1. Continuation Power Flow (20 points): For the two bus system below, calculate the first predicator step (angle, voltage and load at bus 2) starting with a flat start. The load has unity power factor and the step size should be 0.1.

\[ \bar{V}_1 = 1.00 \angle 0^\circ \quad \bar{y}_{12} = -j4 \]

(i) Find \( Y_{bus} \)
\[
Y_{bus} = \begin{bmatrix} -j4 & j4 \\ j4 & -j4 \end{bmatrix}
\]

(ii) Then LFE at bus 2
\[
P_z = V_z \sum_{j=1}^{n} B_{zj} \sin (\delta_z - \delta_j) = 4V_z \sin \delta_z \quad f_z(\delta_z, V_z) = 4V_z \sin \delta_z - P_z
\]
\[
Q_z = V_z \sum_{j=1}^{n} B_{zj} \cos (\delta_z - \delta_j) = -4V_z \cos \delta_z + 4V_z^2 \bar{y}_{zj}(\delta_z, V_z)
\]

(iii) Find Jacobian for LFE at first step
\[
J = \begin{bmatrix} dP_z/d\delta_z & dP_z/dV_z \\ dQ_z/d\delta_z & dQ_z/dV_z \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}
\]

(iv) Then iteration is
\[
\begin{bmatrix} \delta \\ V \\ \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - 0.1 \begin{bmatrix} 4 & 0 & -1 \\ 0 & 4 & 0 \\ -1 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
\]
(v) Substituting with \( \mathbf{J}^{-1} = \begin{bmatrix} \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix} \)

\[
= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0.02 \\ 0.0 \\ 0.1 \end{bmatrix} 
\]

\[
= \begin{bmatrix} -0.02 \\ 1 \\ -0.1 \end{bmatrix} ; \quad P_z \text{ negative for load}
2. State Estimation (25 points): Consider the 6 bus system with the one-line diagram as sketched below. Use the DC load flow approximation with all bus voltages equal to one p.u. to answer the following questions.

- Select and find the location for a minimal number of measurements that provides for observability. For your selected measurements, generate the measurement matrix $H$.
- Select and place the minimal number of measurements needed to detect all errors in your measurement system.
- For this set of measurements that can detect all errors, what is the rank of $HP$? Show clearly your reasoning.
- Extra credit: For this set of measurements that can detect all errors, what is the rank of the projection $D$ (i.e., the projection from the measurements $\mathbf{z}$ to the estimated errors, $\hat{\mathbf{e}}$)? Show clearly your reasoning.

(i) Assume bus 1 is slack, then state is $[v_2 \ v_3 \ v_4 \ v_5 \ v_6]$. Minimal measurement set is 5, e.g., all flows except 1-6

\[
\begin{bmatrix}
p_{12} \\
p_{23} \\
p_{34} \\
p_{45} \\
p_{56}
\end{bmatrix} = H \begin{bmatrix}
v_2 \\
v_3 \\
v_4 \\
v_5 \\
v_6
\end{bmatrix},
\quad H = \begin{bmatrix}
-5 & 0 & 0 & 0 & 0 \\
0 & -5 & 0 & 0 & 0 \\
0 & 10 & -10 & 0 & 0 \\
0 & 0 & 5 & -5 & 0 \\
0 & 0 & 0 & 5 & -5
\end{bmatrix}
\]

(ii) Add one more branch measurement from 6-1, which forms loop with all measurements.

(iii) Rank $H$ is 5. System is observable so $H$ full rank (detectability doesn't change rank of $H$)

(iv) Rank of $H$ is 1. All measurement errors are detectable but none identifiable $\Rightarrow$ rank is 1.
3. Iterative solution to equations (15 points): It is desired to find the solution to the equation below. Assume a starting guess of $x=0.25\pi$ and perform one iteration using the Newton-Raphson method. BE SURE TO SHOW YOUR WORK.

$$5 = x^3 - 4\cos x$$

(i) $f(x) = 0 = x^3 - 4\cos x - 5$

\[ \frac{df(x)}{dx} = 3x^2 + 4\sin x \]

(ii) Iterative form

\[ x^{(i)} = x^{(0)} - \frac{f(x^{(0)})}{\frac{df(x^{(0)})}{dx}} \]

\( \Rightarrow x^{(1)} = 2.335 \)

(Will converge to 1.6656)
4. Power flow solutions – Fast Decoupled Newton-Raphson algorithm (20 points): For the system below, use the fast decoupled Newton-Raphson algorithm to find the first update for the voltage magnitude and angles at all buses. Assume a flat start.

Bus 1
(slack bus) \( \bar{V}_1 = 1.00 \angle 0^0 \)
\( y_{12} = -j10 \)

Bus 2
\( P_2 = 1.25 \quad V_2 = 1.00 \)
\( y_{23} = -j5 \)

Bus 3
\( P_3 = -2.0 \quad Q_3 = -0.5 \)
(PQ bus)
\( y_{30} = j2 \)

(i) Write \( \bar{Y}_{bus} \)
\[ \bar{Y}_{bus} = j \begin{bmatrix} -25 & 10 & 15 \\ 10 & -15 & 5 \\ 15 & 5 & -18 \end{bmatrix} \]

(ii) Then \( B' \) and \( B'' \)
\[ B' = \begin{bmatrix} 15 \quad -5 \\ -5 \quad 20 \end{bmatrix} \]
\[ B'' = \begin{bmatrix} 18 \end{bmatrix} \]

(iii) With flat start no \( \delta \) flaws so
\[ \begin{bmatrix} \Delta P_2^{(0)} \\ \Delta P_3^{(0)} \end{bmatrix} = \begin{bmatrix} 1.25 \\ -2.0 \end{bmatrix} \]

(iii) Reactive at bus 3 \( (V_i = 1.0, \delta_i = 0) \)
\[ Q_3 = -0.5 = -B_{32} - B_{31} - B_{33} = -2.0 \quad \Rightarrow \quad \Delta Q_3^{(0)} = 1.5 \]

(iv) Angle updates
\[ \begin{bmatrix} \delta_2^{(1)} \\ \delta_3^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1.25 \\ -2.0 \end{bmatrix} \begin{bmatrix} 0.055 \\ -0.086 \end{bmatrix} \]

(v) Voltage updates
\[ V_3^{(1)} = 1 + (B'')^{-1} \begin{bmatrix} 1.5 \end{bmatrix} = 1.083 \]

(i) With flat start Jacobian simplifies to

\[
J = \begin{bmatrix}
15 & -5 & 0 \\
-5 & 20 & 0 \\
0 & 0 & 16
\end{bmatrix}
\]

So still decouples at this step.

\[\begin{bmatrix}
\Delta P_x^{(0)} \\
\Delta P_z^{(0)} \\
\Delta Q_3^{(0)}
\end{bmatrix} = \begin{bmatrix} 1.25 \\ -2.0 \\ 1.5 \end{bmatrix} \quad \text{from problem 4}
\]

(iii) Then from (41)

\[\begin{bmatrix}
S^{(i)}_2 \\
S^{(i)}_3
\end{bmatrix} = \begin{bmatrix} 0.055 \\ -0.086 \end{bmatrix}
\]

(iv) But

\[V^{(i)}_3 = 1.0938\]