8th International Conference on Probabilistic Methods Applied to Power Systems, Iowa State University, Ames, Iowa, September 12-16, 2004

Slack Bus Treatment in Load Flow Solutions with Uncertain Nodal Powers

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Abstract — **This paper addresses the problem introduced by the slack bus in load flow solutions with uncertain nodal powers. While balancing powers in the system the slack bus will also absorb all uncertainty. The results obtained are of no practical interest unless realistic constraints are imposed on slack power production/consumption. Two methods of dealing with these constraints are investigated suitable for implementation within the recently developed boundary load flow.**

Index Terms — **Fuzzy sets, load flow analysis, power system planning, slack bus, uncertainty.**

I. Introduction

The most common formulation of the load flow problem requires all input variables (PQ at loads, PV at generators) to be specified as deterministic ('crisp') values. Each set of specified values corresponds to one system state, which is deemed representative for some set of system conditions. Thus, when the input conditions are uncertain, as is predominantly the case in planning, there is a need for numerous scenarios to be analyzed. A load flow approach that could directly incorporate uncertainty into the solution process has been long recognized as useful. The results from such analysis would be expected to give solutions over the range of the uncertainties, i.e., solutions that are sets of values or regions instead of single operating points.

To date, two families of uncertain load flow algorithms have evolved. The first one is the probabilistic load flow (PLF), which considers loads and generations as random variables with some probability distributions (e.g., [1] - [4]). The results of the load flow, i.e., voltages, power flows, and so on, are also random variables with resultant probability distributions obtained using probabilistic techniques. The second is the fuzzy load flow family of algorithms where input variables are represented as fuzzy numbers (e. g., [5] - [7]). Fuzzy numbers are described by possibility distributions and can be considered to be intervals with indistinct boundaries. The results obtained are also fuzzy numbers with resultant possibility distributions. The authors have recently extended these concepts to the so-called boundary load flow [8].

Both families of uncertain load flow algorithms use the same definition of the problem as the traditional deterministic approach. That is, load buses are defined as PQ buses, generator buses as PV buses, and one bus is assumed to be a slack bus to balance the active and reactive power in the system. The 'slack' bus (or 'swing' bus) is defined as $V\theta$ bus. While this definition of the load flow problem is appropriate for a deterministic solution (although it may still be helpful to define a distributed 'slack' among several buses), it has an inherent drawback when dealing with uncertain input variables: the slack bus must absorb all uncertainties arising from the solution and thus, will have the widest nodal power possibility (probability) distributions in the system. If even moderate amounts of uncertainty are allowed in a large system, the resulting distributions will frequently contain values well beyond the generating margins of the slack generator.

This problem has been neglected so far in the literature except for the case of a linearized fuzzy DC load flow [7]. In that work, three approaches, conceptually the same, use an iterative corrective procedure in order to satisfy constraints imposed on the slack bus. Recently, the authors have developed a methodology that enables an accurate solution from a non-linear AC fuzzy load flow [8]. It follows the concept of boundary load flow (BLF) solutions, where solutions are based on an optimization procedure for implicitly defined vector functions. Numerical results obtained from test systems have shown the feasibility of this approach, but they also have shown the problems associated with the inappropriate definition of the slack bus.

This paper extends the previous work and investigates different ways of incorporating the constraints imposed on the slack bus in the framework of boundary load flow solutions. Two methods of dealing with this problem are considered: 1) slack bus to PV bus and PV bus to slack bus conversion, and 2) distributed slack bus modeling. The results obtained from different test systems as well as the specifics in different approaches are discussed and compared.

II. Boundary Load Flow Solutions

The BLF was presented for the first time in [2] within the context of PLF. In that paper, an approximate solution for the ranges of values for state and output variables, given the ranges of values of input variables from their probability distributions, was found. The ranges of variables were then used to determine multiple points of linearization for the load flow equations in order to improve the accuracy of the PLF solutions, particularly for the tail regions of the probability distributions.

This work was supported in part by the National Science Foundation (NSF) under Grants No. DGE-0108076 and No. EEC 02-24810

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The authors have developed a methodology where an accurate solution for a non-statistical interval load flow is obtainable [8]. In the following, a brief explanation of this methodology is given.

The load flow problem is defined by two sets of nonlinear equations:

$$\mathbf{Y} = \mathbf{g}(\mathbf{X}) \tag{1}$$

and

$$\mathbf{Z} = \mathbf{h}(\mathbf{X}), \tag{2}$$

where:

- $\mathbf{X}$ is the vector of unknown state variables (voltage magnitudes and angles at PQ buses; and voltage angles and reactive power outputs at PV buses),
- $\mathbf{Y}$ is the vector of predefined input variables (real and reactive injected nodal powers at PQ buses; and voltage magnitudes and real power outputs at PV buses),
- $\mathbf{Z}$ is the vector of unknown output variables (real and reactive power flows in the network elements), and
- $\mathbf{g}$, $\mathbf{h}$ are the load flow vector functions.

The boundary values are the extreme points found by allowing the inputs to vary over their range. In our notation, we want to find the extreme values for the elements of $\mathbf{X}$ and $\mathbf{Z}$ implicitly expressed in (1) and (2), in terms of the elements of $\mathbf{Y}$ which, in turn, are constrained. Thus, finding the boundary values in a load flow problem is a process of locating the constrained extrema of implicitly defined vector functions of vector arguments.

Because $\mathbf{X}$ cannot be explicitly expressed in terms of $\mathbf{Y}$, the solution of the system of equations (1) is found by an iterative process. Given an initial trial solution, $\mathbf{X'}$, the error is calculated as:

$$\Delta\mathbf{Y} = \mathbf{Y} - \mathbf{Y'} = \mathbf{Y} - \mathbf{g}(\mathbf{X'}). \tag{3}$$

If a Newton-Raphson (N-R) based scheme is used, (1) is linearized around $\mathbf{X'}$ and an update for the new solution is found as:

$$\Delta\mathbf{X} = \mathbf{K}\cdot\Delta\mathbf{Y}, \tag{4}$$

where $\mathbf{K}$ is the inverse of the Jacobian of $\mathbf{g}$ evaluated at $\mathbf{X'}$. The element K_{ij} of this matrix is the partial derivative of X_i with respect to Y_j. Similarly, if we linearize (2) and substitute for $\Delta\mathbf{X}$ from (4) we will obtain:

$$\Delta\mathbf{Z} = \mathbf{S}\cdot\Delta\mathbf{X} = \mathbf{L}\cdot\Delta\mathbf{Y}, \tag{5}$$

where $\mathbf{S}$ is the Jacobian of $\mathbf{h}$ at the given point of linearization. The matrix $\mathbf{L} = \mathbf{S}\cdot\mathbf{K}$ is a sensitivity coefficient matrix and the element L_{ij} is the partial derivative of Z_i with respect to Y_j.

Each row of $\mathbf{K}$ and $\mathbf{L}$ represents the gradient vector of the corresponding state and output variable X_i and Z_i, respectively. Similar to derivative based optimization procedures, by iteratively following the direction of the gradient, extreme points (possibly local) of the state or output variable can be found.

Only the signs of the partial derivatives that comprise the gradient are used in the solution since our experience has shown that the values of the partials are not useful for efficiently determining the updates. Further, a procedure is needed to maintain feasibility of the solution, i.e., ensure the input variables remain within the constraints for all iterations. The iterative procedure is reviewed in the following.

Suppose that the *minimum* value of X_i is sought. If K_{ij} is positive (negative), then decrease (increase) the value of Y_j by some fixed step size. After repeating for all Y_j we obtain a new point of $\mathbf{Y}$ from which a new $\mathbf{X}$ from (1) can be found. From this new point, the above steps are repeated until one of the following is true for all input variables:

- the partial derivative is positive and the associated variable is at a minimum;
- the partial derivative is negative and the associated variable is at a maximum;
- the partial derivative is zero.

If the final condition does not hold for any variable, then the solution is clearly a local constrained extremum. Because of the nonlinearity of (1) and (2), this point may not be the only extrema. In practice, we have found the physical nature of the load flow problem leads to either a unique solution or a relatively small number of extrema.

When one or more of the partial derivates are zero, the solution point lies somewhere on the boundary surface. Such a point is either a local constrained extremum or a saddle point. Though it is unlikely that by preceding in a downhill direction one will end up trapped in a local maximum or a saddle point, theoretically such a possibility exists. Here, previous values of X_i are recorded at each step and if X_i fails to decrease, then the step length is modified.

Finally, in the special case when all the partial derivatives are zero, a solution cannot be obtained due to the singularity of the Jacobian. Such a point typically indicates infeasibility of the load flow and a loading limit for the system considered. A singularity of the Jacobian may also occur even if not all of the partial derivatives are zero. In such cases, the ranges of values of the input variables are too great and one must repeat the calculations with reduced variations for some or all of the variables. Note, the procedure described here must be repeated for each state and output variable considered, and therefore, is computationally intensive.

III. Slack Bus Treatment

The concept of slack bus, as is well-known, is a mathematical necessity but has no physical relationship to any generator bus. Exception arises when a small system is linked to a much bigger system via a single tie line (single bus). In this case, one can represent the large system with an equivalent generator, which can hold the voltage constant and generate as much power as needed, i.e. the slack bus characteristics. Similarly in a distribution network fed by a substation, the transmission network acts as a slack bus with respect to the distribution network.

The slack bus allows the solution of the nonlinear set of equations (1) to be feasible. Since the power losses in the network are not known in advance, its role is to pick up the 'slack' and balance the active and reactive power in the system. This usually does not represent a problem in a well defined deterministic load flow problem. However, in the case with uncertain nodal powers, the slack bus also must absorb all the resulting uncertainties from the solution. As a result, it has the widest nodal power possibility (probability) distributions in the system. This will frequently result in operating points well beyond its generating margins. This also defeats the purpose behind the study of uncertainties, which is to investigate the impact on practical operating scenarios. In the following, two ways of satisfying the constraints imposed on the slack bus are explained.

A. Slack Bus - PV Bus Conversion

This method is analog to that of PV bus to PQ bus conversion for PV buses with reactive power limits. During the course of solution of a load flow, when a PV bus's produced (or consumed) reactive power extends beyond its limits, it is fixed at the violated limit and its voltage magnitude is relaxed. Thus, the PV bus has been converted to a PQ bus, bus with specified active and reactive power. Later, during the solution, if the bus voltage shows tendency to return and the reactive power again falls within the limits, the bus will be converted back from PQ to PV.

Following the same approach as in PV bus to PQ bus conversion above, if the slack bus real power generation (or, theoretically, consumption) extends beyond its predefined limits, it is fixed at the violated limit. Some other PV bus's active power generation (or consumption) then must be relaxed in order to be able to solve the load flow problem. The PV bus to choose seems to be a matter of preference, but it is logical to pick the one that has the highest margin from the current production (consumption) to either its lower or upper limit, depending on which limit was violated at the slack bus.

With the choice of a PV bus to relax, it is now possible to redefine the load flow problem in (1) by swapping only the equation for the real power at the chosen PV bus with the equation for the slack bus real power, without changing the unknown state variables. In other words, the slack bus becomes a PVθ bus and the PV bus becomes just a V bus. We still have a system of *n* equations with *n* unknowns, only the known and unknown variables have changed and Jacobian loses some symmetry. In this case, the system of equations corresponding to (4) will have the following form:

$$\begin{bmatrix} \Delta P_{Slack} \\ \Delta \mathbf{P}_{PV-1} \\ \Delta \mathbf{P}_{PQ} \\ \Delta \mathbf{Q}_{PQ} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial P_{Slack}}{\partial \boldsymbol{\theta}^{\mathrm{T}}_{PV+PQ}} & \dfrac{\partial P_{Slack}}{\partial \mathbf{V}^{\mathrm{T}}_{PQ}} \\ \dfrac{\partial \mathbf{P}_{PV-1}}{\partial \boldsymbol{\theta}^{\mathrm{T}}_{PV+PQ}} & \dfrac{\partial \mathbf{P}_{PV-1}}{\partial \mathbf{V}^{\mathrm{T}}_{PQ}} \\ \dfrac{\partial \mathbf{P}_{PQ}}{\partial \boldsymbol{\theta}^{\mathrm{T}}_{PV+PQ}} & \dfrac{\partial \mathbf{P}_{PQ}}{\partial \mathbf{V}^{\mathrm{T}}_{PQ}} \\ \dfrac{\partial \mathbf{Q}_{PQ}}{\partial \boldsymbol{\theta}^{\mathrm{T}}_{PV}} & \dfrac{\partial \mathbf{Q}_{PQ}}{\partial \mathbf{V}^{\mathrm{T}}_{PQ}} \end{bmatrix} \cdot \begin{bmatrix} \Delta \boldsymbol{\theta}_{PV+PQ} \\ \Delta \mathbf{V}_{PQ} \end{bmatrix} \tag{6}$$

where:

- PV is the set of all PV buses,
- PV-1 is the set of all PV buses without the one with relaxed real power,
- PQ is the set of all PQ buses,
- $PV+PQ$ is the set of all PV and PQ buses,
- **P, Q** are the real and reactive nodal power vector functions,
- **V, θ** are the vectors of unknown state variables (voltage magnitudes and angles), and
- $\partial/\partial(\cdot)^{\mathrm{T}}$ denotes Jacobian of the corresponding vector function.

The problem formulation as in (6) keeps the reference angle at the slack bus (usually 0º). Another approach will be to relax the voltage angle of the slack bus and declare the voltage angle of the PV bus with relaxed real power as the reference (i.e. known). This can simply be done by replacing it in (6) with the now unknown angle at the slack and retaining its current value. This will result in a complete slack to PV bus and PV to slack bus conversion. In this case the system of equations has the usual symmetry, with the slack bus completely swapped.

In the second approach, the original slack will change its voltage angle from the initial value during the course of solution. However, since angles are relative to each other, we can force it back to the initial value if desired, by subtracting that difference from each voltage angle obtained from the solution. In this way, we will obtain exactly the same solution as with the previous formulation.

Regardless of the treatment of the reference angle, the new slack bus takes over the balancing of power and, initially, its production (consumption) will be either decreased or increased, depending on the limit violation at the previous slack bus. During the course of solution, the production of the new slack bus will change and it is possible that one of its limits gets violated also. In this case, the procedure is repeated with some other PV bus capable of taking over the slack. If there is no such bus available, i.e., all PV buses are on their limits, the problem is infeasible.

B. Distributed Slack Bus

Instead of assigning the excess load (or, generation) to only one PV bus as in the previous method, we can also choose a number of PV buses that will share it in a predetermined manner. Two methods of sharing are: 1) proportional to the current injections, and 2) proportional to the margin between the current injections and the lower or upper limits, accordingly. Of course, there are many other combinations that may be used if deemed appropriate for some particular application. In any case, there is no bus type conversion with this method. If the slack bus production (consumption) extends beyond its limits, it is relieved by redistributing the excess load (generation) to the other PV buses. The reference angle remains the same during the load flow solution process.

It was noted previously that in order to maintain the feasibility of the problem, the available generation should always match the load requirement. Cases when this is not always true are not considered here. For example, a case with excess generation (if each generator has some minimum limit and their sum is bigger than the total load) requires a different unit commitment. A case with too little generation requires a procedure for load shedding and/or some kind of adequacy assessment.

IV. Case Studies

Let us now apply the described methods for slack bus treatment in finding boundary load flow solutions of the small IEEE 14-bus test system shown in Fig. 1. The system data and the base case descriptions can be found elsewhere (for example, [10]).

Table I presents the results for boundary values of voltage magnitudes when all specified nodal powers in the network vary in the range [50% - 150%] of the base case values. Shown are columns with minimal, base case, and maximal voltages. The slack bus real power generation was not restricted in this case and was found to vary in the range [0.8372 - 4.0599] p.u. However, this range is outside the limit of the slack bus generator which is [0.1 - 2.5] p.u.

Let us now include constraints for the slack bus and use Slack bus – PV bus conversion when they are violated. The new range of values for the slack bus generator is now restricted to [0.8372 - 2.5] and the new results for voltage magnitudes are shown in Table II.

Table III and Table IV present real and reactive power flows in all elements of the system for the constrained and unconstrained case, respectively.

The results show that the biggest differences between the two cases occur when the system is heavily loaded, as expected. In other words, the biggest differences occur in the values of minimal voltages and maximal power flows, which are usually the most interesting results. The reason for this is that in the most stressed scenario the BLF, in its search for an optimum, tries to supply almost all of the power from the slack. Thus, when its limits are respected the system is less stressed and conditions in the system improved. For example, the real power flow in the most heavily loaded branches between buses 1 and 2 fell from 2.826 to 1.905 p.u.

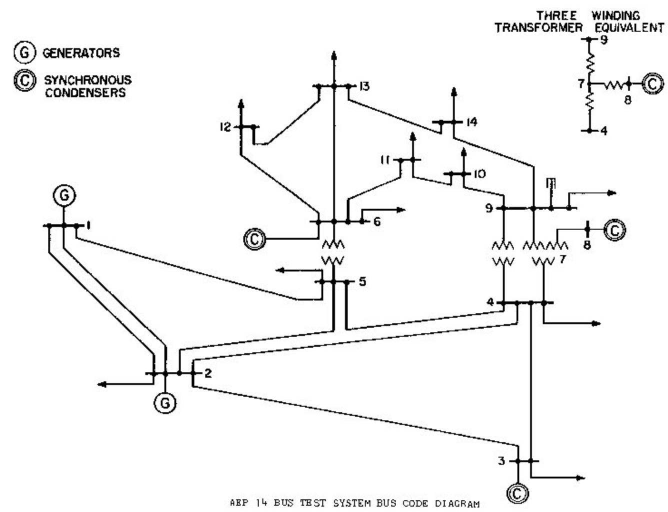

Fig. 1. IEEE/AEP 14-bus test system.

TABLE I BOUNDARY VALUES [P.U.] FOR THE IEEE/AEP 14-BUS SYSTEM VOLTAGE MAGNITUDES – UNCONSTRAINED CASE

bus voltage	nodal power variation [50% - 150%]		
	V_{min}	$V_{100\%}$	V_{max}
V_1	1.0600	1.0600	1.0600
V_2	1.0105	1.0450	1.0450
V_3	0.9645	1.0100	1.0100
V_4	0.9583	1.0186	1.0330
V_5	0.9649	1.0203	1.0328
V_6	1.0094	1.0700	1.0700
V_7	0.9904	1.0620	1.0762
V_8	1.0314	1.0900	1.0900
V_9	0.9920	1.0563	1.0797
V_{10}	1.0013	1.0513	1.0746
V_{11}	1.0189	1.0571	1.0709
V_{12}	0.9843	1.0552	1.0638
V_{13}	0.9902	1.0504	1.0628
V_{14}	0.9581	1.0358	1.0633

TABLE II BOUNDARY VALUES [P.U.] FOR THE IEEE/AEP 14-BUS SYSTEM VOLTAGE MAGNITUDES – CONSTRAINED CASE

bus voltage	nodal power variation [50% - 150%]		
	V_{min}	$V_{100\%}$	V_{max}
V_1	1.0600	1.0600	1.0600
V_2	1.0441	1.0450	1.0450
V_3	1.0100	1.0100	1.0100
V_4	0.9969	1.0186	1.0330
V_5	1.0007	1.0203	1.0328
V_6	1.0507	1.0700	1.0700
V_7	1.0327	1.0620	1.0762
V_8	1.0721	1.0900	1.0900
V_9	1.0249	1.0563	1.0797
V_{10}	1.0248	1.0513	1.0746
V_{11}	1.0414	1.0571	1.0709
V_{12}	1.0268	1.0552	1.0638
V_{13}	1.0261	1.0504	1.0628
V_{14}	0.9959	1.0358	1.0633

It should be noted here that simple arithmetic calculations for finding the boundary values of power flows can not be applied, due to the nonlinearity of the problem. For example, in the constrained case the maximal real power production from the slack is constrained to the value of 2.5 p.u. This value is less than the sum of the maximal real power flows in the branches incident to the slack, i.e., branches 1-2 and 1-5. These results correspond to different conditions and different load flow solutions and can not be simply lumped together.

It is interesting to note how the slack bus changed during the course of solution for the minimal values of voltage magnitudes. In all the cases for the buses with unspecified voltages (PQ buses), the initial slack bus 1 was swapped with bus 3, then 3 was swapped with 2, and 2 was finally swapped with 1 again. This is just a result of this particular system structure and the problem solution approach and does not represent a general pattern.

Similar results are obtained when distributed slack bus modeling approach is used. However, they are appear to be slightly more optimistic. The most extreme power flows in the branches tend to be smaller and minimal voltages at the buses with the smallest values tend to be higher. This can be attributed to the BLF algorithm and not to the treatment of the slack bus. Namely, it is more difficult to locate the exact extremum when several variables simultaneously vary than when only one or few vary. (Although Fig. 1 shows only one generator at bus 2 besides the slack generator at 1, according to the data file, the other PV buses: 3, 5, and 8, also have some real power generating limits.)

The results from the analysis of bigger test systems show similar differences in uncertainty, only scaled to the system size. For example using the IEEE 118-bus test system, the unconstrained slack bus real power generation has maximal value of 13.98 p.u., for specified nodal powers variation in the range [90% - 110%] of the corresponding base case values. This is far away from its limit of 8.05 p.u. So, when constrained with either of the two methods described, its value is held to 8.05 p.u. The branch with the biggest power flow in the unconstrained case is, not surprisingly, one connected to the slack bus (branch 69 – 68). Its real power flow is 5.44 p.u. In the constrained case this value is much smaller, 2.67 p.u. The branch with the highest power flow in this case is not connected directly to the slack bus (branch 9-10) and its real power flow is 4.89 p.u. Also, the minimal voltages in the constrained case are higher or at least equal to the minimal voltages in the unconstrained case.

TABLE III BOUNDARY VALUES [P.U.] FOR THE IEEE/AEP 14-BUS REAL AND REACTIVE POWER FLOWS – UNCONSTRAINED CASE

power flow	nodal power variation [50% - 150%]		
	S_{min}	$S_{100\%}$	S_{max}
S_{1-2}	0.4189 + j 0.1439	1.5683 + j-0.2039	2.8260 + j 0.1056
S_{1-5}	0.3015 + j-0.0094	0.7555 + j 0.0350	1.2372 + j 0.1138
S_{2-3}	0.3472 + j 0.0658	0.7319 + j 0.0357	1.1395 + j 0.0881
S_{2-4}	0.2660 + j-0.0716	0.5614 + j-0.0229	0.8681 + j 0.0496
S_{2-5}	0.1891 + j-0.0300	0.4151 + j 0.0076	0.6524 + j 0.0688
S_{3-4}	-0.5526 + j-0.1079	-0.2333 + j 0.0281	0.0929 + j 0.1721
S_{4-5}	-0.9871 + j 0.0323	-0.6122 + j 0.1567	-0.2374 + j 0.2676
S_{4-7}	0.1115 + j-0.1583	0.2809 + j-0.0942	0.4507 + j-0.0436
S_{4-9}	0.0637 + j-0.0409	0.1609 + j-0.0032	0.2585 + j 0.0367
S_{5-6}	0.2068 + j 0.0888	0.4406 + j 0.1282	0.6781 + j 0.1727
S_{6-11}	-0.0212 + j-0.0222	0.0734 + j 0.0347	0.1720 + j 0.0905
S_{6-12}	0.0331 + j 0.0039	0.0778 + j 0.0249	0.1240 + j 0.0472
S_{6-13}	0.0716 + j 0.0133	0.1774 + j 0.0717	0.2877 + j 0.1310
S_{7-8}	0.0000 + j-0.2305	-0.0000 + j-0.1691	-0.0000 + j-0.0844
S_{7-9}	0.1115 + j-0.0371	0.2809 + j 0.0580	0.4507 + j 0.1481
S_{9-10}	-0.0606 + j-0.0228	0.0524 + j 0.0431	0.1648 + j 0.1085
S_{9-14}	-0.0128 + j-0.0101	0.0944 + j 0.0367	0.2023 + j 0.0838
S_{10-11}	-0.1312 + j-0.0703	-0.0377 + j-0.0153	0.0550 + j 0.0415
S_{12-13}	-0.0236 + j-0.0120	0.0161 + j 0.0074	0.0570 + j 0.0279
S_{13-14}	-0.0341 + j-0.0269	0.0563 + j 0.0169	0.1517 + j 0.0634

TABLE IV BOUNDARY VALUES [P.U.] FOR THE IEEE/AEP 14-BUS REAL AND REACTIVE POWER FLOWS – CONSTRAINED CASE

power flow	nodal power variation [50% - 150%]		
	S_{min}	$S_{100\%}$	S_{max}
S_{1-2}	0.4189 + j-0.2203	1.5683 + j-0.2039	1.9055 + j 0.1056
S_{1-5}	0.3015 + j-0.0094	0.7555 + j 0.0350	0.9042 + j 0.1138
S_{2-3}	0.3472 + j 0.0163	0.7319 + j 0.0357	1.2907 + j 0.0881
S_{2-4}	0.2660 + j-0.0716	0.5614 + j-0.0229	0.7572 + j 0.0589
S_{2-5}	0.1891 + j-0.0300	0.4151 + j 0.0076	0.5965 + j 0.0746
S_{3-4}	-0.5526 + j-0.1079	-0.2333 + j 0.0281	0.0929 + j 0.1812
S_{4-5}	-0.8735 + j 0.0323	-0.6122 + j 0.1567	-0.2374 + j 0.2483
S_{4-7}	0.1111 + j-0.1424	0.2809 + j-0.0942	0.4507 + j-0.0436
S_{4-9}	0.0635 + j-0.0405	0.1609 + j-0.0032	0.2585 + j 0.0367
S_{5-6}	0.2068 + j 0.0879	0.4406 + j 0.1282	0.6605 + j 0.1727
S_{6-11}	-0.0212 + j-0.0216	0.0734 + j 0.0347	0.1646 + j 0.0905
S_{6-12}	0.0331 + j 0.0038	0.0778 + j 0.0249	0.1227 + j 0.0472
S_{6-13}	0.0716 + j 0.0133	0.1774 + j 0.0717	0.2824 + j 0.1310
S_{7-8}	0.0000 + j-0.2312	-0.0000 + j-0.1691	0.0000 + j-0.0844
S_{7-9}	0.1111 + j-0.0371	0.2809 + j 0.0580	0.4507 + j 0.1552
S_{9-10}	-0.0543 + j-0.0228	0.0524 + j 0.0431	0.1648 + j 0.1080
S_{9-14}	-0.0086 + j-0.0101	0.0944 + j 0.0367	0.2023 + j 0.0835
S_{10-11}	-0.1245 + j-0.0703	-0.0377 + j-0.0153	0.0550 + j 0.0410
S_{12-13}	-0.0236 + j-0.0120	0.0161 + j 0.0074	0.0559 + j 0.0279
S_{13-14}	-0.0341 + j-0.0269	0.0563 + j 0.0169	0.1451 + j 0.0634

V. Conclusions

The necessary inclusion of slack bus in the load flow problem definition has an inherent drawback when dealing with uncertain nodal powers. While serving its purpose of balancing powers in the system, it also absorbs all uncertainties. The result is a solution that is usually of no practical interest. To overcome this problem, we have investigated two ways of treating the slack bus so that the solution obtained also satisfies its' constraints. In the boundary load flow context both methods should give approximately the same results since the objective is that of the same global constrained optimum. Still, this very much depends of the actual implementation within the BLF algorithm as the already difficult task is further complicated with an inclusion of yet another constraint. We are investigating further methods to improve the robustness of these methods and make them applicable to planning practical large scale systems.

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VII. Biographies

Aleksandar Dimitrovski received his B.Sc. and Ph.D. in Power Engineering from University "Sv. Kiril i Metodij" in Skopje, Macedonia, and M.Sc. in Computer Science Application from University of Zagreb, Croatia. He is an Assistant Professor in power systems at the University "Sv. Kiril i Metodij", currently on leave at Washington State University. His subjects of interests include advanced computing techniques in power system analysis.

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