A full demand response model in co-optimized energy and reserve market

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**A B S T R A C T**

It has been widely accepted that demand response will play an important role in reliable and economic operation of future power systems and electricity markets. Demand response can not only influence the prices in the energy market by demand shifting, but also participate in the reserve market. In this paper, we propose a full model of demand response in which demand flexibility is fully utilized by price responsive shiftable demand bids in energy market as well as spinning reserve bids in reserve market. A co-optimized day-ahead energy and spinning reserve market is proposed to minimize the expected net cost under all credible system states, i.e., expected total cost of operation minus total benefit of demand, and solved by mixed integer linear programming. Numerical simulation results on the IEEE Reliability Test System show effectiveness of this model. Compared to conventional demand shifting bids, the proposed full demand response model can further reduce committed capacity from generators, starting up and shutting down of units and the overall system operating costs.

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1. Introduction

The reliability and efficiency of power system operation have always been a high priority in competitive electricity markets. Reliable operation of power system necessitates a balance between the generation and demand at all times. This is challenging given the fact that both generation and demand can change rapidly and unexpectedly, e.g., due to reasons of loss of generation units, transmission lines outages and sudden load changes. When renewable energy resources, such as wind and solar, are introduced, this problem becomes even more difficult. As flexibility of conventional generators is restricted by technical constraints, such as ramp rates, maintaining power system reliability using only generation side flexibility becomes technically too constrained and potentially compromises efficiency [1].

Demand response (DR) is another approach to meet the need for flexibility. In fact, the importance of DR has been recognized and in several countries, it is implemented for obtaining reliable and efficient electricity markets [2–4]. DR can reduce the load at peak periods, which reduces the underutilization of generators with marginal costs [5]. In addition, DR can benefit individual customers by reducing their electricity charges through shifting consumption to lower price hours. Besides participation in energy markets, advances in control and communication technologies offer the possibility for DR to participate in reserve markets and provide contingency reserves during emergency conditions of the system by changing the normal consumption [6]. The additional scheduling flexibility introduced by DR facilitates more reliable and efficient power system operation, reduces transmission line congestion and mitigates price fluctuations and generally leads to significant gains in overall system benefits [7–9].

In order to better utilize DR, demand response providers (DRPs) have been introduced in the electricity markets as aggregators of small and widely dispersed customer responses [10]. The DRPs act as medium between Independent System Operators (ISOs) and small customers, bid the aggregated customer responses in the energy and/or reserve markets and schedule the responsive demand according to the result of market clearing. By this means, the flexibility of all customers, even small ones, can be exploited. Large customers satisfying certain requirements, such as minimum curtailment level, can participate as sole entities in the programs. Considerable efforts have been devoted to incorporate DR into the market clearing process to achieve the most efficiency. In [6], a market model in which generators and consumers can submit offers and bids on both energy and reserve are proposed, but the network and multi-period constraints are neglected. In addition, the reserve constraint is deterministic in this model.

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\[ \begin{align*}
&i \quad \text{index of generators, running from 1 to } \text{NG} \\
&j \quad \text{index of demand, running from 1 to } \text{ND} \\
&t \quad \text{index of time periods, running from 1 to } \text{NT} \\
&k \quad \text{index of transmission lines, running from 1 to } \text{NK} \\
&m \quad \text{index of energy blocks offered by generators (demand), running from 1 to } \text{NI} (N) \\
&\omega \quad \text{index of scenarios of generators, running from 1 to } \text{NW} \\
\end{align*} \]

**Binary variables**

\[ u_{it} \quad \text{1 if unit } i \text{ is scheduled on during period } t \text{ and 0 otherwise} \]
\[ u_{jt} \quad \text{1 if demand } j \text{ is scheduled on during period } t \text{ and 0 otherwise} \]

**Continuous variables**

\[ p_{it}(m) \quad \text{power output scheduled from the } m\text{-th block of energy offer by unit } i \text{ during period } t. \text{ Limited to } p_{it}^{\text{max}}(m) \]
\[ d_{jt}(m) \quad \text{power consumption scheduled from the } m\text{-th block of energy bid by demand } j \text{ during period } t. \text{ Limited to } d_{jt}^{\text{max}}(m) \]
\[ P_{it} \quad \text{power output scheduled for unit } i \text{ during period } t \]
\[ D_{jt} \quad \text{power consumption scheduled for demand } j \text{ during period } t \]
\[ R_{it}^{\text{up}} \quad \text{scheduled up-spinning reserve for unit } i \text{ during time period } t \]
\[ R_{it}^{\text{down}} \quad \text{scheduled down-spinning reserve for unit } i \text{ during time period } t \]
\[ R_{jt}^{\text{up}} \quad \text{scheduled up-spinning reserve for demand } j \text{ during period } t \]
\[ R_{jt}^{\text{down}} \quad \text{scheduled down-spinning reserve for demand } j \text{ during period } t \]
\[ P_{i(to)} \quad \text{power output of unit } i \text{ during period } t \text{ in scenario } \omega \]
\[ D_{j(to)} \quad \text{power consumption of demand } j \text{ during period } t \text{ in scenario } \omega \]
\[ r_{i(to)}^{\text{up}} \quad \text{deployed up-spinning reserve from unit } i \text{ during time period } t \text{ in scenario } \omega \]
\[ r_{j(to)}^{\text{up}} \quad \text{deployed up-spinning reserve from demand } j \text{ during time period } t \text{ in scenario } \omega \]
\[ r_{j(to)}^{\text{down}} \quad \text{deployed down-spinning reserve from demand } j \text{ during time period } t \text{ in scenario } \omega \]
\[ r_{i(to)}^{\text{down}}(m) \quad \text{deployed spinning reserve from } m\text{-th block of energy offer by unit } i \text{ during period } t \text{ in scenario } \omega \]
\[ r_{j(to)}^{\text{down}}(m) \quad \text{deployed spinning reserve from } m\text{-th block of energy bid by demand } j \text{ during period } t \text{ in scenario } \omega \]
\[ L_{i(to)} \quad \text{involuntary load shedding from demand } j \text{ during period } t \text{ in scenario } \omega. \text{ Limited to } L_{jt}^{\text{max}} \]

**Constants**

\[ \lambda_{i(m)} \quad \text{marginal cost of the } m\text{-th block of energy offer by unit } i \text{ during period } t \]
\[ \lambda_{j(m)} \quad \text{marginal benefit of the } m\text{-th block of energy bid by demand } j \text{ during period } t \]
\[ A_{i} \quad \text{operating cost of unit } i \text{ at the point of } p_{i}^{\text{min}} \]
\[ B_{j} \quad \text{consumption benefit of demand } j \text{ at the point of } D_{jt}^{\text{min}} \]
\[ C_{it}^{\text{up}} \quad \text{capacity cost offer of unit } i \text{ during period } t \text{ for providing up-spinning reserve} \]
\[ C_{jt}^{\text{up}} \quad \text{capacity cost offer of demand } j \text{ during period } t \text{ for providing up-spinning reserve} \]
\[ C_{jt}^{\text{down}} \quad \text{capacity cost offer of demand } j \text{ during period } t \text{ for providing down-spinning reserve} \]
\[ \pi_{\omega} \quad \text{value of lost load from demand } j \text{ during period } t \]
\[ \omega_{\text{max}} \quad \text{maximum consumption of demand } j \text{ during period } t \]
\[ \omega_{\text{min}} \quad \text{minimum consumption of demand } j \text{ during period } t \]
\[ \Delta t \quad \text{maximum bidding price of demand } j \text{ during period } t \]
\[ y_{1} \quad \text{price elasticity of demand } j \text{ during period } t \]
\[ \gamma_{\omega} \quad \text{recovery rate of demand } j \]
\[ \Delta t \quad \text{duration of time period } t \]
\[ \omega \quad \text{maximum energy consumption of demand } j \text{ during the scheduling horizon} \]
\[ D_{jt}^{\text{fix}} \quad \text{fixed demand of demand } j \text{ during period } t \]
\[ G_{k} \quad \text{generation shift factor to line } k \text{ from unit } i \]
\[ P_{k}^{\text{max}} \quad \text{transmission limit of line } k \]
\[ H_{\omega} \quad \text{health indicator of system during period } t \text{ in scenario } \omega \]

A probabilistic reserve model with demand-side participation is proposed in [11]. In this model, demand is able to submit bids of load reduction in the energy market and load shedding upon request through the reserve market. In [12], the price responsive demand shift bidding of consumers is introduced in a day-ahead market with network constraints. The ACOPF model is used in the formulation without considering unit commitment. The DR modeled with inter-temporal characteristics is incorporated into security constrained unit commitment (SCUC) for economic and security purposes in [13]. The DR is modeled as shiftable load and only participates in energy market in [12,13]. In [14], spinning reserve provided by DRPs and its associated cost function is formulated in a mixed integer linear form and incorporated in a two-stage stochastic SCUC. In [15], the demand recovery effect after deployment of spinning reserve from DR is further considered. It should be noted that DRPs only participate in spinning reserve market in [14,15].

The main contribution of this paper is to propose a full demand response model in which the DRP can submit duplex bids. Specifically, a consumer can bid energy as a price responsive shiftable demand in energy market, meanwhile, bids reserved with its energy bids in reserve market. It should be noted that the principle of demand providing spinning reserve is different from that of generators. A consumer has the potential to provide spinning reserve as long as it is scheduled in the energy market, while a generator can provide spinning reserve only if it is not scheduled at full capacity. Based on this point, we make use of the flexibility of demand in the day-ahead time horizon through price responsive shiftable demand bids before any realization of system contingencies. In addition, the potential of rescheduling the scheduled demand in energy market when system experiences a contingency is further utilized by spinning reserve bids. In other words, the price responsive shiftable demand bids are DRPs’ response to price in the first stage, while the spinning reserve bids are DRPs’ response to
contingencies in the second stage. For this reason, the proposed full demand response model can exploit more flexibility from demand than previous mentioned models. The proposed demand response model is incorporated in a co-optimized day-ahead energy and spinning reserve market in which the expected net cost under any credible system state, i.e., expected total cost of operation minus total benefit of demand, is minimized. The most economic solution with a probabilistic spinning reserve scheme is obtained by balancing the energy plus spinning reserve cost and cost of expected energy not served (EENS) [17]. Unlike the deterministic spinning reserve requirement, the spinning reserve is quantified based on an internal cost/benefit analysis in the proposed model. By using a high value of lost load (VOLL), involuntary load shedding is treated as the last resort to maintain system reliability, which is consistent with practical industry operation.

This paper is organized as follows. In Section 2, the full model of demand response is described. The co-optimized day-ahead energy and spinning reserve market clearing problem is formulated in Section 3. After that, results of case studies are presented in Section 4. Finally, the conclusions are given in Section 5.

2. Model of full demand response

Consumers have the opportunity to reduce their electricity bills by adjusting their activities according to the market clearing results, particularly, clearing prices. However, not all consumers are able or willing to adjust their demand even with sufficient price incentives. For this reason, demand is best divided into price taking and price responsive demands. This section proposes a full demand response model in which the DRP can submit duplex bids. The duplex bidding mechanism includes price responsive shiftable demand bidding in energy market and spinning reserve bidding in the reserve market. The two bids are coupled since the amount of spinning reserve that a demand can provide depends on the accepted energy bids in the energy market.

The full demand response model can be interpreted as an extension of the price responsive shiftable demand bids proposed in [16]. In the energy market, DRPs on behalf of consumers who have the flexibility to shift their consumption in certain time periods can submit bids that are sensitive to electricity prices in the energy market. Hence, DRPs can shift their demand from high electricity price periods to low electricity price periods and reduce the electricity charges over the entire scheduling horizon in day-ahead market. In addition, DRPs, on behalf of consumers who have the means to reschedule their scheduled consumption in the energy market when a contingency happens, can submit bids in the spinning reserve market. The spinning reserve bids are coupled with the price responsive shiftable demand bids since the spinning reserve that a demand can provide are constrained by the accepted energy bids. The more demand that is scheduled, the more spinning reserve it can provide. The price responsive shiftable demand bids make use of the flexibility of demand in the day-ahead stage before the realization of system contingencies, while the spinning reserve bids utilize the flexibility of rescheduling the scheduled demand in energy market when system experiences a contingency in the second stage. The duplex bids correspond to two stages of flexibility.

A typical duplex bidding model is shown in Fig. 1. It includes price responsive shiftable demand bids in energy market and spinning reserve bids in the reserve market. For price responsive shiftable demand bids, the following characteristics can be included:

- total energy consumption over the scheduling horizon,
- demand pickup/drop rate, and
- minimum up/down time limits.

The negative slope bidding curve of Fig. 1 indicates that DRPs would only accept a demand level for which the bidding price \( \lambda_{jt} \) is less than or equal to the market price (i.e., LMP). The benefit function of a DRP can be determined by the integral of the bidding curve from 0 to \( D_{jt} \)

\[
B_{jt}(D_{jt}) = (\lambda_{jt}^{\max} + \alpha_{jt} D_{jt}^{\min}) D_{jt} - 0.5 \alpha_{jt} (D_{jt})^2
\]

(1)

The quadratic benefit function of a DRP can be approximated by piecewise linear functions similar to generation units as shown in Fig. 2 [6]. These slopes of the approximated demand benefit curve are actually the bidding prices of the blocks of energy bids by a DRP. The detailed mathematical model is shown in Section 3.2.

Coupled with the price responsive shiftable demand bids, the spinning reserve bids that DRPs can submit are flexible. In particular, the following characteristics can be included:

- price–volume bid at a specific period (\( C_{jt}^{\min}, R_{jt}^{\max} \)),
- maximum spinning reserve at any period,
- maximum daily curtailment, and
- demand pickup/drop rate.

Similar to generation units, the cost of providing spinning reserve by DRPs includes two costs: capacity and deployment. The deployment cost can be obtained from the benefit function shown in

\[
B_{jt}(D_{jt})
\]

(2)

Fig. 2. Piecewise linear approximation of the demand benefit function.
Section 3.4. In addition, the demand recovery effect after deploying spinning reserve is considered.

3. Formulation of SCUC with full DR model

This section describes a market clearing process that accepts duplex bids from both generators and DRPs. The market operator collects all offers and bids and then performs a multiperiod optimization to determine the optimal production and consumption schedules as well as the required amount of spinning reserve [17]. The market clearing problem is formulated as a two-stage stochastic MILP. The first stage involves the network-constrained unit commitment with probabilistic spinning reserve requirements in the base case without contingency and the second stage involves the recourse in scenarios where contingencies occur. The contingencies in this paper only include random outages of generating units, although it can be easily extended to take into account random outages of transmission lines. A set of contingent scenarios is constructed based on "n − 1" or "n − 2" contingency rules. More generally, Monte Carlo simulation method can be used to generate the set of contingent scenarios and an effective scenario reduction method is essential for reducing the dimension and computational burden of the problem [18].

In this two-stage model, decisions made in the first stage include commitment status of units and demand, scheduled power output and consumption and up- and down-spinning reserve from both generators and DRPs. Decisions made in the second stage include deployment of spinning reserve and involuntary load shedding. It should be noted the decisions on commitment status of units and demand are only made in the first stage and does not change in any scenario. The deployment of spinning reserve and involuntary load shedding are determined for each scenario in order to survive the contingency. The constraints include not only the first stage and second stage constraints but also the linking constraints, which bind the market decisions to the actual power system operation [19].

3.1. Objective function

The objective function minimizes the expected net cost (i.e., combined energy cost, spinning reserve cost, the expected cost of deploying spinning reserve in all scenarios weighted by probabilities of corresponding scenarios less the total benefit of demand). The expected net cost (ENC) is given by

\[
EN C = \sum_{t=1}^{NT} \sum_{i=1}^{NG} \sum_{m=1}^{NI} [\lambda_d(m)p_d(m) + A_iu_{it}]
\]

\[
- \sum_{t=1}^{NT} \sum_{j=1}^{NG} \sum_{m=1}^{NI} [\lambda_j(m)d_j(m) + B_ju_{jt}] + \sum_{t=1}^{NT} \sum_{i=1}^{NG} S_d(u_{it}, u_{i,t-1})
\]

\[
+ \sum_{t=1}^{NT} \sum_{i=1}^{NG} \sum_{j=1}^{NJ} (C_{iu}^d + C_{iu}^u) + \sum_{t=1}^{NT} \sum_{j=1}^{NJ} (C_{j}^d + C_{j}^u)
\]

\[
+ \sum_{o=1}^{NW} \sum_{i=1}^{NT} \sum_{j=1}^{NG} \sum_{m=1}^{NI} \lambda_m(r_{foi}(m))
\]

\[
+ \sum_{t=1}^{NT} \sum_{j=1}^{NG} \sum_{m=1}^{NI} \lambda_j(m)r_{fi}(m) + \sum_{t=1}^{NT} \sum_{j=1}^{NG} VOLL_{jt}^d
\]

(2)

The objective function (2) includes eight terms (from line 1 to line 8), among which lines 1–5 are related to first-stage decisions and lines 6–8 are related to second-stage decisions. Specifically, the first and second lines treat the energy cost of generators and benefit of DRPs. The third line is the start up cost of generators; the fourth and fifth lines are the cost of scheduling up- and down-spinning reserve from generators and DRPs separately; the sixth and seventh lines are the cost associated with the actual deployment of up and down spinning reserve by generators and DRPs separately; and the last line is the cost of involuntary load shedding. All terms are in mixed-integer linear form except the startup cost of generators (line 3), which can be recast into mixed-integer linear form as in [20].

It should be pointed out that the benefit function (line 2) of demand in Eq. (2) only applies to the price responsive part of demand. Theoretically, the integral of bidding curve of price taking part is infinite, since the marginal value of price taking demand is infinite. Without loss of generality, the marginal value of price taking demand is assumed a large positive constant. So, the benefit function of price taking part of demand is also constant and can be taken out of the optimization.

3.2. First-stage constraints

The first-stage constraints are associated with the base case, including the following:

\[
D_j = \sum_{m=1}^{NI} d_j(m) + u_{jt}D_{jt}^{s}\quad \forall j, \forall t
\]

(3)

\[
0 \leq d_j(m) \leq D_j^{d}\quad \forall j, \forall t, \forall m
\]

(4)

\[
\sum_{t=1}^{NT} D_j \Delta t \leq E_j \quad \forall j
\]

(5)

\[
0 \leq D_j \leq D_j^{d}\quad \forall j, \forall t
\]

(6)

\[
r_{jt}^d \leq D_j - D_j^{d}u_{jt} \quad \forall j, \forall t
\]

(7)

\[
r_{jt}^d \leq D_j^{d}u_{jt} - D_j \quad \forall j, \forall t
\]

(8)

\[
\sum_{i=1}^{NG} p_i = \sum_{j=1}^{ND} (D_j + D_{jt}^f) \quad \forall t
\]

(9)

\[
\sum_{i=1}^{NG} \sum_{j=1}^{ND} GSF_{jk}p_i - \sum_{j=1}^{ND} GSF_{jk}(D_j + D_{jt}^f) \leq F_k \quad \forall k, \forall t
\]

(10)

Constraints (3) and (4) approximate the benefit function of DRPs by blocks [21]. Constraint (5) specifies the total energy consumption over the scheduling horizon and (6) specifies the maximum power consumption of demand \( j \) during period \( t \). The coupling between energy bids and reserve bids of the duplex bid is enforced by constraints (7) and (8). To be specific, constraint (7) ensures that the up-spinning reserve that a demand can provide should be less than the difference of its scheduled and minimum consumption if it is scheduled. Similarly, the limit of down-spinning reserve that a demand can provide is specified in (8). The market equilibrium is enforced by (9). DC power flow is used to represent the transmission constraints as in (10). Additionally, each unit or DRP is subject to its own operating constraints, including minimum up and down time, initial condition, capacity limits and ramp limits. See [22] for details about mathematical formulations of these constraints.
3.3. Second stage constraints (depending on scenario \( \omega \))

The second-stage constraints are associated with each contingency scenario, including the following:

\[
\begin{align*}
\sum_{i=1}^{NG} P_{ito} = \sum_{j=1}^{ND} (D_{ito} + D^F_{j} - L_{ito}) & \quad \forall t, \forall \omega \quad (11) \\
\sum_{i=1}^{NG} GSF_{kJ} P_{ito} = \sum_{j=1}^{ND} GSF_{kJ} (D_{ito} + D^F_{j} - L_{ito}) & \leq f^{\text{max}}_k \quad \forall k, \forall t, \forall \omega \quad (12) \\
0 \leq L_{ito} \leq f^{\text{max}}_j & \quad \forall j, \forall t, \forall \omega \quad (13) \\
\sum_{i=1}^{NT} D_{ito} \geq \gamma_j \sum_{i=1}^{NT} D^U_{j} & \quad \forall j, \forall \omega \quad (14)
\end{align*}
\]

The power balance of each scenario is enforced by (11). DC power flow is used to represent the transmission constraints in each scenario as in (12). Inequality (13) constrains the maximum involuntary load shedding of each DRP and (14) specifies the recovery limit of demand \( j \). The recovery rate \( \gamma_j \) is a characteristic of the individual demand \( j \), determined by the properties of activity facilitated by the use of electricity.

3.4. Constraints linking first and second stage variables

The linking constraints, which bind the market decisions to the actual power system operation, include the following:

\[
\begin{align*}
P_{ito} &= P_{i} \xi_{ito} + r^U_{ito} - r^D_{ito} \quad \forall i, \forall t, \forall \omega \quad (15) \\
D_{ito} &= D^U_{j} + r^U_{ito} - r^D_{ito} \quad \forall j, \forall t, \forall \omega \quad (16) \\
r^U_{ito} &\leq r^U_{j} \xi_{ito} \quad \forall i, \forall t, \forall \omega \quad (17) \\
r^D_{ito} &\leq r^D_{j} \xi_{ito} \quad \forall i, \forall t, \forall \omega \quad (18) \\
r^U_{ito} &\leq \begin{cases} r^U_{j} (1 - H_{ito}), & \forall j, \forall t, \forall \omega \\ 0, & \forall i, \forall t, \forall \omega \end{cases} \quad (19) \\
r^D_{ito} &\leq \begin{cases} r^D_{j} (1 - H_{ito}), & \forall j, \forall t, \forall \omega \\ 0, & \forall i, \forall t, \forall \omega \end{cases} \quad (20)
\end{align*}
\]

The actual generation output and demand consumption are enforced by (15) and (16). In order to consider the reliability of generators, \( \xi_{ito} \) is a binary parameter where 1 represents the healthy state of a generator and 0 a unit that is unavailable. The constraints on deployed up- and down-spinning reserve are enforced by (17)-(20), in which health indicator, \( H_{ito} \), is a binary parameter where 1 represents no contingency occurs up to time period \( t \). Introducing \( H_{ito} \) means the spinning reserve can only be deployed after a contingency happens.

In order to take into account the cost of deployment of spinning reserve, the deployed spinning reserve from both demand and generation is decomposed into corresponding energy blocks:

\[
r^U_{ito} - r^D_{ito} = \sum_{m=1}^{NJ} r^U_{ito}(m) \quad \forall j, \forall t, \forall \omega \quad (21) \\
r^U_{ito}(m) \leq d^U_{j}(m) \quad \forall m, \forall j, \forall t, \forall \omega \quad (22) \\
d^U_{j}(m) - \max_{m} d^U_{j}(m) \leq r^D_{ito}(m) \quad \forall m, \forall j, \forall t, \forall \omega \quad (23) \\
r^U_{ito} - r^D_{ito} = \sum_{m=1}^{NJ} r^U_{ito}(m) \quad \forall i, \forall t, \forall \omega \quad (24) \\
r^U_{ito}(m) \leq p^\text{max}_{j}(m) - p_j(m) \quad \forall m, \forall i, \forall t, \forall \omega \quad (25) \\
-p_d(m) \leq r^D_{ito}(m) \quad \forall m, \forall i, \forall t, \forall \omega \quad (26)
\]

Eq. (21) decomposes the deployed spinning reserve of demand \( j \) into blocks. Constraints (22) and (23) enforce that the blocks of spinning reserve are subtracted (or added in case of down-spinning reserve) to blocks of energy [17]. Then, the cost of deployment of spinning reserve of demand \( j \) in scenario \( \omega \) can be represented as \( \sum_{m=1}^{NJ} \lambda_j(m) r^U_{ito}(m) \), which is included in the objective function (2). Similarly, the deployed spinning reserve from generators can be decomposed by (24)-(26) and the cost of deployment of spinning reserve from generators can also be included.

4. Numerical simulation results

4.1. Test system data

In this section, the proposed full demand response model is demonstrated on a modified IEEE Reliability Test System [23]. In the modified system, there are 26 thermal generators but the hydro units have been removed. The ramp rates, failure rates and quadratic cost coefficients are taken from [24]. For simplicity, the quadratic cost curves in [24] are converted into piece-wise linear cost curves. In addition, we assume that all units offer capacity cost of up- and down-spinning reserve at the rates of 10% of their highest incremental cost of producing energy [25]. The deployment cost of spinning reserve from units is calculated according to the actual cost of energy.

The analysis is conducted for a 24-h scheduling horizon and the forecast demand is from [24]. The demand forecast error is neglected. Each demand contains two parts: fixed and responsive. The proposed demand response model is for the responsive component delegated to a DRP at each bus. The price elasticity of DRPs’ offer price is set at 0.1$/MWh [12]. The maximum and minimum energy consumption of all DRPs is set at 200 MW and 0 MW, respectively. The maximum energy bidding price of DRPs is set at 45$/MWh. Based on these parameters, DRPs’ benefit functions are calculated, and then linearized into 3 sections. DRPs offer capacity cost of spinning reserve at the rate of 20% of the highest marginal benefit of energy. The deployment cost of spinning reserve from DRPs is calculated according to the actual benefit of energy. VOLL is set at 4000$/MWh [14].

A two-state Markov model is used to simulate the uncertainties of units outages [26]. For simplicity, only single contingency events are taken into account since multiple order contingency events have relatively small probabilities while requiring far more computational resources to include their impact. Note the difference between operations and a planning calculation, where in planning one needs to consider the possibility of multiple outages and units under maintenance to ensure reliability.

In order to show the advantages of the proposed full demand response model over the price responsive shiftable demand model, the dispatch results of SCUC with the proposed model are compared with that of SCUC with a shiftable demand model. The recovery rate of each load after deployment as spinning reserve is set at 0.7. The model is coded in MATLAB and solved using the MILP solver CPLEX 12.2. With a pre-specified duality gap of 0.1%, the running time of each case is about 8 min on a 2.66 GHz Windows-based PC with 4 G bytes of RAM.

4.2. Comparison of unit commitment results

The results of unit commitment with 10% full demand response and 10% shiftable demand are compared in Fig. 3. As can be seen, the full demand response model, with fewer units committed since DRPs can also provide spinning reserve in the proposed full demand response model, leads to lower committed capacity from generators. Specifically, DRPs provide a comparable amount of spinning
reserve with the proposed full demand response model during hours 1–8 and 22–24 as shown in Fig. 10. With this additional reserve, unit 21 does not need to be committed during hours 1–7 and 23–24 as well as unit 22 during hours 7–8 and 22–24. In contrast, these units have to be committed to provide spinning reserve during these hours with the shiftable demand model. The same effect can be seen when the full demand response and shiftable demand are increased to 20% as in Fig. 4. With 20% of shiftable demand, unit 22 is committed during hours 2–22 and decommitted during other hours. Compared to the proposed full demand response model, unit 22 does not need to be committed and the unit commitment status is the same for all 24 h. In this way, the frequency of starting up and shutting down of the units is decreased.

The results of unit commitment with different percentage of shiftable demand are compared in Fig. 5. With 20% of shiftable demand, unit 22 is additionally committed during hours 2–6 but decommitted during hours 23–24. This is because more demand can be shifted from peak load hours 10–21 to valley load hours 2–6 as shown in Fig. 7. Theoretically, with high enough percentage of shiftable demand, the demand profile will be flat and the unit commitment status will be the same for all 24 h. This effect is more obvious with different percentage of full demand response as in Fig. 6. With 20% of full demand response, unit 21 is committed for all 24 h and unit 22 does not need to be committed at all. In this case, the unit commitment status is actually the same for all 24 h.
4.3. Comparison of total demand

The total demand profiles with different percentages of full demand response and shiftable demand are compared in Fig. 7. As can be seen, with 10% full demand response model, the total demand during peak hours (hours 9–21) is higher than that with 10% of shiftable demand, i.e., the shiftable demand can shift more load from peak hours to valley hours. This is because it is more economic to maintain some responsive load and provide spinning reserve during peak hours relative to shifting load to valley hours and starting up additional units during the valley hours. When the responsive demand penetration increases to 20%, the demand profile with shiftable demand is higher than that with full demand response during hours 3–22. This is due to unit 22 shutting down at hours 23 and 24 as shown in Fig. 4, hence demand reduced during hours 23 and 24 is shifted to other hours.

4.4. Comparison of spinning reserve and EENS

The spinning reserve and EENS profiles with 10% full demand response and 10% shiftable demand are shown in Fig. 8. As can be seen, with full demand response model, the spinning reserve level is lower than that with shiftable demand at hours 7–9, 14, 17–20 and 22–24. Consequently, the EENS with full demand response is higher than that with shiftable demand. Although demand can provide spinning reserve with the full demand response model, the spinning reserve that generators can provide is reduced under the same unit commitment since less demand is shifted as shown in Fig. 7. In addition, the spinning reserve from DRPs is more expensive than that from generators. This explains why the spinning reserve level with full demand response is slightly lower than with shiftable demand during hours 9, 14 and 17–20. During hours 7–8 and 22–24, additional units are committed with shiftable demand as shown in Fig. 3. This results in the spinning reserve level with full demand response is further lower than with shiftable demand in these hours. The same effect can be seen in Fig. 9 during hours 2–22, when the full demand response and shiftable demand are increased to 20%.

The comparison of spinning reserve provided by generators and demand with 10% demand response is presented in Fig. 10. The DRPs provide spinning reserve in both peak and valley load hours. There are two reasons for demand providing more spinning reserve
during valley hours than that during peak hours. Firstly, there is more demand response during valley load hours since responsive demand is shifted from peak to valley. Secondly, units 21 and 22 are committed at hours 8 and 9 respectively, as a result, additional inexpensive spinning reserve is available from these two generators.

Note that not all the responsive demand is scheduled as spinning reserve at the same time. It may be more economic to schedule spinning reserve from generators instead of scheduling more spinning reserve from DRPs.

### 4.5. Comparison of classified and total costs

The classified costs with different models of demand response are presented in Table 1. It can be seen that the energy cost and reserve scheduling cost of generators are significantly decreased when the proposed full demand response model is introduced. Specifically, the energy cost (including start-up cost) is reduced by 1.56% and 2.11% when the percentage of demand response is 10% and 20% respectively. This is because the proposed full demand response model leads to less committed capacity and less starting up and shutting down of the units. This ensures that more units are operating continuously and efficiently. With the proposed model, DRPs can provide spinning reserve instead of generators. Consequently, the reserve scheduling and deployment cost of DRPs cannot be neglected. It should be noted that the involuntary load shedding cost with proposed model can be higher than that with shiftable demand. This is because fewer units are committed since DRPs can also provide spinning reserve. In addition, the cost of spinning reserve from DRPs is more expensive than that from generators. This results in less spinning reserve and higher involuntary load shedding cost. Nevertheless, compared with the very high VOLL, the increase in the cost of spinning reserve has a very small effect on the amount of involuntary load shedding.

The total operating costs of the system with different percentages of full demand response and shiftable demand are compared in Table 2. The total cost with 10% full demand response model is reduced by 0.06% relative to that with 10% shiftable demand. With 20% full demand response, the total cost can be reduced by 0.17%.

With 20% full demand response, the total cost reduction rates under different capacity cost of spinning reserve from DRPs are shown in Fig. 11. As can be seen, when DRPs offer capacity cost of spinning reserve at the rate of 20% of the highest marginal benefit of energy, the total cost can be reduced by 0.17%. However, when DRPs offer capacity cost of spinning reserve at the rate of 10% of the highest marginal benefit of energy, the total cost can be reduced by 1.77%, which is significant. In this case, the average capacity cost of spinning reserve from DRPs is reduced to 4.17 $/MWh, which is still much higher than that from generators (2.12 $/MWh).

### 5. Conclusions

In this paper, a new duplex demand response model which allows one demand to bid in both energy market and spinning reserve market is developed. A co-optimized day-ahead energy and spinning reserve market is proposed to maximize the expected social welfare, i.e., minimize the expected net cost under any credible system state. The problem is solved by mixed integer linear programming. Numerical simulation results on the IEEE Reliability Test System show the effectiveness of this model. Compared to conventional demand shifting bids, the proposed full demand response model can further reduce the need to commit capacity from generators, on/off cycling of generators and fluctuations in system reliability. In particular, simulation results have shown that the proposed full demand response model outperforms conventional demand shifting bids in operating efficiency. That is, the second stage flexibility of demand in the proposed model leads to higher operating efficiency.

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