Example 10.5

The one-line diagram of a simple power system is shown in Figure 10.16. The neutral of each generator is grounded through a current-limiting reactor of 0.25/3 per unit on a 100-MVA base. The system data expressed in per unit on a common 100-MVA base is tabulated below. The generators are running on no-load at their rated voltage and rated frequency with their emfs in phase.

Determine the fault current for the following faults.

(a) A balanced three-phase fault at bus 3 through a fault impedance $Z_f = j0.1$ per unit.

(b) A single line-to-ground fault at bus 3 through a fault impedance $Z_f = j0.10$ per unit.

(c) A line-to-line fault at bus 3 through a fault impedance $Z_f = j0.1$ per unit.

(d) A double line-to-ground fault at bus 3 through a fault impedance $Z_f = j0.1$ per unit.

<table>
<thead>
<tr>
<th>Item</th>
<th>Base MVA</th>
<th>Voltage Rating</th>
<th>$X^1$</th>
<th>$X^2$</th>
<th>$X^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
<td>100</td>
<td>20 kV</td>
<td>0.15</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>$G_2$</td>
<td>100</td>
<td>20 kV</td>
<td>0.15</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>$T_1$</td>
<td>100</td>
<td>20/220 kV</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$T_2$</td>
<td>100</td>
<td>20/220 kV</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$L_{12}$</td>
<td>100</td>
<td>220 kV</td>
<td>0.125</td>
<td>0.125</td>
<td>0.30</td>
</tr>
<tr>
<td>$L_{13}$</td>
<td>100</td>
<td>220 kV</td>
<td>0.15</td>
<td>0.15</td>
<td>0.35</td>
</tr>
<tr>
<td>$L_{23}$</td>
<td>100</td>
<td>220 kV</td>
<td>0.25</td>
<td>0.25</td>
<td>0.7125</td>
</tr>
</tbody>
</table>
The one-line diagram for Example 10.5.

The positive-sequence impedance network is shown in Figure 10.17.

To find Thévenin impedance viewed from the faulted bus (bus 3), we convert the delta formed by buses 123 to an equivalent Y as shown in Figure 10.17(b).

\[ Z_{1s} = \frac{(j0.125)(j0.15)}{j0.525} = j0.0357143 \]

\[ Z_{2s} = \frac{(j0.125)(j0.25)}{j0.525} = j0.0595238 \]
\[
Z_{3s} = \frac{(j0.15)(j0.25)}{j0.525} = j0.0714286
\]

Combining the parallel branches, the positive-sequence Thévenin impedance is
\[
Z_{33}^1 = \frac{(j0.2857143)(j0.3095238)}{j0.5952381} + j0.0714286
= j0.1485714 + j0.0714286 = j0.22
\]

This is shown in Figure 10.18(a).

(a) Positive-sequence network

(b) Negative-sequence network

**FIGURE 10.18**
Reduction of the positive-sequence Thévenin equivalent network.

Since the negative-sequence impedance of each element is the same as the positive-sequence impedance, we have
\[
Z_{33}^2 = Z_{33}^1 = j0.22
\]

and the negative-sequence network is as shown in Figure 10.18(b). The equivalent circuit for the zero-sequence network is constructed according to the transformer winding connections of Figure 10.6 and is shown in Figure 10.19.

To find Thévenin impedance viewed from the faulted bus (bus 3), we convert the delta formed by buses 123 to an equivalent Y as shown in Figure 10.19(b).

\[
\begin{align*}
Z_{1s} &= \frac{(j0.30)(j0.35)}{j1.3625} = j0.0770642 \\
Z_{2s} &= \frac{(j0.30)(j0.7125)}{j1.3625} = j0.1568807 \\
Z_{3s} &= \frac{(j0.35)(j0.7125)}{j1.3625} = j0.1830257
\end{align*}
\]

Combining the parallel branches, the zero-sequence Thévenin impedance is
\[
Z_{33}^0 = \frac{(j0.4770642)(j0.2568807)}{j0.7339449} + j0.1830275 \\
= j0.1669725 + j0.1830275 = j0.35
\]
FIGURE 10.19
Zero-sequence impedance diagram for Example 10.5.

FIGURE 10.20
Zero-sequence network for Example 10.5.

The zero-sequence impedance diagram is shown in Figure 10.20.

(a) Balanced three-phase fault at bus 3.

Assuming the no-load generated emfs are equal to 1.0 per unit, the fault current is

\[
I_3^a(F) = \frac{V_3^a(0)}{Z_{33} + Z_f} = \frac{1.0}{j0.22 + j0.1} = -j3.125 \text{ pu}
\]

\[
= 820.1 \angle -90^\circ \text{ A}
\]

(b) Single line-to-ground fault at bus 3.

From (10.62), the sequence components of the fault current are
\[ I_3^0 = I_3^1 = I_3^2 = \frac{V_3^a(0)}{Z_{33}^1 + Z_{33}^2 + Z_{33}^0 + 3Z_f} = \frac{1}{1.0} = \frac{j0.22 + j0.22 + j0.35 + 3(j0.1)}{-j0.9174 \text{ pu}} \]

The fault current is
\[
\begin{bmatrix} I_3^0 \\ I_3^1 \\ I_3^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_3^0 \\ I_3^1 \\ I_3^2 \end{bmatrix} = \begin{bmatrix} 3I_3^0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -2.7523 \\ 0 \\ 0 \end{bmatrix} \text{ pu}
\]

(c) Line-to-line fault at bus 3.

The zero-sequence component of current is zero, i.e.,
\[ I_3^0 = 0 \]

From (10.75), the positive- and negative-sequence components of the fault current are
\[ I_3^1 = -I_3^2 = \frac{V_3^a(0)}{Z_{33}^1 + Z_{33}^2 + Z_f} = \frac{1}{j0.22 + j0.22 + j0.1} = -j1.8519 \text{ pu} \]

The fault current is
\[
\begin{bmatrix} I_3^1 \\ I_3^2 \\ I_3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j1.8519 \\ j1.8519 \end{bmatrix} = \begin{bmatrix} 0 \\ -3.2075 \\ 3.2075 \end{bmatrix}
\]

(d) Double line-to-line fault at bus 3.

From (10.88), the positive-sequence component of the fault current is
\[ I_3^1 = \frac{V_3^a(0)}{Z_{33}^1 + Z_{33}^2 + 3Z_f} = \frac{1}{j0.22 + \frac{j0.22(j0.35 + j0.3)}{j0.22 + j0.35 + j0.3}} = -j2.6017 \text{ pu} \]

The negative-sequence component of current from (10.87) is
\[ I_3^2 = \frac{-V_3^a(0) - Z_{33}^1 I_3^1}{Z_{33}^2} = -\frac{1 - (j0.22)(-j2.6017)}{j0.22} = j1.9438 \text{ pu} \]

The zero-sequence component of current from (10.86) is
\[ I_3^0 = \frac{-V_3^a(0) - Z_{33}^1 I_3^1}{Z_{33}^0 + 3Z_f} = -\frac{1 - (j0.22)(-j2.6017)}{j0.35 + j0.3} = j0.6579 \text{ pu} \]
and the phase currents are

\[
\begin{bmatrix}
I_3^a \\
I_3^b \\
I_3^c
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 \\
1 & a^2 & a \\
1 & a & a^2
\end{bmatrix} \begin{bmatrix}
j0.6579 \\
-j2.6017 \\
j1.9438
\end{bmatrix} =
\begin{bmatrix}
0 \\
4.058\angle165.93^\circ \\
4.058\angle14.07^\circ
\end{bmatrix}
\]

The fault current is

\[I_3(F) = I_3^b + I_3^c = 1.9732\angle90^\circ\]

10.8 UNBALANCED FAULT ANALYSIS USING BUS IMPEDANCE MATRIX

We have seen that when the network is balanced, the symmetrical components impedances are diagonal, so that it is possible to calculate \(Z_{bus}\) separately for zero-, positive-, and negative-sequence networks. Also, we have observed that for a fault at bus \(k\), the diagonal element in the \(k\) axis of the bus impedance matrix \(Z_{bus}\) is the Thévenin impedance to the point of fault. In order to obtain a solution for the unbalanced faults, the bus impedance matrix for each sequence network is obtained separately, then the sequence impedances \(Z_{kk}^0, Z_{kk}^1, \text{ and } Z_{kk}^2\) are connected together as described in Figures 10.11, 10.13, and 10.15. The fault formulas for various unbalanced faults is summarized below. In writing the symmetrical components of voltage and currents, the subscript \(a\) is left out and the symmetrical components are understood to refer to phase \(a\).

10.8.1 SINGLE LINE-TO-GROUND FAULT USING \(Z_{bus}\)

Consider a fault between phase \(a\) and ground through an impedance \(Z_f\) at bus \(k\) as shown in Figure 10.21. The line-to-ground fault requires that positive-, negative-, and zero-sequence networks for phase \(a\) be placed in series in order to compute the zero-sequence fault current as given by (10.62). Thus, in general, for a fault at bus \(k\), the symmetrical components of fault current is

\[
I_k^0 = I_k^1 = I_k^2 = \frac{V_k(0)}{Z_{kk}^1 + Z_{kk}^2 + Z_{kk}^0 + 3Z_f}
\]

(10.90)

where \(Z_{kk}^1, Z_{kk}^2, \text{ and } Z_{kk}^0\) are the diagonal elements in the \(k\) axis of the corresponding bus impedance matrix and \(V_k(0)\) is the prefault voltage at bus \(k\). The fault phase current is

\[I_k^{abc} = AI_k^{012}\]

(10.91)