Quadratic Programming

\[
\min_x \quad c^T x + \frac{1}{2} x^T Q x
\]

with \( Ax = b \) \( x \geq 0 \) (ignore inequalities)

(typical generator costs are quadratic)

- Assume \( Q \) is a positive definite and symmetric (easily satisfied and then the problem is convex)

- Apply K-T conditions

  - Form Lagrangian
    \[
    \mathcal{L}(x, \lambda) = c^T x + \frac{1}{2} x^T Q x + \lambda^T (Ax - b) + \mu^T x
    \]

  - Take derivatives
    \[
    \frac{\partial \mathcal{L}}{\partial x} = 0 \Rightarrow x^T Q + \lambda^T A + \mu = 0
    \]
    \[
    \Rightarrow A^T \lambda + Q x + \mu = -c
    \]
- Also $2\|x\|_1 / 2 x = 0$
  \[ Ax = b \]

- Now include the $\leq 0$ term so $2\|x\|_1 / 2 x$
  \[ Qx + A^T \mu = -c \]

if $x > 0$ then $\mu = 0$ and if $x = 0$ then $\mu \geq 0$

- Equivalent LP

\[
\min \; \sum \mu_i \\
\text{with } Qx + A^T \mu = -c \\
Ax = b \quad \mu \geq 0 \quad x \geq 0
\]

- We can solve any QP or LP with essentially the same algorithm.
Homework this week
- Solve both LP / QP
- Impose various constraints
- You can use linprog and quadprog in Matlab to solve.

Electricity Markets
- Open bidding of prices submitted by generation companies (GenCo's)
  - Electricity price is given by the Market Clearing Price (MCP)
    - The price to supply the next increment
  - Assume no network constraints and no losses
    - ED and if GenCo's bid costs
      - then the result is the same
      - Price is the system lambda (price of next increment)
- This price is simply the dual variable from the constraint $\sum P_i = \sum P_i$ (ignoring other constraints).

- Most markets use some form of Locational Marginal Prices (LMPs),
  - Cost for supplying the next increment at a given node.
  - That price is in theory the dual variable associated with the power flow constraint at that bus.

- Practically, no market does a full or “true” OPF to determine LMPs (some variation of a DC OPF is used).

LMP = Energy + Congestion + Loss
Load Balancing

- Load is always changing & schedule is always inexact
- Maintain energy balance to maintain frequency
- Steady-state analysis
- What happens when there is a load change (increase)
  - Energy pulled from spinning units
    - Frequency decreases
    - For this course, all units see the same frequency change
  - To arrest frequency decline, governor acts to increase power output
- Droop characteristic - fully local control

slope = -R \left( \frac{Hz}{MW} \right)

R = Regulation constant
(neglecting any deadband)

\Delta P_m = -\frac{1}{R} \Delta f + \Delta P / f

+ assume 0

Typically R say 5% so a small change in f has
a fairly large change in power output.
- Multi-machine case

\[ \Delta P_m = \sum_{i=1}^{n} \Delta P_{mi} \]

- Assume no reference change so

\[ \Delta P_{mi} = -\frac{1}{R_i} \Delta f \]

\[ \sum_{i=1}^{n} \Delta P_{mi} = -\Delta f \sum_{i=1}^{n} \frac{1}{R_i} \]

Let \( \beta \) = frequency response characteristic

\[ \sum_{i=1}^{n} \frac{1}{R_i} = \frac{\sum_{i=1}^{n} P_i}{\sum_{i=1}^{n} R_i} \]

\[ \Delta P_m = -\beta \Delta f \]

and assuming a steady-state is reached

\[ \Delta P_{\text{load}} = \Delta P_m \]

the \( \Delta f \) can be found.
Example: Three units all with $P = 5\%$ and unit ratings are 1600 MVA, 750 MVA, 500 MVA.
Load increases by 200 MW.

- Use a 1000 MVA base

\[
R_1 = 0.05 \\
R_2 = 0.05 \left( \frac{1000}{750} \right) = 0.067 \\
R_3 = 0.05 \left( \frac{1000}{500} \right) = 0.10
\]

\[
\beta = 1 + \frac{1}{R_1} + \frac{1}{R_2} = 45 \text{ p.u.}
\]

- Then

\[
\Delta f = -\frac{1}{R_3} \Delta P_m = -\frac{1}{45} (0.2) \quad = -4.4 \times 10^{-3} \text{ p.u.}
\]

\[
\Delta f = (-.0002667 \text{ Hz})
\]

- Each unit picks up power based on \( \Delta P_m = -\frac{1}{R_i} \Delta f \)

\[
\Delta P_{m1} = 38.9 \text{ MW} \quad \Delta P_{m2} = 66.7 \text{ MW} \quad \Delta P_{m3} = 45.4 \text{ MW}
\]

\( \Rightarrow \) Each unit increases power based on capacity.
This stops frequency decline, but
- Frequency has not been restored
- Schedules are now off
- Picked up load based on capacity not economics

Need a wide area approach to address these issues. Load Frequency Control (LFC).