

## Homework-3 Solutions

### 1) Depth-First Search

Various rules can be used for this problem. Here, the rule is: to visit adjacent nodes in alphabetical order. To prevent loops; nodes are to be visited only once.

Tacoma: Start Node

Adjacent nodes in alphabetical order:

**Bremerton** – Olympia – Renton – Seattle

Bremerton

Adjacent nodes in alphabetical order:

**Port Angeles** – Seattle

Port Angeles

Adjacent nodes in alphabetical order:

Bremerton (visited) – **Shelton**

Shelton

Adjacent nodes in alphabetical order:

**Olympia** – Port Angeles (visited)

Olympia

Adjacent nodes in alphabetical order:

Shelton (visited) – Tacoma (visited) – **Yakima**

Yakima

Adjacent nodes in alphabetical order:

**Ephrata** – Mosses Lake – Olympia (visited) – Pasco – Renton – Wenatchee

Ephrata

Adjacent nodes in alphabetical order:

**Mosses Lake** – Wenatchee – Yakima (visited)

Mosses Lake

Adjacent nodes in alphabetical order:

Ephrata (visited) – **Pasco** – Spokane – Yakima

Pasco

Adjacent nodes in alphabetical order:

Mosses Lake (visited) – **Walla Walla** – Yakima (visited)

Walla Walla

Adjacent nodes in alphabetical order:

Pasco (visited) – **Pullman**

**Path: Tacoma – Bremerton – Port Angeles – Shelton – Olympia – Yakima – Ephrata – Mosses Lake – Pasco – Walla Walla – Pullman**

## 2) Dijkstra's Shortest Path

Attached is an m file in which Dijkstra's algorithm is implemented. In addition to the provided code, the following is a step-by-step solution to this question.

Tacoma: Start Node

Adjacent Nodes

Renton = 27  
 Olympia = 31  
 Seattle = 34  
 Bremerton = 34

		Renton	Olympia	Seattle	Bremerton
0	Tacoma	27	31	34	34

Renton: at 27mi

Adjacent Nodes

Everett =  $27 + 39 = 66$   
 Seattle =  $27 + 12 = 39/34 \rightarrow 34$   
 Yakima =  $27 + 142 = 169$   
 Tacoma visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima
0	Tacoma	27	31	34	34		
27	Renton	27	31	34	34	66	169

Olympia at 31mi

Adjacent Nodes

Shelton =  $31 + 22 = 53$   
 Yakima =  $31 + 177 = 208/169 \rightarrow 169$   
 Tacoma visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Shelton
0	Tacoma	27	31	34	34			
27	Renton	27	31	34	34	66	169	
31	Olympia	27	31	34	34	66	169	53

Seattle at 34

Adjacent Nodes

Everett =  $34 + 29 = 63/66 \rightarrow 63$

Bremerton =  $34 + 34 = 68/34 \rightarrow 34$

Tacoma Visited

Renton visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Shelton
:	:	:	:	:	:	:	:	:
31	Olympia	27	31	34	34	66	169	53
34	Seattle	27	31	34	34	63	169	53

Bremerton at 34

Adjacent Nodes:

Port Angeles =  $34 + 76 = 110$

Tacoma Visited

Seattle visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Shelton	Port Angeles
:	:	:	:	:	:	:	:	:	:
34	Seattle	27	31	34	34	63	169	53	
34	Bremerton	27	31	34	34	63	169	53	110

Shelton at 53

Adjacent Nodes:

Port Angeles =  $53 + 99 = 152/110 \rightarrow 110$

Olympia Visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Shelton	Port Angeles
:	:	:	:	:	:	:	:	:	:
34	Bremerton	27	31	34	34	63	169	53	110
53	Shelton	27	31	34	34	63	169	53	110

Everett at 63

Adjacent Nodes:

Bellingham = 63 + 62 = 125  
 Wenatchee = 63 + 122 = 185  
 Renton Visited  
 Seattle Visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Shelton	Port Angeles	Bellingham	Wenatchee
:	:	:	:	:	:	:	:	:	:	:	:
53	Shelton	27	31	34	34	63	169	53	110		
63	Everett	27	31	34	34	63	169	53	110	125	185

Port Angeles at 110

Adjacent Nodes:

Bremerton Visited  
 Shelton Visited

Bellingham at 125

Adjacent Nodes:

Everett Visited

Yakima at 169

Adjacent Nodes:

Wenatchee = 169 + 106 = 275/185 -> 185  
 Mosses Lake = 169 + 100 = 269  
 Pasco = 169 + 86 = 255  
 Ephrata = 169 + 94 = 263  
 Olympia Visited  
 Renton Visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Shelton	Port Angeles	Bellingham	Wenatchee	Mosses Lake	Pasco	Ephrata
:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
63	Everett	27	31	34	34	63	169	53	110	125	185			
110	Port Angeles	27	31	34	34	63	169	53	110	125	185			
125	Bellingham	27	31	34	34	63	169	53	110	125	185			
169	Yakima	27	31	34	34	63	169	53	110	125	185	269	255	263

Wenatchee at 185

Adjacent Nodes:

Ephrata = 185 + 49 = 234/264 -> 234  
 Yakima visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Shelton	Port Angeles	Bellingham	Wenatchee	Mosses Lake	Pasco	Ephrata
:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
169	Yakima	27	31	34	34	63	169	53	110	125	185	269	255	
185	Wenatchee	27	31	34	34	63	169	53	110	125	185	269	255	234

Ephrata at 234

Adjacent Nodes:

Mosses Lake = 234 + 19 = 253/269 -> 253  
 Wenatchee visited  
 Yakima visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Shelton	Port Angeles	Bellingham	Wenatchee	Mosses Lake	Pasco	Ephrata
:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
185	Wenatchee	27	31	34	34	63	169	53	110	125	185	269	255	234
234	Ephrata	27	31	34	34	63	169	53	110	125	185	253	255	234

Mosses Lake at 253

Adjacent Nodes:

Pasco = 253 + 72 = 325/255 -> 255  
 Spokane = 253 + 105 = 358  
 Yakima Visited  
 Ephrata Visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Shelton	Port Angeles	Bellingham	Wenatchee	Mosses Lake	Pasco	Ephrata	Spokane
:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
234	Ephrata	27	31	34	34	63	169	53	110	125	185	253	255	234	
253	Mosses Lake	27	31	34	34	63	169	53	110	125	185	253	255	234	358

Pasco at 255

Adjacent Nodes:

Walla Walla = 255 + 48 = 303

Mosses Lake Visited

Yakima visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Shelton	Port Angeles	Bellingham	Wenatchee	Mosses Lake	Pasco	Ephrata	Spokane	Walla Walla
:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
253	Mosses Lake	27	31	34	34	63	169	53	110	125	185	253	255	234	358	
255	Pasco	27	31	34	34	63	169	53	110	125	185	253	255	234	358	303

Walla Walla at 303

Adjacent Nodes

Pullman = 303 + 115 = 418

Pasco visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Shelton	Port Angeles	Bellingham	Wenatchee	Mosses Lake	Pasco	Ephrata	Spokane	Walla Walla	Pullman
:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
255	Pasco	27	31	34	34	63	169	53	110	125	185	253	255	234	358	303	
303	Walla Walla	27	31	34	34	63	169	53	110	125	185	253	255	234	358	303	418

Spokane at 358

Adjacent Nodes:

Pullman = 358 + 76 = 434/418 -> 418

Mosses Lake Visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Shelton	Port Angeles	Bellingham	Wenatchee	Mosses Lake	Pasco	Ephrata	Spokane	Walla Walla	Pullman
:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
255	Pasco	27	31	34	34	63	169	53	110	125	185	253	255	234	358	303	
303	Walla Walla	27	31	34	34	63	169	53	110	125	185	253	255	234	358	303	418

Putting all previous steps in one table:

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Shelton	Port Angeles	Bellingham	Wenatchee	Mosses Lake	Pasco	Ephrata	Spokane	Walla Walla	Pullman
0	Tacoma	27	31	34	34												
27	Renton	27	31	34	34	66	169										
31	Olympia	27	31	34	34	66	169	53									
34	Seattle	27	31	34	34	63	169	53									
34	Bremerton	27	31	34	34	63	169	53	110								
53	Shelton	27	31	34	34	63	169	53	110								
63	Everett	27	31	34	34	63	169	53	110	125	185						
110	Port Angeles	27	31	34	34	63	169	53	110	125	185						
125	Bellingham	27	31	34	34	63	169	53	110	125	185						
169	Yakima	27	31	34	34	63	169	53	110	125	185	269	255				
185	Wenatchee	27	31	34	34	63	169	53	110	125	185	269	255	234			
234	Ephrata	27	31	34	34	63	169	53	110	125	185	253	255	234			
253	M-Lake	27	31	34	34	63	169	53	110	125	185	253	255	234	358		
255	Pasco	27	31	34	34	63	169	53	110	125	185	253	255	234	358	303	
303	Walla Walla	27	31	34	34	63	169	53	110	125	185	253	255	234	358	303	418

**Path: Tacoma – Renton – Yakima – Pasco – Walla – Pullman (418 Miles)**

**3) Linear Programming: See attached m file.**

**4) A\* Algorithm**

Attached is an m file in which A\* algorithm is implemented. In the provided code, latitude difference is chosen as the evaluation function. In addition to the provided code, the following is a step-by-step solution to this question. Here; the evaluation function is chosen to be the direct distance i.e. the mileage on a straight line between each city and the goal city (Pullman)

City	Estimated Distance to Pullman (miles)
Bellingham	288
Bremerton	270
Ephrata	120
Everett	255
Moses Lake	108
Olympia	273
Pasco	101
Port Angeles	313
Pullman	0
Renton	245
Seattle	250
Shelton	286
Spokane	64
Tacoma	260
Walla Walla	71
Wenatchee	152
Yakima	162

a) Conservation

The mileage on a straight line between two cities is always less or equal to the mileage of any other route between those two cities. Therefore, the evaluation function is always conservative (estimated distance  $\leq$  actual distance)



b) Consistency:

For the evaluation function to be consistent, then:

$$k(m, n) \geq |h(m) - h(n)|$$

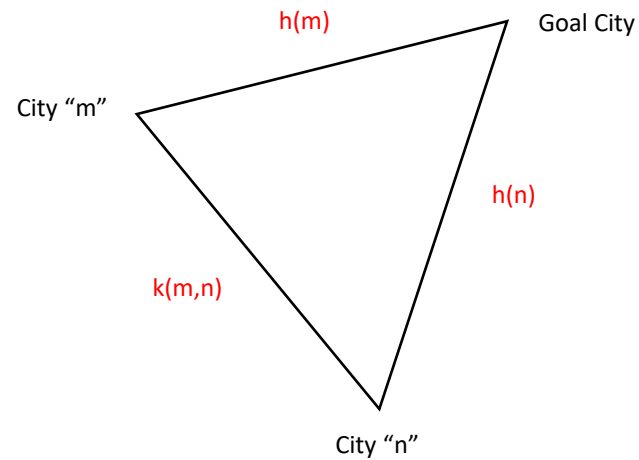
Where

$k(m, n)$  is the true distance between nodes  $m$  and  $n$

$h(m)$  is the estimate from node  $m$  to the goal node

$h(n)$  is the estimate from node  $n$  to the goal node

Assuming that  $k(m, n)$  is the shortest possible distance between nodes  $m$  and  $n$ , then a triangle can be drawn as follows:



Triangles have the following property: the length of a side of a triangle is always less than the sum of the lengths of the other two sides and always greater than the difference of the lengths of the other two sides. Therefore,  $k(m, n)$  is always greater than  $h(m) - h(n)$ . Since consistency holds for the shortest possible distance between nodes  $m$  and  $n$ , then it will always hold for all possible values of  $k(m, n)$ .

c)

Tacoma: Start Node

Adjacent Nodes

Renton = 27 + 245 = 272

Olympia = 31 + 273 = 304

Seattle = 34 + 250 = 284

Bremerton = 34 + 270 = 304

		Renton	Olympia	Seattle	Bremerton
0	Tacoma	272	304	284	304

Renton: at 27mi

Adjacent Nodes

Everett = 27 + 39 + 255 = 321

Seattle = 27 + 12 + 250 = 289/284 -> 284

Yakima = 27 + 142 + 162 = 331

Tacoma visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima
0	Tacoma	272	304	284	304		
27	Renton	272	304	284	304	321	331

Seattle at 34

Adjacent Nodes

Everett = 34 + 29 + 255 = 318/321 -> 318

Bremerton = 34 + 34 + 270 = 338/304 -> 304

Tacoma Visited

Renton visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima
0	Tacoma	272	304	284	304		
27	Renton	272	304	284	304	321	331
34	Seattle	272	304	284	304	318	331

Bremerton at 34

Adjacent Nodes:

Port Angeles = 34 + 76 + 313 = 423

Tacoma Visited

Seattle visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Port Angeles
:	:	:	:	:	:	:	:	:
34	Seattle	272	304	284	304	318	331	
34	Bremerton	272	304	284	304	318	331	423

Olympia at 31mi

Adjacent Nodes

Shelton = 31 + 22 + 286 = 339

Yakima = 31 + 177 + 162 = 370/331 -> 331

Tacoma visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Port Angeles	Shelton
:	:	:	:	:	:	:	:	:	:
34	Bremerton	272	304	284	304	318	331	423	
31	Olympia	272	304	284	304	318	331	423	339

Everett at 63

Adjacent Nodes:

Bellingham = 63 + 62 + 288 = 413

Wenatchee = 63 + 122 + 152 = 337

Renton Visited

Seattle Visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Port Angeles	Shelton	Bellingham	Wenatchee
:	:	:	:	:	:	:	:	:	:	:	:
31	Olympia	272	304	284	304	318	331	423	339		
63	Everett	271	304	284	304	318	331	423	339	413	337

Yakima at 169

Adjacent Nodes:

Wenatchee = 169 + 106 + 152 = 427/337 -> 337  
 Mosses Lake = 169 + 100 + 108 = 377  
 Pasco = 169 + 86 + 101 = 356  
 Ephrata = 169 + 94 + 120 = 383  
 Olympia Visited  
 Renton Visited

	Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Port Angeles	Shelton	Bellingham	Wenatchee	Mosses Lake	Pasco	Ephrata
:	:	:	:	:	:	:	:	:	:	:	:	:	:
63	Everett	272	304	284	304	318	331	423	339	413	337		
169	Yakima	272	304	284	304	318	331	423	339	413	337	377	356 383

Wenatchee at 185

Adjacent Nodes:

Ephrata = 185 + 49 + 120 = 354/383 -> 354  
 Yakima visited

	Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Port Angeles	Shelton	Bellingham	Wenatchee	Mosses Lake	Pasco	Ephrata
:	:	:	:	:	:	:	:	:	:	:	:	:	:
169	Yakima	272	304	284	304	318	331	423	339	413	337	377	356 383
185	Wenatchee	272	304	284	304	318	331	423	339	413	337	377	356 354

Shelton at 53

Adjacent Nodes:

Port Angeles = 53 + 99 + 313 = 465/423 -> 423  
 Olympia Visited

	Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Port Angeles	Shelton	Bellingham	Wenatchee	Mosses Lake	Pasco	Ephrata
:	:	:	:	:	:	:	:	:	:	:	:	:	:
185	Wenatchee	272	304	284	304	318	331	423	339	413	337	377	356 354
53	Shelton	272	304	284	304	318	331	423	339	413	337	377	356 354

Ephrata at 234

Adjacent Nodes:

Mosses Lake = 234 + 19 + 108 = 361/377 -> 361

Wenatchee visited

Yakima visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Port Angeles	Shelton	Bellingham	Wenatchee	Mosses Lake	Pasco	Ephrata
:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
53	Shelton	272	304	284	304	318	331	423	339	413	337	377	356	354
234	Ephrata	272	304	284	304	318	331	423	339	413	337	361	356	354

Pasco at 255

Adjacent Nodes:

Walla Walla = 255 + 48 + 71 = 374

Mosses Lake = 255 + 72 + 108 = 435/361 -> 361

Yakima visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Port Angeles	Shelton	Bellingham	Wenatchee	Mosses Lake	Pasco	Ephrata	Walla Walla
:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
234	Ephrata	272	304	284	304	318	331	423	339	413	337	361	356	354	
255	Pasco	272	304	284	304	318	331	423	339	413	337	361	356	354	374

Mosses Lake at 253

Adjacent Nodes:

Spokane = 253 + 105 + 64 = 422

Yakima Visited

Ephrata Visited

Pasco Visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Port Angeles	Shelton	Bellingham	Wenatchee	Mosses Lake	Pasco	Ephrata	Walla Walla	Spokane
:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
255	Pasco	272	304	284	304	318	331	423	339	413	337	361	356	354	374	
253	Mosses Lake	272	304	284	304	318	331	423	339	413	337	361	356	354	374	422

Walla Walla at 303

Adjacent Nodes

Pullman

= 303 + 115 = 418

Pasco

visited

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Port Angeles	Shelton	Bellingham	Wenatchee	Mosses Lake	Pasco	Ephrata	Walla Walla	Spokane	Pullman
:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
253	Mosses Lake	272	304	284	304	318	331	423	339	413	337	361	356	354	374	422	
303	Walla Walla	272	304	284	304	318	331	423	339	413	337	361	356	354	374	422	418

Putting all previous steps in one table:

		Renton	Olympia	Seattle	Bremerton	Everett	Yakima	Port Angeles	Shelton	Bellingham	Wenatchee	Mosses Lake	Pasco	Ephrata	Walla Walla	Spokane	Pullman
0	Tacoma	272	304	284	304												
27	Renton	272	304	284	304	321	331										
34	Seattle	272	304	284	304	318	331										
34	Bremerton	272	304	284	304	318	331	423									
31	Olympia	272	304	284	304	318	331	423	339								
63	Everett	272	304	284	304	318	331	423	339	413	337						
169	Yakima	272	304	284	304	318	331	423	339	413	337	377	356	383			
185	Wenatchee	272	304	284	304	318	331	423	339	413	337	377	356	354			
53	Shelton	272	304	284	304	318	331	423	339	413	337	377	356	354			
234	Ephrata	272	304	284	304	318	331	423	339	413	337	361	356	354			
255	Pasco	272	304	284	304	318	331	423	339	413	337	361	356	354	374		
253	Mosses Lake	272	304	284	304	318	331	423	339	413	337	361	356	354	374	422	
303	Walla Walla	272	304	284	304	318	331	423	339	413	337	361	356	354	374	422	418

**Path: Tacoma – Renton – Yakima – Pasco – Walla – Pullman (418 Miles)**