State Estimation

We will say the network is observable if there are enough measurements to determine the system state. Those measurements have to be in the right locations.

Focus on linearized equations: (i.e. DC load flow) so the state is the bus angles.

Recall is the nodal incidence matrix. A connected network is observable iff

\[ \forall S \neq H \beta = 0 \implies A S = 0 \]

(where $\beta = H \beta + x$)

This says that a state gives rise to zero measurements implies the line angles are zero.
The following are equivalent:

(i) The network is observable
(ii) \( H \) is of full rank.
(iii) \( H^T H \) is non-singular

Example:

1. Be the slack \( \mathbf{b} = 0 \)
2. \( x_{ij} = 1 \), \( R = I \)
3. \( H = \begin{bmatrix} -1 & -1 \\ -1 & 0 \end{bmatrix} \) full rank

Clearly need at least two measurements.

How about errors? Given \( H \) how will the errors distribute?

Our best estimate \( \hat{S} = (H^T R^{-1} H) H^T R^{-1} \hat{z} \)

the error is \( \hat{e} = \hat{z} - H \hat{S} \)

Substitute \( \hat{e} = \hat{S} \hat{z} - H (H^T R^{-1} H) H^T R^{-1} \hat{z} = D \hat{z} \)

with \( D = (I - H (H^T R^{-1} H) H^T R^{-1}) \)
Example (cont.)

\[ D = \mathbb{I} - H (H^T H)^{-1} H^T \] since \( D' = I \)

\[ = \mathbb{I} - \left( \frac{H^T H}{I} \right) \] since \( H \) is invertible

\[ = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \]

Errors will never be seen by the estimate. Both measurements are needed (i.e., critical) for observability.

Example (cont.)

- Now add a measurement of flow from 1 to 3

So \( H = \begin{bmatrix} -1 & -1 \\ -1 & 0 \end{bmatrix} \) and \( H^T H = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \)
\[
\begin{bmatrix}
\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\
\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\
\frac{1}{3} & -\frac{1}{3} & \frac{2}{3}
\end{bmatrix}
\]

and
\[
\begin{bmatrix}
\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\
-\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
-\frac{1}{3} & \frac{1}{3} & \frac{1}{3}
\end{bmatrix}
\]

\(D\) is rank 1 and any error will appear equally from the best estimate.

Errors are detectable but not identifiable.

\( \Rightarrow \) Develop an approach to know which measurements have large errors - defect and identify.