phase.

d. Find the total real and reactive power.

3.20 Repeat Problem 3.19 assuming that the load is wye connected.

3.21 A three-phase load draws 120 kW at a power factor of 0.85 lagging from a 440-V bus. In parallel with this load is a three-phase capacitor bank that is rated 50 kVAR. Find (a) the total line current and (b) the resultant power factor.

3.22 A three-phase motor draws 40 kVA at 0.65 power factor lagging from a 230-V source. A capacitor bank is connected across the motor terminals to make the combined power factor 0.95 lagging.
a. Determine the required kVAR rating of the capacitor bank.

b. Determine the line current before and after the capacitors are added.

3.23 Two balanced wye-connected loads are connected in parallel with each other. The first draws 15 kVA at 0.8 PF lagging, and the second requires 20 kW at 0.9 PF leading. The two loads are supplied by a balanced three-phase, wye-connected, 2400-V source.

a. Determine the phasor current drawn by each load.

b. Find the real and reactive power absorbed by each load.

c. Compute the phasor current supplied by the source.

d. Calculate the total real and reactive power drawn by the combined load.

e. What is the overall power factor?

3.24 The motor of Problem 3.17 is connected to a substation bus through a three-phase feeder with an impedance 0.5 + j1.5 Ω per phase. Find the line-to-line voltage at the bus if the voltage at the motor terminals is 440 V.

3.25 A three-phase substation bus supplies two wye-connected loads that are connected in parallel through a three-phase feeder with an impedance of 0.5 + j2.0 Ω per phase. Load 1 draws 50 kW at 0.866 lagging power factor, and load 2 draws 36 kVA at 0.9 leading power factor. The line-to-line voltage at the loads is 460 V. Find the following:

a. Impedance of each load per phase

b. Total line current flowing through the feeder

c. Line-to-line voltage at the substation bus

d. Total real and reactive power supplied by the bus

3.26 A delta-connected load consists of three identical impedances $Z_Δ = 45/60°$ Ω per phase. It is connected to a three-phase, 208-V source by a three-phase feeder with conductor impedance $Z_{ab} = (1.2 + j1.6)$ Ω per phase.

a. Calculate the line-to-line voltage at the load terminals.

b. A delta-connected capacitor bank with a reactance of 60 Ω per phase is connected in parallel with the load at its terminals. Find the resulting line-to-line voltage at the load terminals.

3.27 The total power being absorbed by a balanced three-phase load is measured using the two-wattmeter method. The phase sequence is abc. The current coils of the wattmeters are connected in lines a and b. Let the readings of the two meters be $P_a$ and $P_b$, respectively. The line voltage is 2400 V, and the load is 30 kVA.

a. Show a wiring diagram.

b. Calculate $P_a$ and $P_b$ if the power factor is 1.0.

c. Calculate $P_a$ and $P_b$ if the power factor is 0.2 lagging.

d. Calculate $P_a$ and $P_b$ if the power factor is 0.5 leading.

e. Sketch the phasor diagram showing all voltages and currents for each case.

3.28 Using 100 MVA and 115 kV as base values, express 110 kV, 75 MVA, 375 A, and 26.5 Ω in per-unit and percent values.

3.29 The per-unit impedance of a single-phase electric load is 0.5. The base power is 200 kVA, and the base voltage is 12 kV.

a. Find the per-unit impedance of the load if 400 kVA and 24 kV are selected as base values.

b. Find the ohmic value of the impedance.

3.30 A three-phase, 350-MVA, 13.8-kV AC generator has a synchronous reactance of 1.20 per unit. The generator is connected to a circuit for which the specified bases are 100 MVA, 13.2 kV.

a. Find the per-unit value of the generator synchronous reactance on the specified bases.

b. Find the ohmic value of the synchronous reactance.

3.31 A single-phase source is connected to an electrical load. The load draws a 0.6 pu current at 1.10 pu voltage while taking a real power of 0.4 pu at a lagging power factor. Choose a base voltage of 8 kV and a base current of 125 A. Calculate the following:

a. Real power in kW

b. Reactive power in kVAR

c. Power factor

d. The ohmic values of the resistance and the reactance of the load