Congestion Influence on Bidding Strategies in an Electricity Market

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***Abstract*—Much of the research on bidding strategies in an electricity market has focused on idealized situations where participants have limited market power and the transmission system is not constrained. Yet, congestion may act to effectively give a bidder market power, and consequently the ability to influence the market clearing price. In such a noncompetitive situation, the bidding strategies of market participants will change. In this paper, the electrical power market is modeled as an oligopoly market and the Cournot quantity model is applied to the bidding strategy problem. The bidding process with congestion management is modeled as a three level optimization problem. A statistical methodology is then proposed as a solution for large systems.**

***Index Terms*—Auctions, bidding strategies, congestion management, market clearing price, mixed strategy equilibrium, Nash equilibrium.**

I. Introduction

UNDERSTANDING how market participants bid into the electricity market is of fundamental importance for designing electricity markets. Generally, the objective of market participants is to maximize their expected profit. Since the expected profit of each participant depends upon the joint actions of others, effective decision-making requires that each participant evaluate the effects not only of their own actions, but also of the actions undertaken by the others. The complexity of these interactions make it difficult to determine ahead of time the strategies that market players will employ in bidding.

There have been numerous attempts to model bidding strategies using optimization methods. For example in [1], a Lagrangian relaxation method was used to determine the utilities' optimal bidding and self-scheduling, and based on the New England ISO, a closed form solution was found assuming a simple bidding model. In [2], the power market was treated as an oligopoly, and by "guessing" competitor's bidding curves, a stochastic optimization model was built. Richter and Sheble applied a genetic algorithm to GENCO strategies and schedules, in which an intelligent bidding strategy was developed using a GP-Automata algorithm [3]. In [4], the combination of different pricing systems and curtailment methods was analyzed so as to understand methods to prevent taking advantage of network congestion. Hao studied the bidding strategies in a clearing price auction [5]. Based on the clearing price auction, the author drew the conclusion that the market participants have incentives to mark up their bids above their production cost and the amount of mark up depends on the probability of how frequently they win the bid.

Ni, *et al.*, presented a unified optimization algorithm for the bidding strategy problem given a mix of hydro, thermal and pumped storage units [6]. Their algorithm manages bidding risk and self-scheduling requirements. Gan and Bourcier modeled the market as a single-period auction oligopoly market and examined the influence of suppliers' capacity constraints [7]. Shrestha, *et al.*, analyzed the effect of minimum generator output [8]. Others, such as Ferrero, *et al.* [9], applied game theory to analyze transactions. In their work, spot price was used to calculate the payoff matrix and both Nash equilibrium and characteristic functions were applied to the bidding analysis. Congestion charges were not considered in their work. Hobbs and Kelley applied game theory to electric transmission pricing [10]. Bai, *et al.* applied the Nash Game equilibrium concept to the transmission system [11]. Yu, *et al.* [12] investigated transmission limits and the influence of wheeling charges on competitive and gaming behavior. It was shown that wheeling charges and transmission line limits stimulate gaming phenomena.

While providing valuable insight into transmission system impacts, none of these efforts have fully incorporated transmission constraints into the bidding strategies. In practice, congestion management is separate from the bidding process and as such difficult to analyze in a single bidding framework. When congestion occurs, a noncompetitive situation (i.e., deviation from price-taking behavior) is far more likely to occur. Much of the literature has ignored congestion or included it as part of the bidding process, as in [13]. That is, most researchers have included the congestion as constraints within the market clearing process. This is not representative of typical market rules.

In this paper, the bidding strategy problem is modeled as a three level optimization problem, and the congestion's influence is explicitly expressed in the profit function. Game theory is applied to the optimal bidding strategies problem based on a U.K. pricing system. Congestion's influence is modeled and the curtailment due to congestion is calculated via a separate least curtailment method [14]. Numerical examples clarify congestion's influence on price and bidding strategies. Subsequently, these results are modified to reflect behavior based on a statistical study of bidding in the California market.

II. Hierarchical Model of Bidding Process

In a power clearing market, each participant submits a bidding curve to a power exchange, or similar organization. The

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exchange will decide the market clearing price (MCP) based on these bids. The security coordinator then checks to ensure that the resulting bidding schedule is feasible. When there is a security problem, curtailment will be performed. If the uniform price and least curtailment algorithm are used, the bidding problem can be represented by a hierarchical optimization problem developed in this section. Other congestion management approaches will require a slight modification to this development but the approach is similar.

The electricity power market is in practice an oligopoly [15]. In an oligopoly market, competition among the market participants is inherently a setting of strategic interaction. Thus, the appropriate tool for analysis is game theory. In the electric power market, the participants submit their bids first, and then the MCP is found by matching the aggregate demand to the aggregate supply. The bidding strategies have a clear influence on the MCP and price cannot be treated as a simple function of demand. The Cournot quantity model [16] is applied to the bidding strategy decision problem here. Other oligopoly market models, such as Bertrands price competition model, may be more appropriate in specific situations. The Cournot model assumes that generators compete more by quantity than by price and generally holds well when capacity changes more slowly than price.

A. Market Clearing Price

To determine MCP, the exchange looks at the aggregated supply bid curve and the aggregated demand curve with the highest accepted bid the MCP. Assume for simplicity, the bidding curves are given as continuous curves of the form

$$IC_i(p_i) = b_i + a_i p_i \tag{1}$$

where $IC_i(p_i)$ is the incremental price for generating at p_i by the ith generator, and a_i and b_i are the bidding coefficients. Here, we further assume that these two parameters have the following relations with the generator costs:

$$k_i = \frac{b_i}{b_{ic}} = \frac{a_i}{a_{ic}} \tag{2}$$

where a_{ic} and b_{ic} are parameters from the generator's actual cost function. The true costs are given by

$$C_{ic}(p_i) = c_{ic} + b_{ic}p_i + \frac{1}{2}a_{ic}p_i^2. \tag{3}$$

Thus, the bidding parameter k_i represents the proportion above (or below) marginal cost that a generator i decides to bid (i.e., the markup). Certainly, more complex functions for strategies are possible (e.g., the use of random variables [17]). Here, the focus is on the congestion's influence and mark-up provides more insight to the direct impact. Further, the strategies that might pursued by consumers are ignored and instead a simple demand benefit function $B_i(p_i)$ is used to model their role as

$$B_i(p_i) = b_i p_i - 0.5 a_i p_i^2 \tag{4}$$

where p_i is the load consumed at bus i. The market clearing problem is represented by the following social welfare maximization problem (ignoring losses):

$$\begin{aligned} \max_{p_i} \quad & \sum_{i\in L} B_i(p_i) - \sum_{i\in G} C_i(p_i) \\ \text{s.t.} \quad & \sum_{i\in L} p_i = \sum_{i\in G} p_i \\ & p_i^{\min} \le p_i \le p_i^{\max}, \quad \forall\, i \in G \end{aligned} \tag{5}$$

where L and G represent the set of loads and generators, respectively, and p_i is the load in megawatts the ith player delivers or receives in the bidding. The cost function $C_i(p_i)$ here is derived from bidding curves

$$C_i(p_i) = b_i p_i + 0.5 a_i p_i^2. \tag{6}$$

Solving (5) yields the MCP, the generator outputs p_i^* and demands that provides maximum benefit. The MCP is simply

$$\text{MCP} = \max_{i\in G} IC_i(p_i^*). \tag{7}$$

B. Congestion Management

When the bidding process is finished, the system security is analyzed. If there exists a security problem, curtailments must be carried out, either by modifying the generation dispatch or reducing load. While there are many different kinds of curtailment algorithms, here, the separate curtailment algorithm [14] is applied. Assuming a dc load flow model [18] (those equations are omitted for brevity) and no load curtailment (since demand side bidding is not considered here), this is formulated as

$$\begin{aligned} \min_{\Delta p_i} \quad & \Delta P^T \cdot W \cdot \Delta P \\ \text{s.t.} \quad & \sum_{i\in G} \Delta p_i = 0 \\ & |P_{ij}| \le P_{ij}^{\max} \end{aligned} \tag{8}$$

where $\Delta P = [\Delta p_1, \Delta p_2, \ldots, \Delta p_n]^T$ is the vector of the supplier's curtailment, so that $\Delta p_i > 0$ means the ith supplier must increase its output, the P_{ij} are the line flows; and W is a diagonal weight matrix whose elements denote the participant's willingness to pay to avoid curtailment. In this paper, the weights are set to 1, (i.e., the objective of curtailment is the least curtailment). When a generator's output is reduced, it should be compensated for possible lost profits by receiving some payment. This is found here as

$$RC_i = -\Delta p_i \left[\text{MCP} - (a_i + b_i p_i + b_i \Delta p_i)\right]. \tag{9}$$

The supplier is compensated based on the philosophy that their bid represents their actual costs and so this payment will account for the actual loss of profit. Again, there are other approaches to compensation but the approach here can accommodate such methods. The assignment of costs to consumers and the transmission company is not germane to the development here.

C. Bidding Strategy

When uniform pricing is applied in the system, all power originally purchased and actually run is paid at MCP. Thus, the profit function of participant i is

$$\text{Profit}_i = \text{MCP} \cdot (p_i + \Delta p_i) - C_{ic}(p_i + \Delta p_i) + RC_i. \quad (10)$$

MCP used here is the solution to (5), Δp_i is the curtailment due to the congestion from (8) and RC_i is found from (9) and $C_{ic}(p_i+\Delta p_i)$ is the generator's production cost. For participant i, the best strategy is the bidding parameter k_i that will maximize profit. When the congestion problem is taken into account, the ith player's problem is represented by the following maximization problem:

$$\max_{k_i} \quad \text{Profit}_i \quad (11)$$

while satisfying (5)–(8). Note, the true production costs from (3) should be used in the solution.

D. Problem Formulation

The bidding strategy problem is now seen more clearly as a hierarchical optimization problem. For simplicity, the generator capacity limits are omitted at first. The inner solution for (7) is

$$\begin{aligned} \text{MCP} =& b_i + a_i p_i, \forall\, i \in G, \\ =& b_j - a_j p_j, \forall\, j \in L. \end{aligned} \quad (12)$$

Simple algebraic manipulations show

$$\text{MCP} = \frac{\sum\limits_{i\in L} \frac{b_i}{a_i} + \sum\limits_{i\in G} \frac{b_{ic}}{a_{ic}}}{\sum\limits_{i\in G} \frac{1}{k_i a_{ic}} + \sum\limits_{i\in L} \frac{1}{a_i}} \quad (13)$$

with

$$p_i = \frac{\text{MCP} - k_i b_{ic}}{k_i a_{ic}}. \quad (14)$$

The revenue from curtailment simplifies to

$$RC_i = k_i a_{ic} \Delta p_i^2. \quad (15)$$

The individual's bidding problem (11) can be solved directly by substituting (12)–(15) if Δp_i is known. Unfortunately, Δp_i will not be known until after congestion management. In many power markets, the power transfer distribution factor (PTDF) [19] is used to decide the curtailment/redispatch. Here, we use the GSF generation shift factor (GSF) [19], which is essentially the same except the focus is on the sensitivity between the generation and transmission line. Assume the GSF is denoted by $\rho_{jk,i}$

$$\rho_{jk,i} = \frac{\Delta P_{jk}}{\Delta p_i} \quad (16)$$

where ΔP_{jk} is the flow change on line j-k, and j and k are the initial bus and terminal bus of the line. When the dc power flow is employed, the GSFs are constants related to the system topology parameters. The curtailment of each generator can be represented as a linear function of overflow of the congested line and the GSFs. Let's take one congested path as an example, assume there is only congested path with $P_{jk}^{\max} < P_{jk}$, (8) can be solved and rewritten as (the derivation is given in Appendix A):

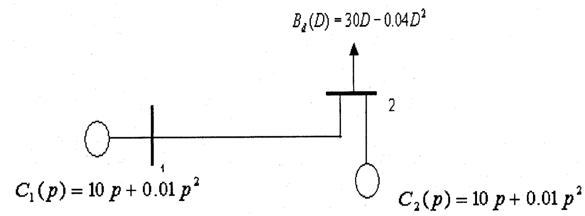

Fig. 1. Example system 1.

$$\Delta p_i = \frac{\rho_{jk,i} - \frac{1}{n}\sum\limits_{l=2}^{n} \rho_{jk,l}}{\sum\limits_{i=2}^{n}\left(\rho_{jk,i}^2 - \frac{1}{n}\rho_{jk,i}\sum\limits_{l=2}^{n}\rho_{jk,l}\right)} \cdot \left(P_{jk}^{\max} - P_{jk}\right). \quad (17)$$

Without congestion, there is no curtailment (i.e., the $\Delta p_i = 0$), so we can rewrite (17) in general form as

$$\Delta p_i = \frac{\rho_{jk,i} - \frac{1}{n}\sum\limits_{l=2}^{n} \rho_{jk,l}}{\sum\limits_{i=2}^{n}\left(\rho_{jk,i}^2 - \frac{1}{n}\rho_{jk,i}\sum\limits_{l=2}^{n}\rho_{jk,l}\right)} \cdot \min\left(0, P_{jk}^{\max} - P_{jk}\right). \quad (18)$$

III. Solution Method

The difficulty in this problem stems from the conditional constraint (18). This differs from an "either-or" type constraint that can be modeled as mixed integer problem since the existence of the constraint depends on the solutions of the problem. Here, a method similar to branch and bound is employed. Given the rivals response, a series of ranges that divide a player's response into congestion and noncongestion situations are found. Thus, the problem divides into a series of relaxations. A simple example illustrates the approach.

Consider the system from [13], there are two supplies and one demand whose parameters are shown in the Fig. 1. The power flow on the only line will be $P_{12} = q_1$. With a limit of power flow on this line of $P_{ij}^{\max}$ (MW), then the conditional constraint can be written as

$$\Delta p_1 = -\Delta p_2 = \min\left(0, P_{12}^{\max} - p_1\right). \quad (19)$$

For player 1, simple substitution (11) in (12) yields

$$p_1 = \frac{\text{MCP} - k_1 a_{1c}}{k_1 b_{1c}} = \frac{\frac{d_1}{d_0} + \frac{a_{1c}}{b_{1c}} + \frac{a_{2c}}{b_{2c}}}{1 + k_1 b_{1c}\left(\frac{1}{k_2 b_{2c}} + \frac{1}{d_0}\right)} - \frac{a_{1c}}{b_{1c}}. \quad (20)$$

If $p_1 > P_{12}^{\max}$, then there is a congestion problem; otherwise, there is no congestion problem. Solving $p_1 = P_{12}^{\max}$, yields

$$k_1^{\max}(k_2) = \frac{\left(\frac{\frac{d_1}{d_0} + \frac{a_{1c}}{b_{1c}} + \frac{a_{2c}}{b_{2c}}}{P_{12}^{\max} + \frac{a_{1c}}{b_{1c}}} - 1\right)}{b_{1c}\left(\frac{1}{k_2 b_{2c}} + \frac{1}{d_0}\right)}. \quad (21)$$

This function $k_1^{\max}(k_2)$ divides the problem into the congested and noncongested strategies. That is, if $k_1 \leq k_1^{\max}(k_2)$, then there is congestion. Thus, the bidding problem is now the following two optimization problems:

$$
\begin{aligned}
\max_{k_1} \quad & \mathrm{MCP} \cdot p_1 - C_1(p_1) \\
\text{s.t.} \quad & \mathrm{MCP} = \frac{\sum\limits_{i \in L} \frac{d_{1i}}{d_{0i}} + \sum\limits_{i \in G} \frac{a_{ic}}{b_{ic}}}{\sum\limits_{i \in G} \frac{1}{k_i b_{ic}} + \sum\limits_{i \in L} \frac{1}{d_{0i}}} \\
& p_1 = \frac{\mathrm{MCP} - k_1 a_{1c}}{k_1 b_{1c}} \\
& k_1 \geq k_1^{\max}(k_2) \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
\max_{k_1} \quad & \mathrm{MCP} \cdot (p_1 + \Delta p_1) - C_1(p_1 + \Delta p_1) \\
& + k_1 b_{1c} \Delta p_1^2 \\
\text{s.t.} \quad & \mathrm{MCP} = \frac{\sum\limits_{i \in L} \frac{d_{1i}}{d_{0i}} + \sum\limits_{i \in G} \frac{a_{ic}}{b_{ic}}}{\sum\limits_{i \in G} \frac{1}{k_i b_{ic}} + \sum\limits_{i \in L} \frac{1}{d_{0i}}} \\
& p_1 = \frac{\mathrm{MCP} - k_1 a_{1c}}{k_1 b_{1c}} \\
& 0 \leq k_1 \leq k_1^{\max}(k_2). \qquad (23)
\end{aligned}
$$

Given player 2's bidding parameters, both (22) and (23) can be solved. The more profitable solution of these two solutions is player 1's best response. Repeating for all of player 2's possible strategies will determine player 1's optimal responses. If this is duplicated for determine player 2's optimal strategies, then the market equilibrium point can be found by comparing solutions. While this procedure appears to be viable, even for larger systems with many players, complex relationships in $k_i^{\max}(k_j)$ may arise that render finding market equilibrium points extremely difficult.

IV. Numerical Results

To analyze congestion's influence on the bidding strategy and price, we first look at the situation when no congestion management is included. Subsequently, transmission system limits are included in the calculation. Comparing these two results highlights the influence of congestion on the optimal bidding strategy.

A. Example 1

Consider the system as shown in Fig. 1 but neglecting transmission line capacity. Figs. 2 and 3 plot the optimal values for k_1 vs. k_2, with and without transmission constraints, respectively. The maximum for k_1 and k_2 is assumed to be 3. A maximum value acts similarly to a price cap and is needed since the demand is relatively inelastic leading to unbounded mark up without the constraint. With no transmission constraints, the pure Nash equilibrium is for both players choose to bid at 1.15 times marginal cost. When an 80-MVA transmission line capacity is included, the optimal strategies change radically.

As the constraint comes into force, this translates into sudden changes in strategy (i.e., a large variation in both k_1 and k_2). A pure Nash equilibrium does not exist. For player 1, values of k_1 in $[1.36, 1.69]$ result in identical profit when player 2 chooses to play at $k_2 = 1.78$. Similarly, for player 2, values of k_2 in

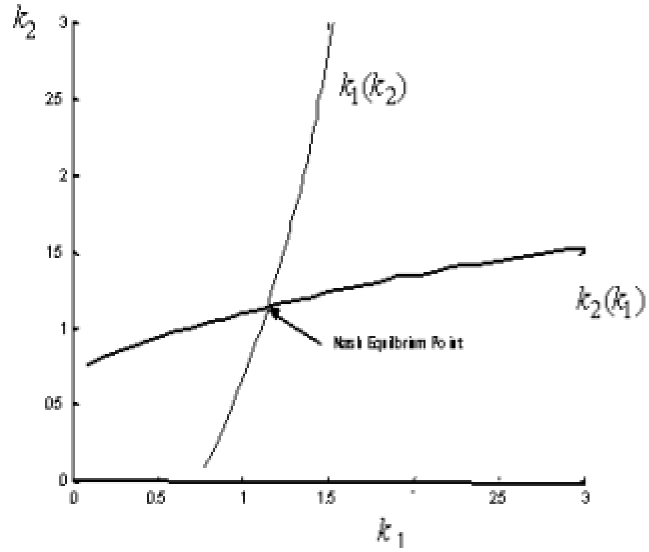

Fig. 2. Example 1—optimal strategies without transmission limit.

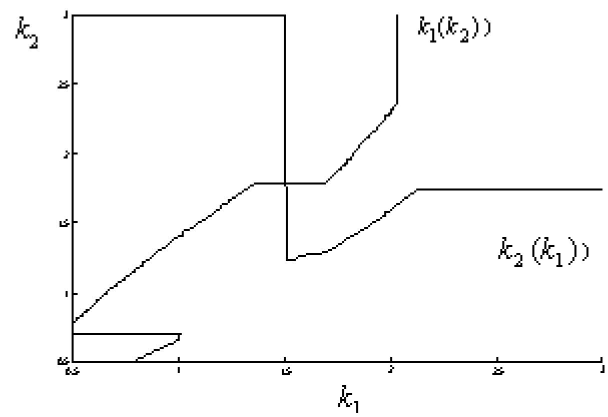

Fig. 3. Example 1—optimal strategies with transmission limit.

$[1.255, 3.0]$ result in identical profit when player 1 chooses to play at $k_1 = 1.553$. This is similar to the result in [1]. Thus, one should consider the possibility of mixed strategy equilibria. The mixed strategy for this problem is: player 1 will choose to play at $k_1 = 1.36$ with probability 0.53 and $k_1 = 1.69$ with probability 0.47, and player 2 will choose to play at $k_2 = 3.0$ with probability of 0.80 and $k_2 = 1.25$ with probability of 0.20. An approach to computation of the mixed strategy equilibrium point is given in Appendix B.

The above simple example shows that generator 2 should bid at the maximum feasible price most of the time. This means that player 2 is willing to forego any sale in the first round bid and take profits from the congestion round. Notice in this system, only player 1 faces a congestion problem (i.e., since $P_{12} = q_1$), the maximum output of generator 1 can only be $P_{12}^{\max}$. There is no transmission limit for generator 2. Thus, no matter how high generator 2 bids; it will finally win some bid when the curtailment is taken into account. When generator 2 expects congestion, the higher bid will tend to increase MCP. The system's potential congestion guarantees player 2 wins $D - P_{12}^{\max} MW$. This "biased" congestion situation (i.e., the congestion imparts

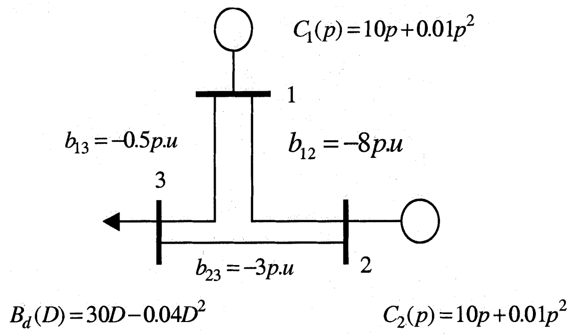

Fig. 4. Example system 2.

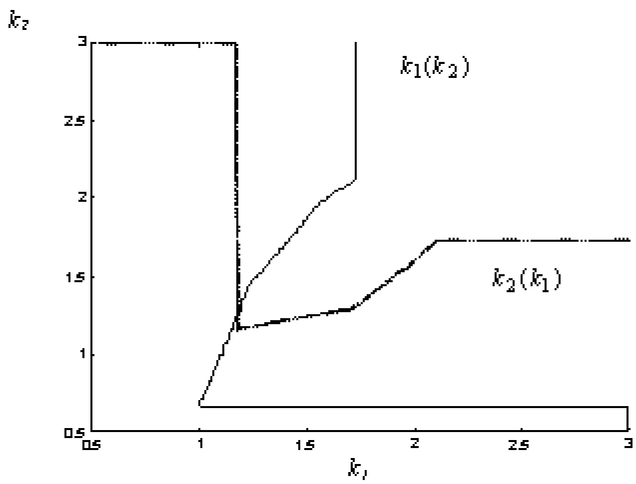

Fig. 5. Example 2—optimal strategies with transmission limit.

more constraints on certain players) gives player 2 significant market power.

B. Example 2

In this example, both of the generators face transmission limits. Let the parameters of generators and loads remain the same, but the network is now the system [20] shown in Fig. 4. When the transmission system limits are not included, the system will have same Nash Equilibrium at $k_1 = k_2 = 1.15$ as the former system. The line flows will be

$$
\begin{aligned}
P_{12} =& 0.6q_1 - 0.1q_2 \\
P_{23} =& 0.6q_1 + 0.9q_2 \\
P_{13} =& 0.4q_1 + 0.1q_2
\end{aligned}
$$

with limits of

$$
\begin{aligned}
P_{12}^{\max} =& 78 \\
P_{13}^{\max} =& 225 \\
P_{23}^{\max} =& 300.
\end{aligned}
$$

The result of the response of a player versus its rival is shown in Fig. 5. There is a jump in k_2 from 3 to 1.16 when player 1 plays at 1.18. Again, there is no pure Nash Equilibrium, so a mixed strategy equilibrium point is sought. Since k_1 is continuous, $k_1 = 1.18$ with probability 1.0. The best response of player 2 is to choose to play at 3 with probability of only 0.09, and at 1.16 with probability of 0.91.

TABLE I
GENERATOR COST FUNCTIONS

Market Participant	Bus	Cost Coefficients $a(i)$	$b(i)$	$c(i)$	Max [MW]	Min
A	1	0	2	0.02	0	80
	2	0	1.75	0.0175	0	80
	22	0	1	0.0625	0	50
B	13	0	3	0.025	0	30
	23	0	3	0.025	0	40
	27	0	3.25	0.00834	0	55

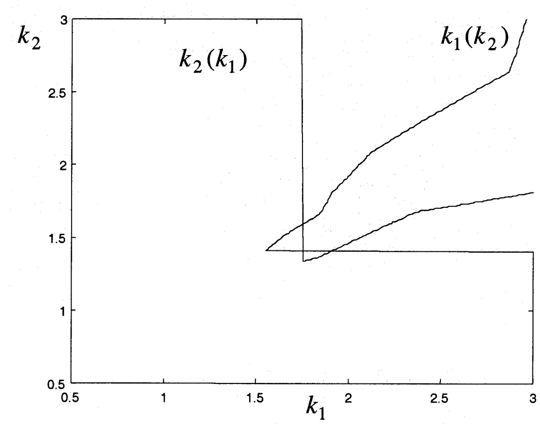

Fig. 6. Example 3—IEEE-30-bus system with line limits.

Relative to the first example, both generators tend to decrease their bidding price as they are both at risk of losing a sale due to congestion problem. Both generators will increase their bidding price if there is any possibility of congestion. Notice also that the more serious the congestion, the higher the bidding price. When the transmission system is "fair" to each market participant (i.e., there is no obvious congestion problem for some participants), the market participants will have more incentive to bid at their marginal cost.

C. Example 3

A modified IEEE-30 bus system from [9] is applied in this example with two dominant market participants. System data and line limits can be found in [21]. Table I lists the respective cost functions.

For simplicity, the benefit function for all demands are assumed to be identical. Specifically

$$B_i(D_i) = 18.136D_i - 0.02D_i^2.$$

When the congestion is not included in the bidding process, there is a pure Nash equilibrium point at (1.27, 1.19). Considering congestion, the result is as shown in Fig. 6. Similar to the previous examples, the pure Nash Equilibrium point disappears with the introduction of the congestion's influence. The mixed strategy equilibrium point is that player A chooses to play at $k_1 = 1.41$ with probability 1.0; player B chooses to player at $k_2 = 3.0$ with probability equal to 0.23, and at $k_2 = 1.13$ with probability 0.73.

Comparing these results with the simpler cases, player A has more incentive to play high under the influence of congestion while player B tends to remain near the pure Nash equilibrium point. An examination of the congestion at the Nash equilibrium points shows that the transmission line (2–6) is overloaded. The generator at bus 2 belongs to player A while bus 6 is an intermediate bus, which connects with several load buses. Thus in this situation, player A has more possibilities to force congestion and incentive to increase mark up.

Understandably as the system becomes more complex, finding the precise influence of congestion on the bidding strategy becomes becomes more difficult. The possibility of more than one mixed strategy equilibrium point arises and other influences arise which make it is more difficult to apply the results. Thus, Section V introduces a new bidding strategy using statistics but following the basic form as the previous.

V. New Bidding Strategy

The above examples show analytically how congestion influences the bidding strategy problem, and at least for these scenarios, shows pure Nash equilibrium points are less likely. Unfortunately, even for these idealized problems, the optimal strategies are difficult to find. For a larger system with many participants and where precise information about transmission limits is more difficult to determine, it may not be feasible to construct a practical formulation. The authors' analysis of actual bidding behavior in the California market will be used to modify the approach in Section IV. Specifically, the optimal strategy problem is simplified to reflect the information that would be most readily available for all participants. A few observations help clarify the approach.

- Due to the complexity and limited knowledge of the transmission limits by most participants, congestion is modeled as the probability of congestion. This probability is based on the percentage of time that congestion exists during an operating day. The participants are assumed to be aware of this general risk of congestion, and in fact, this can be determined from historical data.
- The generators have different relative locations to the congestion zones. So a given congested path will tend to influence some generators more than others and that may be reflected by either higher or lower bids.

The analysis here looks at a base line when the possibility of congestion is low and compares this to congested time periods. The average bidding price is adopted as the index of bidding strategy and then the correlation coefficient between this index and congestion percent based on the day-ahead market are calculated. This coefficient can then be used as the indicator of adjustments due to the congestion. The details of this analysis will be presented in future reports. Here, we assume that a participant seeking to take advantage of congestion will modify k based on a linear function of the probability of congestion. For the examples here this is given as

$$\Delta k = 0.275 P(\text{congestion}). \tag{24}$$

The following strategy is then employed. The bid will decrease k for all those bids less than the optimal output P^* and increase k for all those bids greater than P^*. By doing do, the bidding output (including MCP and P^*) without considering

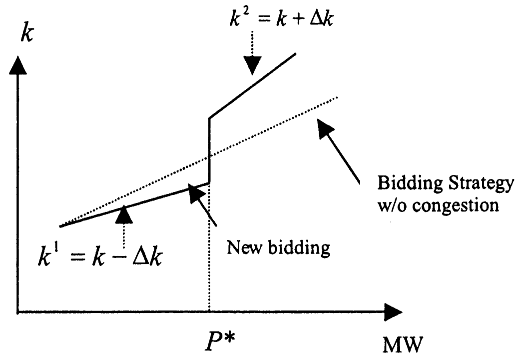

Fig. 7. New bidding strategy.

congestion's influence will remain unchanged (i.e., the optimal strategy is chosen). When there is congestion, compensation will increase due to the difference between *MCP* and the bid price increase, and hence, there will be greater profit. Also, since the higher the congestion possibility, the larger Δk, greater profits are realized at times of high congestion. Fig. 7 shows the new bidding strategy. Notice the result has a similar characteristic to the earlier example.

From the earlier examples, the original optimal point is seen to be $k_1^* = k_2^* = 1.15$ and the corresponding outputs are $P_1^* = P_2^* = 101.08$ MW. When the 80-MVA line power limit is introduced and assuming a simple uniform distribution, the probability of congestion in the system is

$$P(\text{congestion}) = \frac{P_1^* - P_{12}^{\max}}{P_{12}^{\max}} = \frac{101.0781 - 80}{80} = 26.53\%.$$

Thus

$$\begin{aligned}\Delta k &= 7.3\% \\ k^1 &= k_1^* - \Delta k = 1.08 \\ k^2 &= k_1^* + \Delta k = 1.22.\end{aligned}$$

This new bidding strategy is compared with the theoretical mixed Nash equilibrium and shown in Table I. The results show that in the "biased" congestion case, when player 2 chooses to bid at 3.0, the profits of both players will be significantly higher than in the proposed probabilistic approach. This case also requires a significant amount of load curtailment so the result is not surprising. The statistical approach shows similar results to that obtained in the "fair" congestion case (Table II).

VI. Conclusion

Congestion in the transmission system may allow some participants to enjoy effective market power, resulting in higher prices. This work analyzes this mechanism in the framework of game theory. We show that the deviation from idealized price-taker behavior is more serious when some market participants suffer disproportionately from the congestion problem. Based on this theoretical analysis, a probabilistic bidding methodology is proposed that shows similar profits to the game theoretic approach. Due to the complexity of the calculations in the theoretical approach, the statistical analysis methodology has clear advantages. We also believe these strategies reflect actual behavior in existing markets. Our ongoing research is focusing on how bids change given the likelihood of congestion.

TABLE II
STRATEGIES FOR ALL FOUR APPROACHES NOTE: WE LIST BOTH POSSIBILITIES FOR PLAYER2 SINCE THEY HAVE VERY SIMILAR PROBABILITY IN CASE 2; WHILE IN CASE 4, THE BIDDING STRATEGY IS A DISCRETE STRATEGY

	Player 1			Player 2		
	k*	P*(MW)	Profit	k*	P*(MW)	Profit
No congestion	1.15	80	252.42	1.15	122.17	278
"Biased" Congestion (mixed-strategy, two possibilities)	1.36	67.3	320	1.255	114.74	491.39
	1.36	80	761.71	3.0	72.99	833.68
"Fair" Congestion	1.18	94.19	290.18	1.16	105.53	313.16
Statistical (discrete strategy)	(1.08, 1.22)	80	271.5	(1.08, 1.22)	122.17	350.5

APPENDIX A

Equation (17) can be found as follows. Assuming equal weightings rewrite (8) as

$$\begin{aligned} \min_{\Delta p_i} \quad & \Delta P^T \cdot \Delta P \\ \text{s.t.} \quad & \sum_{i \in G} \Delta p_i = 0 \\ & |P_{ij}| \le P_{ij}^{\max}. \end{aligned} \tag{A.1}$$

With a dc power flow, the GSF is constant. So the power flow on each line can be given by

$$P_{jk} = \sum_{\forall\, i} \beta_{jk,i} p_i \tag{A.2}$$

where $\rho_{jk,i} = \Delta P_{jk} / \Delta p_i$ is the GSF. Then, the new flow is

$$P_{jk}^{\text{new}} = \sum_{\forall\, i} \beta_{jk,i}(p_i + \Delta p_i) = P_{jk}^{\text{old}} + \sum_{\forall\, i} \beta_{jk,i} \Delta p_i \tag{A.3}$$

where $\Delta p_i = 0 \quad \forall\, i \notin G$. Now rewrite the flow constraint as

$$\left| P_{jk}^{\text{old}} + \sum_{\forall\, i} \beta_{jk,i} \Delta p_i \right| \le P_{jk}^{\max}. \tag{A.4}$$

Expanding the absolute value gives

$$-P_{jk}^{\max} - P_{jk}^{\text{old}} \le \sum_{\forall\, i} \beta_{jk,i} \Delta p_i \le P_{jk}^{\max} - P_{jk}^{\text{old}}. \tag{A.5}$$

Now applying the Kuhn-Tucker conditions to (8), the inner solution will be

$$2\Delta p_i + \lambda + \sum_{\forall \text{ lines}} \mu_{jk}^{+} \beta_{jk,i} - \sum_{\forall \text{ lines}} \mu_{jk}^{-} \beta_{jk,i} = 0 \tag{A.6}$$

$$\mu_{jk}^{+} \left(\sum_{i \in G} \beta_{jk,i} \Delta p_i - P_{jk}^{\max} + P_{jk}^{\text{old}} \right) = 0, \quad \forall jk \in \text{lines} \tag{A.7}$$

$$-\mu_{jk}^{-} \left(\sum_{i \in G} \beta_{jk,i} \Delta p_i + P_{jk}^{\max} + P_{jk}^{\text{old}} \right) = 0, \quad \forall\, jk \in \text{lines}. \tag{A.8}$$

Note for line flows within limits, the μ must equal to zero. The above can then be solved to find the Δp_i. If the assumption of a single line overflow is made with least curtailment, one obtains (17) through simple algebraic manipulation. The quadratic terms in (17) arise from substituting the linear solution of (A.6)–(A.8).

TABLE III
PAYOFF MATRIX

	L	R
U	(-2,2)	(4,0)
D	(2,1)	(2,4)

APPENDIX B

Computing mixed strategy Nash Equilibriums can be a challenging task; however, there is a trick that can often greatly simplify this task. Note that in a mixed strategy Nash Equilibrium, the expected payoffs for any player will remain the same if he or she switches to any pure strategy that has positive probability of being picked by the equilibrium mixed strategy. Consider a very simple example with the payoff matrix of Table III. There is no pure Nash Equilibrium. To calculate the mixed strategy equilibrium, player 2's probability of play L and R are y and $1 - y$. Then player 1's expected payoff if he chooses either U or D must be equal. Let σ_2 represents player 2's best response, so

$$E_1(U, \sigma_2) = -2y + 4(1 - y)$$

and

$$E_1(D, \sigma_2) = 2y + 2(1 - y).$$

Solving $E_1(U, \sigma_2) = E_1(D, \sigma_2)$, yields $y = 1/3$. Thus, the Nash Equilibrium mixed strategy for player 2 is given by $\sigma_2^* = (1/3, 2/3)$. Similarly, player 1's is found to be $\sigma_1^* = (3/5, 2/5)$. It is easy to apply this process to the problem in this paper.

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