Consider the system of non-linear simultaneous equations \( g(x) = 0 \) where \( g : \mathbb{R}^2 \to \mathbb{R}^2 \) defined by:

\[
\forall x \in \mathbb{R}^2, g(x) = \begin{bmatrix} (x_1)^2 - 0.1x_1x_2 \\ (x_2)^2 - 0.1x_1x_2 \end{bmatrix}.
\]

Write out explicitly the Newton-Raphson update to solve \( g(x) = 0 \). Invert the Jacobian matrix explicitly using the formula for the inverse of a 2 \( \times \) 2 matrix.

Is there a solution to \( g(x) = 0 \)?

Starting at \( x^{(0)} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \), calculate the first three iterates.

Now replace the exact Jacobian by the approximation \( \bar{J}(x) \) obtained by neglecting the off-diagonal terms of the Jacobian. Write down the new update equations.

Starting at \( x^{(0)} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \), calculate the first five iterates based on the approximate Jacobian \( \bar{J}(x) \).

Which method takes less effort overall to achieve a solution satisfying the condition \( \| g(x^{(v)}) \| < 0.3 \): using the exact Jacobian or using the approximate Jacobian? (Hint: The answer is: "it all depends.")

7.4 Consider a plate capacitor, illustrated in Figure 7.12. (We will consider a similar arrangement in the sizing of interconnects in integrated circuits case study in Section 15.5.) The capacitor consists of two conductors separated by a non-conducting dielectric. The upper conductor is of length \( L \) and width \( w \) and has thickness \( T \). The upper conductor is separated from the lower conductor by a dielectric of thickness \( d \) and dielectric constant \( \varepsilon \).