ECE 325: MIDTERM EXAM II  
Spring 2017

NAME:

There are 6 problems be sure to show all work clearly.

1. **Synchronous Machine (20 points):** A three-phase synchronous generator is connected to a load (infinite bus) as sketched in the one-line diagram below.

   Circuit Parameters:
   - Armature resistance: \( R_a \approx 0.0 \)
   - Synchronous reactance: \( X_s = 0.3 \text{ p.u.} \)
   - Internal voltage: \( E_a = 1.2 \text{ p.u.} \)
   - Terminal voltage: \( V_t = 1.0 \text{ p.u.} \)

   (a) At what rotor angle \( \delta \) will the generator operate if the real power output is 1.25 p.u.?
   (b) Find the armature current for this power output in p.u.?
   (c) Sketch the phasor diagram (showing \( V_t, E_a, I_a \) and \( S_{out} \) where \( I_a \) is the armature current and \( S_{out} \) is the apparent power output).

\[ E_a \angle \delta = 1.3 \angle \delta \]

\[ V_t = 1 \quad X_s = 0.5 \quad \frac{X_s}{E_a} = \frac{0.5}{1.3} \]

\[ V_t = 1 \quad \frac{X_s}{E_a} = \frac{0.5}{1.3} \]

\[ S_{out} = 1.25 \]

**a)** \( P_{out} = \frac{E_a V_t \sin \delta}{X_s} \)  
Substituting and solving  
\[ S = 18.21^\circ \]  
\[ S = 28.74^\circ \]

\[ (E_a = 1.2; X_s = 0.3) \]

\[ (E_a = 1.3; X_s = 0.5) \]

**b)** \( I_a = \frac{E_a - V_t}{j X_s} = 1.33 \angle -20.46^\circ \) p.u.

**c)**

\[ S_{out} \]
\[ \overline{E_a} \]
\[ \overline{V_t} \]
\[ \overline{I_a} \]
2. General Rotating Machine (20 points): A two pole cylindrical rotor AC generator can be modeled by the parameters given below. The field is connected to a source with \( i_f(t) = 10 \) A and the stator current output is \( i_s(t) = 10 \sin(337t + \pi/4) \) A. The rotor is spinning at a constant rate of 3600 RPM and the stator is open circuited. Neglect winding and rotational losses.

**Machine Parameters:**
- Armature (stator): \( L_{ss} = 4.5 \) H, \( N_s = 100 \) turns
- Field (rotor): \( L_{rr} = 2 \) H, \( N_r = 50 \) turns
- Coupling: \( L_{sr} = 3 \cos \theta_m \) H

(a) Find the stored magnetic energy in the machine as a function of time.

(b) Find the armature terminal voltage as a function of time.

\[
\begin{align*}
(\text{a}) \quad W_t &= \frac{1}{2} L_{ss} i_s^2 + \frac{1}{2} L_{rr} i_r^2 + L_{sr} i_s i_r \\
&= 2.25 \sin^2 (337t + \pi/4) + 100 + 300 \cos (337t + \theta_m) \sin (337t + \pi/2) \\
&= \\
(\text{b}) \quad &\text{If you assumed } i_s(t) = 0 \text{ then} \\
&\quad e_s(t) = \frac{d}{dt} (L_{sr} i_r) = -\frac{d}{dt} [30 \cos (337t + \theta_m)] \\
&\quad = 11.3 \sin (337t + \theta_m) \text{ kV} \\
&\text{If you used } i_s(t) = 10 \sin (337t + \pi/4) \text{ then} \\
&\text{also have voltage drop across } L_{ss} (L_{ss} \frac{d}{dt} i_s(t)) \\
&\quad e_s(t) = 11.3 \sin (337t + \theta_m) - 18.2 \cos (337t + \pi/4) \text{ kV}
\end{align*}
\]
3. **Three phase Induction Motor (20 points).** For a three phase 381V, 60 Hz, 4 pole induction motor, you are given the per phase parameters below. When operating at full load, this motor has a slip of 3%. Find: (a) the frequency (or frequencies) of the rotor current at full load, (b) the power developed (electrical power converted to mechanical power) when operating at full load, and (c) the mechanical torque under full load. Neglect any rotational losses.

**Motor parameters:**

\[
R_1 = 0.2 \, \Omega \quad R_2 = 1.0 \, \Omega \quad X_m = 80 \, \Omega \\
X_1 = 0.6 \, \Omega \quad X_2 = 2.0 \, \Omega
\]

\[(a) \quad f_r = s f_s = 1.8 \, \text{Hz} \]

\[(b) \quad V_e = 220 \quad |i_2| = 6.54 \]

\[
P_{\text{dev}} = \frac{1 - s}{s} R_2 i_2^2 \cdot 3 \\
= 4.15 \, \text{kW}
\]

\[(c) \quad T_m = \frac{P_{\text{dev}}}{\omega_m} \\
= 27.7 \, \text{N} \cdot \text{m}
\]
4. DC Motor (15 points). A shunt connected 250 V DC motor has a armature resistance of $R_a = 3 \, \Omega$ and field winding resistance of $R_f = 100 \, \Omega$. When the motor draws 4.0 A, it rotates at 1000 RPM. Neglect brush voltage drop and rotational losses and find:

(a) the motor constant $K_a$, and
(b) the mechanical rotational speed (in RPM) when the motors draws 12 A with the same 250 V supply.

\[ \omega_m = \frac{1000 \, \left( \frac{2\pi}{60} \right)}{60} = 104.7 \, \text{rad/s} \]

(a) (i) Find $I_f$: $I_f = 2.5 \, \text{A}$

(ii) Find $E_a$: $I_a = 4.2.5 = 1.5 \, \text{A}$

(iii) Now $E_a = V_f - R_a I_a = 245.5$

(iv) $E_a = K_a I_f \omega_m \Rightarrow K_a = 0.938$

(b) Now $I_a = 9.5 \, \text{A}$ so $E_a = 221.5 \, \text{V}$

\[ \omega_m = \left( \frac{K_a I_f}{E_a} \right)^{-1} = 94.5 \, \text{rad/s} \]

\[ = 902 \, \text{RPM} \quad \text{(Good speed regulation)} \]
5. Mechanical/Electrical Energy Relationships (10 points). A motor is operating at 1000 RPM and measurements show it consumes 1 kW. Find:
(a) the stored rotational energy, if the rotational inertia of the machine is \( J = 0.30 \) kg-m\(^2\).
(Note: kg-m\(^2\) \( \equiv \) N-m-s\(^2\));
(b) the useful mechanical torque developed if the net losses (electrical and mechanical) are 100 W at this operating point.

(a) \[ \overline{W_m} = \frac{1}{2} J \omega_m^2 \] (all values given)
= 1.64 kJ

(b) \[ P_m = \overline{T_m} \omega_m \] so \[ P_m = 900 \] (subtracting losses)
\[ \overline{T_m} = 8.6 \text{ N-m} \]
6. Short Answer (15 points).

(a) What useful performance characteristic does a shunt connected DC motor exhibit?

Good speed regulation

(b) When a synchronous generator is providing real power, is the magnetic field of the rotor aligned with the rotating field of the stator? Explain your answer (not just yes or no).

No, because $\sin(\phi)$ so $S$ must be non-zero for any real power output

(c) A four pole three phase induction motor has a 50 Hz signal applied to the stator windings and runs at 5% slip. What is (are) the rotor current frequency(s)? What is the rotational speed of the motor in RPM?

$f_s = 2.5 \text{ Hz} \quad \text{(just the slip frequency)}$

$n_m = \frac{(1-s)f_s}{\rho/2} \cdot 60 = 1425 \text{ RPM}$