1

Given $\min_{x} \; x_1^2 + 3x_2^2$ and $3x_1 + 2x_2 \cancel{\leq} -4$

Form Lagrange

$$\mathcal{L}(\underline{x}, \lambda) = x_1^2 + 3x_2^2 - \lambda(3x_1 + 2x_2 + 4)$$

then

$$\min_{\underline{x}} \mathcal{L}(\underline{x}, \lambda) = \omega(\lambda)$$

solve using derivatives

$$\partial \mathcal{L}(\cdot)/\partial x_1 = 2x_1 - 3\lambda = 0 \qquad x_1 = \tfrac{3}{2}\lambda$$

$$\partial \mathcal{L}(\cdot)/\partial x_2 = 6x_2 - 2\lambda = 0 \qquad x_2 = \tfrac{1}{3}\lambda$$

then substitute

$$\omega(\lambda) = \left(\tfrac{3}{2}\lambda\right)^2 + 3\left(\tfrac{1}{3}\lambda\right)^2 - \lambda\left(\tfrac{9}{2}\lambda + \tfrac{2}{3}\lambda + 4\right)$$

$$= \frac{-31}{12}\lambda^2 - 4\lambda$$

and again take derivatives so

$$d\omega(\lambda)/d\lambda = -\frac{31}{6}\lambda - 4 = 0 \qquad \lambda^* = -24/31$$

and substitute

$$x_1^* = -\frac{36}{31} \qquad x_2^* = -\frac{8}{31}$$

$$f(\underline{x}^*) = \frac{48}{31} \qquad \omega(\lambda^*) = \frac{48}{31}$$

2 Given

$$\min_{} f(\underline{x}) = 10 - 3x_1 - 2x_2 - x_3$$

$$\ni \; 2x_1 + 3x_2 + 4x_3 \leq 4$$

a) Note:- All equations and constraints are linear and thus General Duality reduces to LP duality. Hence the LP dual can be used

12.

$$\max \omega(\lambda) = 4\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + 10$$

$$2\lambda_1 + \lambda_2 \le -3$$

$$3\lambda_1 + \lambda_3 \le -2$$

$$4\lambda_1 + \lambda_4 \le -1$$

$$\ni \lambda_i \le 0$$

Thus

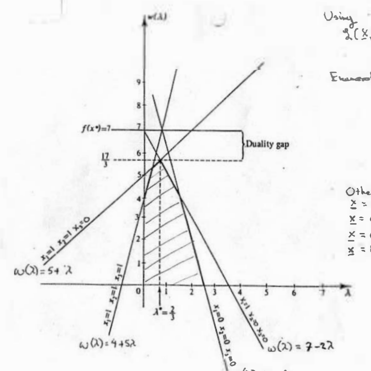
$$\lambda^* = \begin{bmatrix} -2/3 \\ -5/3 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \omega^*(\lambda^*) = \frac{17}{3}$$

$$\Rightarrow x = \begin{bmatrix} 1 \\ 2/3 \\ 0 \end{bmatrix} \quad \text{and} \quad f^* = \frac{17}{3}$$

Using

$$\mathcal{L}(\underline{x},\lambda) = 10 - 3x_1 - 2x_2 - x_3 + \lambda(2x_1+3x_2+4x_3-4)$$

Enumerating possibilities for $\underline{x}$



Others:

$\underline{x} = 001 \quad \omega(\lambda) = 9$

$\underline{x} = 010 \quad \omega(\lambda) = 8-\lambda$

$\underline{x} = 011 \quad \omega(\lambda) = 7+3\lambda$

$\underline{x} = 101 \quad \omega(\lambda) = 6+2\lambda$

Figure 6.3 Duality gap for an integer programme

Note:- only $x_1 = 1,\ x_2 = 0,\ x_3 = 0$ ie. $\omega(\lambda) = 7-2\lambda$ feasible