Multiple Solutions

We know non-linear systems can have multiple solutions.

Given what is specified for the HFE, are there multiple possible solutions? If so, which one is correct?

Simplest example

\[ P = \frac{V_1 V_2 \sin \theta}{X} \]

\[ Q = -V_2^2 B_{22} - V_2 V_1 B_{12} \]

\[ = -\frac{V_2^2}{X} - \frac{V_2 V_1}{X} \]

How many solutions are there for a large system?

- With m equations
- Up to \( 2^m \) solutions
- Most (maybe all but 1) are not viable.

Fix \( V_1 \)

\( P = \text{P-V curve} \)

\( V_1 \Rightarrow \text{High current, low loss} \)

\( V_4 \Rightarrow \text{Low current, low loss} \)
Multiple Solutions

Example

\[ V_3 = V \]

[Diagram of circuit with labeled nodes and branches]

\[ V_1 = V_2 = \frac{1}{\sqrt{2}} \]

\[ P_3 = P \]

\[ P_2 = 0, \quad Q_2 = 0 \]

(i) Find \( Y_{bus} \)

\[ Y_{bus} = j \begin{bmatrix}
-\frac{1}{x} & \frac{1}{x} & 0 \\
\frac{1}{x} & \frac{2}{x} & \frac{1}{x} \\
0 & \frac{1}{x} & -\frac{1}{x}
\end{bmatrix} \]

(iii) Need to find \( V_2, S_2 \)

\[ P_2 = V_2 \sum_{j=1}^{3} V_j (B_{2j} \sin (\theta_2 - \theta_j)) = 0 \]

\[ = V_2 \sqrt{3} B_{z1} \sin \theta_2 + V_2 \sqrt{3} B_{z3} \sin (\theta_2 - \theta_3) = 0 \]

\[ B_{z1} = +\frac{1}{x}, \quad B_{z3} = \frac{1}{x} \]

\[ \Rightarrow \sin (\theta_2) + \sin (\theta_2 - \theta_3) = 0 \]
\[ Q_2 = -V_2 \sum_{j=1}^{3} V_j B_{2j} \cos (\delta_2 - \delta_j) = 0 \]

\[ 0 = -V_2 V B_{21} \cos \delta_2 - V_2 B_{22} \cos \delta_1 - V_2 V B_{23} \cos (\delta_2 - \delta_3) \]

\[ V_2 \left( \frac{B - \frac{3}{x}}{x} \right) = -\frac{V}{x} \left( \cos (\delta_2) + \cos (\delta_2 - \delta_3) \right) \]

\[ V_2 = -\frac{V}{Bx - z} \left( \cos (\delta_2) + \cos (\delta_2 - \delta_3) \right) \]

\[ P_3 = P = V \sum_{j=1}^{3} V_j B_{3j} \sin (\delta_3 - \delta_j) \]

\[ P = V V_2 B_{32} \sin (\delta_3 - \delta_2) \]

\[ P = \frac{V V_2}{x} \sin (\delta_3 - \delta_2) \]

Solve using \( \cos^2 x + \sin^2 x = 1 \)

\[ \cos \delta_2 = \sqrt{1 - \left( \frac{P x}{V_2 V} \right)^2} = \cos (\delta_2 - \delta_3) \]
Then substitute

\[ V_2 = \sqrt{\frac{2}{Bx - 2} \left( \frac{1}{2} \left( V^2 + \sqrt{V^4 - (p_x)^2 (x - 2)^2} \right) \right)^\frac{1}{2}} \]

e.g. given

\[ X = 0.063 \quad B_0 = 6.3 \quad p = 10.66 \quad u = 1.04 \]

\[ \Rightarrow \quad V_2 = 0.96 \angle 43^\circ \quad S_3 = -84.5^\circ \]

\[ \text{or} \quad V_2 = 0.87 \angle -42.9^\circ \quad S_3 = -95.8^\circ \]

Are they possible?

Which will the load flow methods find?

Apply concepts of contraction mapping to see.
\[
\begin{bmatrix}
\delta_2 \\
\delta_3 \\
V_2
\end{bmatrix}^{k+1} = \phi(x^k) = \begin{bmatrix}
\delta_2 \\
\delta_3 \\
V_2
\end{bmatrix}^k - \begin{bmatrix}
1 & 1 & 0 \\
1 & 2 & 0 \\
0 & 0 & \frac{1}{2-BX}
\end{bmatrix}
\begin{bmatrix}
V\sin(\delta_2^k) + \sin(\delta_3^k - \delta_3^k) \\
V\sin(\delta_2^k - \delta_3^k) - \frac{BX}{V_2} \\
-V(\cos(\delta_2^k) + \cos(\delta_3^k - \delta_3^k)) - (BX - 2)V_2^k
\end{bmatrix}
\begin{bmatrix}
\delta_2 \\
\delta_3 \\
V_2
\end{bmatrix}^k
\begin{bmatrix}
V\sin(\delta_2^k) - \frac{BX}{V_2} \\
V\sin(\delta_2^k - \delta_3^k) - 2\frac{BX}{V_2} \\
-\frac{V}{2-BX}(\cos(\delta_2^k) + \cos(\delta_3^k - \delta_3^k)) + V_2^k
\end{bmatrix}
\]

To know if it converges we can just check if
\(\phi(x^n)\) is a contraction over some region.

For a vector valued function if \(\phi(x^n)\) is differentiable then the Jacobian of \(\phi(.)\) must have a norm < 1.
Note also when we get near a solution

$$x^{k+1} = \varepsilon(x^k)$$

If close then the sensitivity should give you something like this (by linearizing)

$$x^k + \Delta x^{k+1} = \varepsilon(x^k) + \left[ \frac{\partial \varepsilon(x^k)}{\partial x^k} \right] \Delta x^k$$

Solution:

$$\Delta x^{k+1} = \left[ \frac{\partial \varepsilon(x^k)}{\partial x^k} \right] \Delta x^k$$

Just a difference equation

$$x^{k+1} = A x^k \implies \text{eigenvalues of within unit circle then stable}$$

$$\implies \text{Jacobian of } \varepsilon \text{ has eigenvalues within unit circle then it will converge.}$$

"Local convergence"