

# OPTIMAL BIDDING STRATEGIES: AN EMPIRICAL CONJECTURAL APPROACH

Tengshun Peng and Kevin Tomsovic

*School of EECS, Washington State University, Pullman, WA*

**Abstract:** The optimal bidding strategy for a generation company is a complex decision making problem involving numerous uncertainties. Generally, all market participants attempt to refine their strategies to earn greater profit. Success in the market requires not only successful forecasting of demand and other market conditions but also anticipating rival behaviours. The main objective of this paper is to suggest one use of historical data for purposes of strategic bidding. A conjectural model based on the market clearing process is presented. In this model, participants estimate a mark-up function for their competitors in the market. Based on these estimates, an optimal bid is found. Numerical examples highlight the methodology. *Copyright © 2003 IFAC*

**Keywords:** Auctions, bidding strategies, conjectural model, risk.

## 1. INTRODUCTION

The optimal bidding strategy problem is a complex decision making problem involving numerous uncertainties. Generally, all market participants attempt to refine their strategies to earn greater profit. Success in the market requires not only successful forecasting of demand and other market conditions but also anticipating rival behaviours. As a result, many researchers have proposed a game theoretic model to address this problem (Ferrero, *et al.*, 1997; Hobbs and Kelley, 1992; Bai, *et al.*, 1997; Peng and Tomsovic, in press).

There is evidence to suggest that the energy market acts mostly like an oligopoly market (David and Wen, 2000). Game theory is often applied in oligopoly markets and certainly gaming has taken place in real markets. In an oligopoly market, a market participant's behaviour will affect the market clearing price (MCP). The participant's bidding strategy problem is to determine the bids to maximize one's own benefit.

In the typical electric power market, market participants submit bids and the intersection between the aggregate supply and aggregate demand curve is the market equilibrium point that determines the MCP and the winning bids. According to economics theory, there are several ways to consider strategic market interactions, including: pure competition, Cournot strategy, Stackelberg model, conjectural model, and so on. Particularly appropriate for the bidding problem here is the Cournot model (Day, *et al.*, 2002).

Due to the complexity of a real market with numerous participants, most of these theoretical models are too unwieldy to apply to a representative system model. Instead, numerous simplifications are needed to make application possible. For example, frequently there are no pure Nash equilibrium points and instead a mixed strategy is introduced (Mas-Colell, *et al.* 1995). Under mixed strategies, the optimal solution should be a probability density function. Unfortunately, this is difficult to apply as guidance to bidding activity. A deterministic decision is needed despite the underlying risks and various probabilistic outcomes.

A conjecture model can be used to consider market participants interactions. The conjecture model acts as a generalized Cournot model in that each market participant attempts to guess a rival's activity corresponding to the price change (Song, *et al.* 2003). Since many power markets have now been in operation for some time (albeit with frequent changes in the market rules), there exists significant historical data that can be analyzed to help participants understand likely competitor behaviour. At a minimum, market participants forecast energy prices based on historical prices. Beyond this, individual market bids are made publicly available in many markets, e.g., in California, all public bids are posted online by the ISO 6 months after the day of submission. Such information provides valuable information about a rival's likely behaviours. Thus, it is feasible to apply a conjecture model to address bidding strategies.

The main objective of this paper is to consider the best use of specific historical data. To begin, a model of the optimal bidding strategy problem is developed. Then, a statistical application method is introduced based on different price forecast technique.

## 2. PROBLEM DEFINITION

In economics, there are two types of conjecture models: the general conjectural variations (GCV) and the conjectured supply function (CSF). While both models require a “guess” of rival activities beforehand, the difference between these two conjectural models lies in the focus on a rival’s price or the rival’s supply function. The CSF model is adopted in this paper as the more appropriate for assessing bidding strategies in an electricity market.

### 2.1 CSF Model

Assume there are  $n$  market participants, let  $k_i$  denote market participant  $i$ ’s decision. Both the market clearing price and the quantity are a function of all market participants’ decision variables denoted here by  $MCP(k_1, k_2, \dots, k_n)$  and  $p_1(k_1, k_2, \dots, k_n)$ , respectively. Assume without loss of generality the bidding decision for individual market participant 1 is desired. The conjecture variation is defined as the belief of the  $i^{th}$  market participant’s response to its rivals. Here, we are interested in the bidding strategy  $k_i$ , and the conjecture variation can be represented by  $k_i(k_1)$ . This means that market participant 1 will guess all other decision variables  $k_i$  as the function of one’s own activity, that is,

$$k_i(k_1) = f_i(k_1), \quad \forall i = 2, \dots, m \quad (1)$$

Since it is not possible to know a rival’s actions precisely, it is necessary to represent the deviation of the  $i^{th}$  firm’s behaviour from this forecast. Here, the error is represented as:

$$k_i(k_1) = f_i(k_1)(1 + e_i), \quad \forall i = 2, \dots, m \quad (2)$$

where  $f_i(k_1)$  are the forecast functions, which are static functions given the current market situation, and  $e_i$  are random variables representing the error in the forecast process. Since this function arises from historical data analysis, it represents the belief that these market participants will continue to bid in the same manner. Absent other information, the  $k_i(k_1)$  are assumed to normally distributed since this is the most tractable mathematically. Then

$$E(k_i(k_1)) = f_i(k_1), \quad \forall i = 2, \dots, m \quad (3)$$

$$Var(k_i(k_1)) = f_i(k_1)^2 var(e_i), \quad \forall i = 2, \dots, m \quad (4)$$

It follows that  $e_i \sim N(0, \sigma_i^2)$ , since given a market situation  $f_i(k_1)$  is deterministic. The decision

problem is to maximize profit by choosing the decision variables  $k_1$  such that:

$$\begin{aligned} \max_{k_1} \quad & \pi = MCP_0 \cdot p_1 - C_1(p_1) \\ \text{s.t.} \quad & k_2 = k_2(k_1) \\ & \vdots \\ & k_n = k_n(k_1) \\ & MCP_0 = MCP(k_1, k_2, \dots, k_n), \\ & p_1 = p_1(k_1, k_2, \dots, k_n) \end{aligned} \quad (5)$$

where  $C_1(p_1)$  is the generation cost function,  $p_1$  represents the MW output awarded in the auction and  $MCP_0$  is the market clear price in \$/MWhr for the time period of interest.

### 2.2 Decision Variable $k_i$

Let the generation cost be represented by a quadratic function as

$$C_i(p_i) = \frac{1}{2} c_i p_i^2 + b_i p_i + a_i \quad (6)$$

Since the constant term  $a_i$  will not affect the maximization result (assuming a unit will be committed), let  $a_i = 0$  with the marginal generation cost function of the form:

$$MC = c_i p_i + b_i \quad (7)$$

In an electrical power market, the market participants submit bidding curves that provide the energy price for different levels of generation output. The market operator will determine the winning bids and the MCP according to the particular set of market rules. The decision problem of interest here is the optimal bidding curve that the market participant submits to the market. Normally, a staircase bidding curve is adopted, and for simplicity here, the linearization process as depicted in figure 1 is applied. This bidding curve is represented by the following linear function:

$$IC(p_i) = \alpha_i p_i + \beta_i \quad (8)$$

Variables  $\alpha_i$  and  $\beta_i$  represent the bidding coefficients participant  $i$  submits, which form the decision vector  $k_i = [\alpha_i \quad \beta_i]$  for problem (5). We make a further assumption that both  $\alpha_i$  and  $\beta_i$  are directly related to the  $MC$  by a constant level of mark-up. Thus, (5) simplifies to determination of a scalar,  $k_i$ . That is let:

$$k_i = \alpha_i / c_i = \beta_i / b_i \quad (9)$$

Then it is easy to see that:

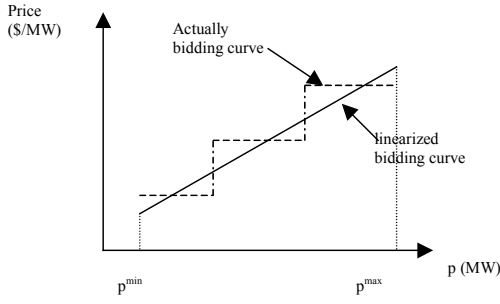


Fig. 1 Linearized Bidding Curve

$$IC(p_i) = \alpha_i p_i + \beta_i = k_i(c_i p_i + b_i) \quad (10)$$

### 2.3 Conjecture Process

While each market participant may have more than one bidding strategy, in the development here we consider only one bidding strategy, and then subsequently address multiple bidding strategies in the numerical examples. Under a single strategy assumption, there are  $m-1$  forecast functions needed by each decision maker to reach a decision with  $m$  market participants.

With the linearized bidding curve and market clearing process assumption, it follows that

$$p_1 = \frac{p(k_1, k_2(k_1), \dots, k_m(k_1))}{k_1 c_1} - \frac{b_1}{c_1} \quad (11)$$

Since the  $e_i$  are random variables, the  $k_i(k_1)$  and  $p(k_1, k_2(k_1), \dots, k_m(k_1))$  are random variables as well. Thus, the profit maximization problem is a maximum expected profit problem.

### 2.4 Risk

Since the rival behaviour is not deterministic and knowledge of their behaviours is imperfect, there is risk in any bidding strategy. From a decision stand point, it is necessary to consider the consequence of competitors' actions that deviate significantly from the forecast function. In general, each market participant is willing to take a certain amount of risk to earn more in return. A related problem arises in investment where risk is often represented by the standard deviation in the expected profit. Then risk is addressed by the so-called Portfolio Selection Problem (Farrell, 1983), which is adopted widely in analysis of stock investments.

Portfolio management can be best characterized as obtaining the highest long-run return at the lowest risk (Farrell, 1983). There are two common formulations of portfolio selection: one, to minimize variance (risk) subject to achieving a specified level of return; and two, to maximize return subject to achieving a specified level of variance. Here, the former form is applied. Rewrite (5) as follows

$$\begin{aligned} \max_{k_1} E(\pi) &= E(MCP \cdot p_1 - C_1(p_1)) \\ \text{s.t.} \quad k_2 &= k_2(k_1) \\ &\vdots \\ k_m &= k_m(k_1) \\ MCP &= MCP(k_1, k_2, \dots, k_m), \\ p_1 &= p_1(k_1, k_2, k_m) \\ \sigma_\pi^2 &\leq \sigma_*^2 \end{aligned} \quad (12)$$

where  $\sigma_*^2$  represents the maximum acceptable risk determined a priori by the market participant. Those wishing to minimize the risk subject to achieving a specified level of return are essentially solving the dual of the above.

Now since the cost function is quadratic and noting that the third and fourth moments of  $e_i$  are  $E(e_i^3) = 0$   $E(e_i^4) = 3\sigma^4$ , simple algebraic manipulation yields:

$$E(\pi) = \left( \frac{1}{k_1 c_1} - \frac{1}{2k_1^2 c_1} \right) (\sigma_p^2 + \mu_p^2) \mu_p - \dots \quad (13)$$

$$\frac{b_1}{c_1} \mu_p + \frac{b_1^2}{2c_1}$$

and

$$\begin{aligned} \sigma_\pi^2 &= \left( \frac{1}{k_1 c_1} - \frac{1}{2k_1^2 c_1} \right)^2 \sigma_p^2 - \dots \\ &+ 2 \left( \frac{1}{k_1 c_1} - \frac{1}{2k_1^2 c_1} \right) \cdot \frac{b_1}{c_1} (E(p^3) - (\sigma_p^2 + \mu_p^2) \mu_p) + \dots \\ &\left( \frac{b_1}{c_1} \right)^2 \sigma_p^2 \end{aligned} \quad (14)$$

where  $\sigma_{mcp^2}^2 = E(MCP^4) - E(MCP^2)^2$ , is the variance of  $MCP^2$ ,  $\sigma_{mcp}^2$  and  $\mu_{mcp}$  are the variance and mean of  $MCP$ , respectively. The following section introduces a price forecasting approach to determine the necessary inputs for solving this problem. The expected profit and variance can be found by standard optimization routines given the statistics for errors in the prediction process.

## 3. PRICE ESTIMATION MODEL

This section places the preceding development in the context of an electricity market. Assume all other exogenous variables, including the load level, transmission limits, and fuel price, are known. The energy price is determined by the market participants' bidding strategies. Using a linear model forecast, the expected price can be represented by:

$$MCP_{\text{exp}} = \left( \sum_{i=1}^m s_i k_i + s_0 \right) (1 + e_{mcp}) \quad (15)$$

Where  $s_i$  are the sensitivity coefficients between price and bidding strategies,  $s_0$  is a constant term corresponding to the exogenous variables, and  $e_{mcp} \sim N(0, \sigma_{mcp}^2)$  represents the error from the price estimation model. Rearranging

$$MCP_{\text{exp}} = \left( s_1 k_1 + \sum_{i=2}^m s_i f_i(k_1)(1 + e_i) + s_0 \right) (1 + e_{mcp}) \quad (16)$$

with  $e_i$  an independent random variable. Now, separating the deterministic and the random variables, and defining the deterministic term

$$p_0 = s_1 k_1 + \sum_{i=2}^m s_i f_i(k_1) + s_0 \quad (17)$$

and the stochastic term,

$$MCP_{\text{exp}} = \left( p_0 + \sum_{i=2}^m s_i f_i(k_1) e_i \right) (1 + e_{mcp}) \quad (18)$$

We have the following:

$$\mu_{mcp} = E(MCP_{\text{exp}}) = p_0 \quad (19)$$

and

$$\sigma_{mcp}^2 = \left( \sum_{i=2}^{m+1} s_i^2 f_i(k_1)^2 \sigma_i^2 \right) (1 + \sigma_{mcp}^2) \quad (20)$$

## 4. NUMERICAL EXAMPLE

### 4.1 Numerical Example 1

First, consider a system with four market participants. The forecast error variances and coefficients are given by:

$$\sigma_2^2 = \sigma_3^2 = \sigma_4^2 = 0.25, \quad \sigma_{mcp}^2 = 100$$

Assume that using statistical data analysis for a given market operation situation, participant 1 forecasts that the rivals have split strategies as given by the following bidding functions:

$$k_2(k_1) = \begin{cases} (1.0 + 0.1k_1)(1 + e_2), & k_1 \leq 2.0 \\ (0.2 + 0.5k_1)(1 + e_2), & k_1 > 2.0 \end{cases}$$

$$k_3(k_1) = \begin{cases} (1.1 + 0.2k_1)(1 + e_3), & k_1 \leq -0.5 \\ (1.2 + 0.3k_1)(1 + e_3), & k_1 > -0.5 \end{cases}$$

$$k_4(k_1) = \begin{cases} (2.0 - 0.1k_1)(1 + e_4), & k_1 \leq 1.8 \\ (2.54 - 0.4k_1)(1 + e_4), & k_1 > 1.8 \end{cases}$$

Further assume the expected price function can be represented by the following linear function:

$$MCP_{\text{exp}} = (7.973k_1 + 4.46k_2 + 82.68k_3 + \dots + 11.6k_4 + 4.014)(1 + e_{mcp})$$

### 4.2 Discussion

If there is no risk constraint, the higher the market participant 1 bids, the higher the expected profit. Thus, the optimal solution is  $k_1^* = 3.0$  (where we assume

that 3.0 is the bidding cap.). Notice the risk will increase as the expected profit increases and this occurs particularly rapidly at the higher bidding prices. In the worst case, there could be significant losses. Worse from the generation company point-of-view, the expected profit changes very little beyond say  $k_1=1.2$  while the risk increases rapidly. It is advantageous to sacrifice some marginal profit to reduce the risk. The optimal decision depends on the risk level a firm would like to take. For example, if the risk tolerance is about 80% of the maximum risk level (shown as the black dashed line in Fig. 2), then the optimal solution is  $k_1^* = 1.2$ .

Intuitively, another method to reduce risk is to employ a split bidding strategy, i.e., bid different type of generators at different strategies. For example, one may choose to bid low cost generators at a fairly low price (say, marginal cost) to ensure some profit while bidding some at a higher cost (so-called economic withholding). By doing so, risk should decrease. For simplification, here assume that market participant 1 employs such a split strategy labelled for convenience as the main and minor bidding strategies. Figure 3 shows the results of such an approach. Since the split strategy is used, the figure is no longer a two-dimensional figure. To simplify the figure, only the main strategy is shown along the x-axis. The corresponding expected profit is the optimal value given this major bidding strategy that market participant 1 found by choosing different levels of the minor bidding strategy.

In this case, if there are no risk constraints, the optimal bidding strategy is  $k_{11}=1.2$  (major) and  $k_{12}=3.0$  (minor). Again, if the maximum risk level is added as a constraint, there are two new optimal bidding strategies. Shown as  $k_{11}^* = 0.75$  and  $k_{11}^* = 2.8$  with in both case  $k_{12}=3.0$ . By withholding some generation economically, expected profit does not increase as rapidly but risk has been reduced.

### 4.3 Numerical Example 2: Simplified two market participant example

In practice, it is difficult to estimate all competitors' strategies accurately and it may not be particularly profitable to do so. That is, it is unnecessary to distinguish the different market participants when

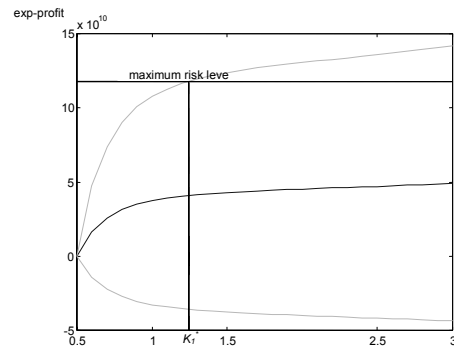


Fig. 2 Example 1 showing expected profit  $\pm\sigma$

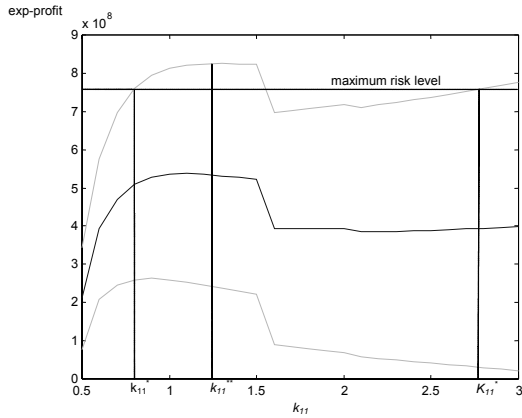


Fig. 3. Example 1 showing expected profit with a split strategy.

they are all acting as rivals. Further in our framework, several high order moments are needed to find the optimal mark-up and these will not be readily available. Accordingly, consider a single opponent with this forecast function:

$$k_2(k_1) = \begin{cases} 1.0 + 0.1k_1 + e_k, & k_1 \leq 1.5 \\ 0.55 + 0.4k_1 + e_k, & k_1 > 1.5 \end{cases} \quad (24)$$

All variances are as before. The profit results are shown in Fig. 4. Again, when there is no risk constraint, the optimal solution is  $k_1^* = 3.0$ . When 80% risk tolerance is applied, the optimal solution is  $k_1^* = 2.4$ . For this simple case, there is a significant difference between these two models simply due to the selection of the expected rival's bidding strategy and other expected variables. In practice, it should be possible to obtain similar results with a two participant example.

## 5. CONCLUSION

This paper presents an approach for determining an optimal bid into the market as a mark-up over actual costs. It requires as input an estimate of competitors' bids in terms of price and possible variation. The solution then maximizes profit while maintaining a tolerable financial risk. Our related work has shown the possibility of forecasting competitor behaviour suggesting the approach is practical. Currently, we are applying these techniques to data from the California market.

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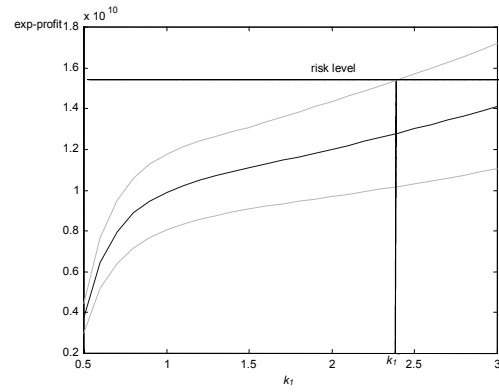


Fig. 4. Example 1 showing expected profit with a single competitor.

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