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Optimal distribution protection design: quality of solution and computational analysis

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Abstract

The fundamental goal of an electric utility is to serve its customers with a reliable and low cost power supply. To achieve this goal, utilities use various reliability indices to evaluate the service reliability, and prioritize capital and maintenance expenditures. Utilities improve distribution reliability either through preventive measures or by appropriate remedial actions in response to a disturbance. Remedial action capabilities include those provided by protective device response, fault locators and other sensors, and various distribution automation functions. The number, type and location of the protective devices on a distribution feeder have a direct effect on the system reliability. In previous work, a binary programming problem was formulated to identify type and location of the protective devices on a distribution feeder in order to optimize reliable performance. The purpose of this study is to evaluate the practicality of such design optimization techniques through: (a) comparison of these solutions to existing utility practices; and (b) analysis of the computational complexity of the algorithms.

Keywords: Distribution reliability; Optimal protection design; Binary programming

1. Introduction

The fundamental goal of an electric utility is to serve its customer with a reliable and low cost power supply. To achieve this goal, utilities use various reliability indices to evaluate its service reliability, and prioritize capital and maintenance expenditures. A recent survey indicates that the most common indices used in the industry are customer based indices. These customer based indices are system average interruption frequency index (SAIFI), system average interruption duration index (SAIDI), customer average interruption duration index (CAIDI) and average system availability index (ASAI) [1]. Also, some utilities use load based indices such as, average system interruption frequency index (ASIFI) and average system interruption duration index (ASIDI). These are detailed in Appendix 1.

Utilities improve distribution reliability either through preventive measures or by appropriate remedial actions in response to a disturbance. Preventive measures include tree trimming on a regular basis, construction design modification, installation of lightning arresters and use of animal guards. Remedial action capabilities include those provided by protective device response, fault locators and other sensors, and various distribution automation functions.

Protection devices prevent or minimize damage to equipment, allow greater public safety measures and improve service reliability. The number, type and location of the protective devices on a distribution feeder have a direct effect on the system reliability. Protective devices are installed on a distribution feeder based on utility guidelines and engineering judgment. In general, the full effectiveness of protective devices can not be utilized, because the utility guidelines are limited in scope and engineering judgment depends on the engineer's experience. In [2], a binary programming problem is formulated to identify type and location of the protective devices on a distribution feeder in order to minimize the SAIFI index. This technique was extended through a goal programming approach to achieve compromises among various engineering objectives [3]. In this work, the design goals are: (a) to minimize the SAIFI and ASIFI indices by identifying types and locations of protective devices; and (b) to achieve a reasonable trade-off between a decrease in the SAIFI index and an increase in MAIFI index by application of a fuse saying scheme. (A fuse saving scheme protects fuses from momentary faults on their load side through proper response of the source side line recloser.)

In this paper, a typical distribution circuit from a major utility is selected to illustrate the proposed approach. The

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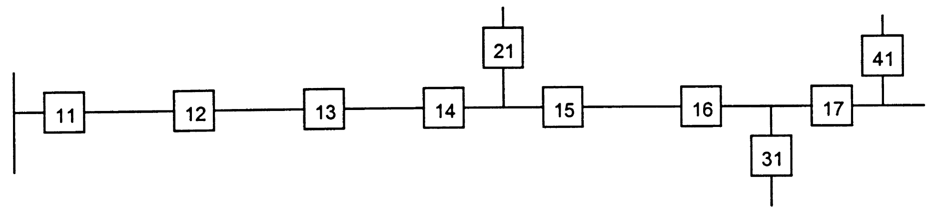

Fig. 1. Main feeder with seven possible locations.

purpose of this study is to evaluate the performance of the techniques and compare the solutions against existing utility practices. The study also analyzes the computational complexity of the algorithms to show the feasibility of the approach and addresses the concern raised by designers and researchers in finding global optima for practical systems.

2. Background

Distribution networks can be classified as radial, spot or secondary. Radial systems are the most common and the focus of this study. Radial networks may consist of both overhead and underground lines using either a three wire or four wire scheme. On a four wire system, a ground neutral wire is run alongside the three phases, whereas on a three wire system, the neutral remains grounded at the substation. Laterals branching from the main three phase feeder can be single phase, two phase or three phase. In addition, most utilities design a system so two radial networks can pick up neighboring loads by tie switches during scheduled or unscheduled outages.

In general, protection of a distribution system consists of a circuit breaker at the substation with line reclosers, sectionalizers, interrupters and fuses along the main feeders and on the laterals. These protective devices must satisfy the following requirements:

- prevent safety hazards to the public by removing a faulted circuit from the network;
- prevent or minimize damage to equipment by clearing an abnormal condition;
- improve service reliability by removing a small section of the circuit for a given fault.

The type and location of distribution protective devices has been largely left to engineering judgment with limited guidelines. In general, a circuit breaker with phase and ground relays is installed at the substation. Line reclosers are installed on the main overhead feeder and on heavily loaded or long laterals. Due to cost limitations, only a limited number of these devices are installed on any given feeder. Interrupters are applied similar to line reclosers, except that they are used for underground circuits. Sectionalizers are overhead line devices, which must be used in conjunction with a line recloser or a breaker. If there is a location for which a line recloser should be installed, but coordination cannot be achieved (with load and source side protection devices) then a sectionalizer can be used instead. Fuses are used on both overhead and underground circuits.

In [2], a binary programming optimization is utilized to identify type and location of the protective devices on a distribution feeder. The proposed technique identifies type and location of the specific number of the protective devices on a distribution feeder in order to minimize the SAIFI index. The distribution feeder is assumed to be radial in construction. In that proposed formulation, a distribution feeder is divided into four categories: a main feeder, lateral one, lateral two or lateral three. A lateral one category is short and will not be fused. The effect of this lateral on reliability can be included in the feeder section from which it branches. A lateral two will only be fused and its effect on the SAIFI index is constant. All other laterals are category three. Thus, only the main feeder and category three laterals are explicit in the optimization. In [3], a goal programming technique is used to find an optimal solution for a multiple objectives. This technique is useful where a utility may wish to find an optimal solution, which considers both SAIFI and ASIFI indices. The explicit formulation of the objectives and constraints can be found in Appendix 2.

3. System description

In this section, an existing distribution feeder system is analyzed using the proposed optimization approach to highlight possible improvements in reliability from optimizing protective device type and location. The selected circuit is 40 miles long with 450 sections covering an area of approximately 15 square miles. This circuit serves a mixture of 2559 residential and industrial customers with a total peak load of 12 002 kW. The existing protection scheme of this circuit consists of 62 fuses, four line reclosers and one circuit breaker. This design was based on standard company guidelines and engineering experience.

The actual circuit diagram has been reduced to simplify presentation of the example. This circuit is divided into one main section, three category three laterals and 61 category two laterals. Based on the number of the customers, the connected load and the failure rate, numerous possible locations for installing protective devices are identified for the main feeder and category three laterals. For the main feeder (Fig. 1), seven points are identified as possible locations to

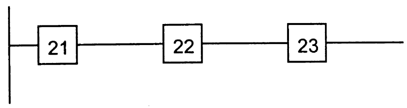

Fig. 2. Category three lateral, $q = 2$, with three possible locations.

install protective devices. Similar sketches are shown for three category three laterals in Figs. 2–4. The number of customers, average connected kW load, permanent failure rates, and temporary failure rates (i.e. failures which last less than a couple of minutes) for each section are shown in Tables 1–4. The contribution of category two laterals to overall customer outage frequency, SAIFI, and to load per unit outage frequency, ASIFI, is constant.

3.1. Case study description

For this circuit, the following problems are to be analyzed:

1. Minimize SAIFI index by identifying type and location of protective devices.
2. Minimize ASIFI index by identifying type and location of protective devices.
3. Find the optimal trade-off between the SAIFI and ASIFI indices by identifying type and location of protective devices.
4. Identify where a fuse saving scheme should be applied based on the optimal trade-off between a decrease in SAIFI index and an increase in the MAIFI index.
5. Minimize cost based on satisfying specific SAIFI and ASIFI objectives.

Since the existing system has four line reclosers, the study will only allow four line reclosers to be installed. Other constraints are:

1. There are an unlimited number of fuses.
2. A fuse can not be installed on the main feeder.
3. A fuse or a three phase device such as line recloser or a sectionalizer must be installed at the tap points.
4. There will be a breaker with its associated relays at position 11.

4. Solution and analysis

The solutions for the various design problems are discussed in the following. Case no. 1 shown in Table 5 is the original design. Table 5 shows SAIFI, ASIFI, recloser/fuse locations, and the remaining locations where no protective device is assigned for each of the five case studies.

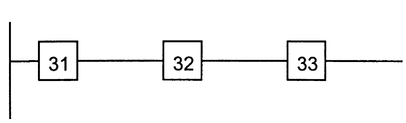

Fig. 3. Category three lateral, $q = 3$, with three possible locations.

These case studies highlight the different aspects of optimizing the design.

4.1. Minimize SAIFI

The objective for this problem was to minimize the SAIFI index subject to the above limitations, which translates to 154 constraints. The optimal solution for this problem is case no. 2. The solution relocates two line reclosers and one fuse from the initial design. The solution for this optimization problem has improved the SAIFI index by 20% or about 0.324 fewer outages per customer per year relative to the original design. The impact of this reduction is directly related to increased customer satisfaction. Minimizing the SAIFI index de-emphasizes the importance of the customers on section 2, because those customers are few in number but large in load. Finally, moving the recloser to the main feeder and closer to the substation allows better protection for those customers served directly from the main feeder.

4.2. Minimize ASIFI

The objective for this problem was to minimize the ASIFI index. The optimal solution is given as case no. 3 from Table 5. By relocating one line recloser and installing two additional fuses, the ASIFI index decreases by 7%. Compared with the existing system configuration, there is a reduction of 0.0895 outages per kW per year. The impact of this reduction depends on the time to repair the faulted line section, the customer costs due to an outage, and the total loss of sales. The repair time depends on individual utility practices and the extent of the trouble and the equipment involved. The customer costs depend on the length of the outage and the nature of customers. In contrast to the previous problem, minimizing the ASIFI index emphasizes the connected load. Notice a recloser on lateral section 2 is installed because of the large load customers. Fuses are placed at locations 31, 32 and 33 because of the smaller loads.

4.3. Optimal tradeoff between the SAIFI and ASIFI indices

The objective of this problem was to find an optimal tradeoff between the SAIFI and ASIFI indices. Weighting of the deviational variables by the Euclidean norm and the fuzzy programming methods are applied. For the Euclidean norm method, the SAIFI and ASIFI minimum goals are required. The values for these goals are obtained from problems no. 1 and no. 2, respectively. There are 156 constraints for this problem. The optimal solution with this method is case no. 4, which relocates one recloser and removes a fuse from the initial system. By removing all fuses from section 3 and relocating a recloser from the main feeder to location 31 from case no. 3, the SAIFI index improves by 2% while increasing the ASIFI index by 4.6%. Relative to case no. 2, replacement of the fuses at location 21 with a recloser from the main feeder reduces

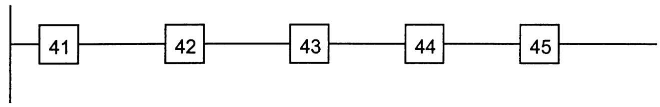

Fig. 4. Category three lateral, $q = 4$, with five possible locations.

the ASIFI index by 33% at the cost of 9.3% increase in the SAIFI index.

The next step was to use the fuzzy programming to find a solution for this problem. In this method, similar to the Euclidean norm method, the minimum SAIFI and ASIFI goals obtained from problems no. 1 and no. 2 were used. In addition, this method requires a maximum, worst case, goal value for each index. The maximum goal value for each index is set to the value calculated from the minimization solution for the other index. The optimal solution is case no. 5 which removes all fuses from section 3 and relocates reclosers from locations 12 and 15 to locations 13 and 31 relative to case no. 3. It is shown that, although the solution improves SAIFI index by 2%, it degrades the ASIFI index by 30.7%. Compared with case no. 2, a fuse is removed from location 21 and two reclosers are relocated from locations 12 and 15 to locations 13 and 21. The solution reduces the ASIFI by 16.6% at the cost of a 9.1% increase in the SAIFI index.

Because of the small deviation from best to worst case in SAIFI for this formulation, the fuzzy programming approach sacrifices improvement in ASIFI for a very small decrement in SAIFI. This is not representative of the true problem objectives. The deviations between the maximum and minimum indices should represent the full range of the problem, but in this case using two competing objectives is inadequate. The result is an inappropriately heavy weight placed on one of the objectives as occurred in this example. In order to represent the problem fully, a third objective of minimal cost could be introduced. The minimal cost solution places a breaker at location 11 and uses no other protective devices on the feeder. While from a practical point of view this is not an acceptable solution, it does provide an unbiased reference for the two objectives. Another approach, followed here, is the introduction of a more practical cost objective (e.g. representative of a management decision), which prefers installation of a small number of protective devices. Now with the three objectives, maximum values are established by performing single objective optimization problems for each index. Using this approach, the fuzzy method results in exactly the same solution as the Euclidean norm. While the fuzzy approach has recently garnered wide interest for its ability to model objectives in natural language, this example indicates care must be taken to apply the method effectively.

4.4. Optimal trade-off between SAIFI and MAIFI

The objective for this problem was to identify where a fuse saving scheme should be applied based on an optimal trade off between a decrease in. the SAIFI index and an increase in the MAIFI index. Case no. 4, which suggests installing a breaker at location 11 and line reclosers at locations 15,21,31 and 41, is used as the base solution for this problem (Fig. 5). There are 18 category two laterals directly connected to section 11. Similarly, there are 11 on section 15, 7 on section 31 and 13 on section 41.

The optimal solution is found to be the application of fuse saving schemes on the line reclosers at 31 and 41. This will increase the MAIFI index by 0.798 and decrease the SAIFI index by 0.0695. If fuse saving schemes are implemented on all of the devices, the maximum increase in the MAIFI index is 5.15 and the maximum decrease in the SAIFI index is 0.166. Both the Euclidean norm and fuzzy programming methods require calculation of the minimum values for MAIFI and SAIFI indices. Fuse saving schemes are applied for all devices when calculating the minimum MAIFI index. For calculation of the SAIFI index, the fuse saving scheme is not applied for any device. For the fuzzy method, the maximum values for these indices are found from these solutions. For this problem, the weighting by the Euclidean norm method and the fuzzy method find the same solution. Notice fuse saving schemes for reclosers at 31 and 41 will not subject a large number of customers to the momentary faults. In contrast, fuse saving on the sections 11 or 15 exposes many customers to momentary interruption for very little improvement in the SAIFI index. Fuse

Table 1
Component data for the main section

Node number	Number of customers	Connected kW load	Permanent failure rate	Temporary failure rate
11	544	1743	0.365	1.460
12	506	1129	0.440	1.758
13	25	650	0.083	0.330
14	157	5883	0.037	0.149
15	138	114	0.275	1.101
16	527	1683	0.123	0.491
17	662	800	0.256	1.024

Table 2
Component data for category three lateral $q = 2$

Node number	Number of customers	Connected kW load	Permanent failure rate	Temporary failure rate
21	0	0	0.195	0.780
22	0	0	0.183	0.732
23	2	5782	0.003	0.013

saving at 21 is inappropriate, as there are no category two laterals on that section.

4.5. Minimal cost to achieve SAIFI and ASIFI objectives

The last design problem is to find the minimum cost to improve SAIFI and ASIFI indices. The acceptable level is chosen as the calculated values for these indices from cases no. 2 and no. 3. There are 154 constraints for this problem. The cost of purchasing and installing a recloser is assumed to be three times greater than a fuse. The solution for this problem is case no. 6. By adding a recloser at location 12, removing a recloser from 23, and replacing a fuse with a recloser at 31, the desired levels of ASIFI and SAIFI indices can be reached.

5. Computational considerations

Computational analysis addresses the efficiency of an algorithm. It is essential to ensure that a proposed algorithm can solve any practically large problem in an acceptable time period. While it is important to analyze any algorithm for its computational efficiency, since the proposed algorithm uses binary programming, the computational issues are obviously a concern. It will be demonstrated that a large problem can be solved in a reasonable time period using this algorithm. The complexity will be analyzed and then the dimensions of a practical problem will be input to the complexity calculations.

Today, most approaches to solving integer programming are based on the branch and bound algorithm. This algorithm is an efficient enumeration procedure for examining feasible integer solutions [4]. The basic procedure of the branch and bound algorithm is three steps: branching; bounding; and fathoming. A branching step divides a problem into sub-problems by specifying two ranges of values for the branching variables. Bounding follows from solving a ‘relaxed’ sub-problem. For example in binary programming, relaxation can be performed by replacing a binary constraint with the requirement that the variable lies in [0,1]. In fathoming, tests are applied to each subproblem. determine which problems merit flirther consideration. This procedure is repeated until the optimal solution is found. The branch and bound algorithm relies primarily on the efficiency of solving linear programming problems, as each branch requires two linear programming solutions. The computational time of linear programming depends on the dimension and sparsity of a constraint matrix, **A** of dimension $m \times n$, where m is the number of constraints and n is the number of variables. While theoretically linear programming may be computationally hard, most practical problems using simplex algorithm require $3m/2$ iterations and seldom more than $3m$ iterations [5]. It has been shown in practical cases that the computational time may increase exponentially if $n \geq 2m$. It will be shown that m is always much greater than n in the proposed formulation.

For a given optimization problem, great latitude exists both in variables and constraint definitions. In practice, some general guidelines may be useful [6]. To begin, the number of integer variables should be small. It may be possible to accept non-integer solutions for larger values, e.g. integer variables whose value exceeds 20 may be considered continuous. Further restriction of the number of sub-problems is obtained by placing as tight bounds as possible on all integer variables. Similarly, increasing the number of constraints may limit the number of feasible solutions and thus, the number of branches. Notice this is in contrast to general linear programming problems. Finally, the ordering of branches greatly determines the efficiency of the search. If possible, important variables can be defined which will be selected with higher priority for branching. While integer programming is NP hard, many practical problems exhibit much better performance. Still, it is difficult to establish estimates of the computational time for general pure integer programming problems [7]. Certain classes of problems, notably node packing, can be solved in $O(l^k)$ where k is a fixed constant and $l = m{\cdot}n$.

In the proposed formulation, the distribution circuit main feeder and laterals are divided into various categories. This specification eliminated cross products of location variables

Table 3
Component data for category three lateral $q = 3$

Node number	Number of customers	Connected kW load	Permanent failure rate	Temporary failure rate
31	101	60	0.151	0.607
32	130	105	0.113	0.454
33	146	113	0.084	0.337

Table 4
Component data for category three lateral $q = 4$

Node number	Number of customers	Connected kW load	Permanent failure rate	Temporary failure rate
41	26	174	0.236	0.943
42	34	54	0.190	0.761
43	26	98	0.215	0.858
44	7	2	0.236	0.944
45	87	306	0.153	0.612

on the main feeder and cafegory three laterals, which results in a significant reduction in the number of variables and constraints. As the number of cross products variable reduces, less number of constraints will be required in order to change these variables to linear variables.

For this work, a commercial mathematical programming package based on the branch and bound method, CPLEX®, is used [8]. A Pentium® 100 MHz processor is utilized to solve the case study problems and problems of various dimension. The problem size, non-zero entries in the constraint matrix and the associated computational times are shown in Tables 6 and 7. Analysis of the data confirm the earlier stated expectation that computational time increases $O(l^k)$ where k is less than 1.0.

5.1. Worst case analysis

The preceding section showed that the developed approach was extremely fast on a practical feeder. That experiment followed the specification of an experienced protection engineer. Since many researchers have doubted the practicality of the branch and bound algorithm for distribution design problems, this section shows that even careless application of the approach would be computationally feasible.

For our algorithm, the number of the variables and constraints follow from the number of possible protective device locations on the main feeder and category three laterals, as well as limitations imposed by coordination, design, application and cost concerns.

Consider Fig. 6 as a main feeder with n_1 possible locations for installing line reclosers. With a circuit breaker installed at the first location, there are $(n_1 - 1)$ variables assigned for these locations, $x_{121}, x_{131},\ldots, x_{1(n1)1}$. The second set of variables is the nine product terms arising from two adjacent reclosers, $x_{121}x_{131}$, $x_{131}x_{141},\ldots$, $x_{1(n1-1)1}x_{1(n1)1}$. The third set of variables is the product of three adjacent reclosers and so on. This continues until the last set of variables, which are the product of all reclosers $x_{121}x_{131},\ldots$, $x_{1(n1-1)1}x_{1(n1)1}$. The total number of variables can be stated as

$$n_{\text{main}} = \sum_{k=1}^{n_1} (n_1 - k) \tag{1}$$

where Eq. (1) can be rewritten as

$$n_{\text{main}} = n_1^2 - \sum_{k=1}^{n_1} k \tag{2}$$

the second term of Eq. (2) is an arithmetic series [9], and it can be easily seen that

$$n_{\text{main}} = \frac{1}{2} n_1(n_1 - 1) \tag{3}$$

which is the total number of the variables from the main feeder. Of these variables, $1/2(n_1 - 1)(n_1 - 2)$ are products of the location variables. To replace these non-linear terms, three constraints are added to each product term. Thus the total number of constraints is

$$m_{\text{main}} = \frac{3(n_1 - 1)(n_1 - 2)}{2} \tag{4}$$

Coordination constraints may arise from various protective devices on the main feeder, between the main feeder devices and protective devices on the tap point of category three laterals, and between protective devices on each category three lateral. The maximum number of coordination constraints is shown in Table 8. For design and application constraints, there are a total of $\sum_{i=1}^{\alpha} n_i$, constraints, which prevent attempts to install two different types of device at the same location and $\sum_{i=1}^{\alpha}[n_i(n_i - 1)]/2$ number of the constraints to disallow a fuse on the source side of a recloser. Finally, there are two constraints for the cost

Table 5
Various case study results

Case	SAIFI	ASIFI	Reclosers	Fuses	No device
1	1.5941	1.3444	15,21,23,41	31	12,13,14,16,17,22,32,33,42,43,44,45
2	1.2703	1.9659	12,15,31,41	21	13,14,16,17,22,23,32,33,42,43,44,45
3	1.4146	1.2549	12,15,21,41	31,32,33	13,14,16,17,22,23,42,43,44,45
4	1.3881	1.3126	15,21,31,41	None	12,13,14,16,17,22,23,32,33,42,43,44,45
5	1.3854	1.6398	13,21,31,41	None	12,14,15,16,17,22,23,32,33,42,43,44,45
6	1.2691	1.2312	12,15,21,31,41	None	13,14,16,17,22,23,32,33,42,43,44,45

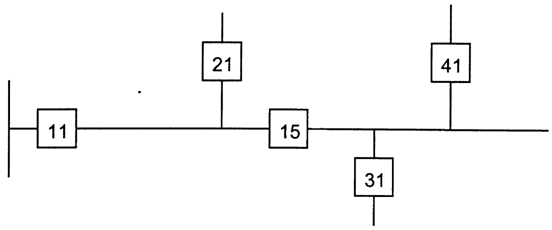

Fig. 5. Automatic protective device locations for case no. 4.

limitations and limiting the number of three-phase and single-phase devices to be installed.

Calculation of the number of variables for a category three is the same as the main feeder with exception that there is no breaker at the first location and for each possible locations two single variables will be assigned: one for a recloser and one for a fuse. Thus for the category three laterals, the number of variables is $\sum_{i=1}^{\alpha} n_i(n_i + 1)$ and the number of the constraints is $\sum_{i=1}^{\alpha} 3n(n_i - 1)$, where n_i is the number of the possible protective device locations on the category three lateral i, and α is the number of the category three laterals. The total number of variables and constraints for the main feeder and category three laterals are summarized in Table 9.

Thus, assuming that the algorithm is linear in terms of $m \cdot n$, the computational algorithm will be $O(\alpha^2 n^4)$, where n is the largest number of locations on the main feeder or a category three lateral. This is an acceptable degree of complexity for this problem.

5.2. Example worst case analysis

To put these dimensions in perspective, consider a hypothetical large feeder 75 miles long. This feeder is broken down into a main feeder 12 miles long, five category three laterals 6 miles long each, and numerous category two laterals. As an extreme example, assume an inexperienced engineer will consider possible locations on the main feeder every 1/2 mile and possible locations on the category three laterals every mile. Under these considerations and the worst case coordination problems, the maximum number of the variables and constraints can be found (Table 10). For coordination, it is assumed that no two devices will coordinate with each other to complicate the solution process, although this is not a practical assumption.

For this example, there are 486 variables and 1954 constraints. Since for our proposed formulation, m is always much greater than n the chance of exponential increase for computational time for solving simplex algorithm is small [5]. Using the worst computational time from Table 6 and assuming $k = 1$ (a worst case) on the running term $O(l^k)$, the solution time will be less than 27 s.

This detailed analysis shows clearly the computational tractability of the protection design problem. The authors suggest

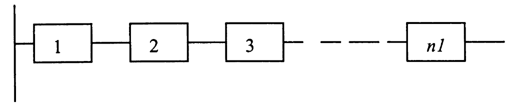

Fig. 6. A mainfeeder with n_1, possible locations.

Table 6
Computational time for various size problems

Case	m	n	Non-zeros	Computational units[a]
1	4	6	12	1.00
2	23	16	83	1.91
3	25	17	87	3.43
4	155	75	507	38.83

[a] Normalized by case no. 1

Table 7
Computational time for case study problems

Case	m	n	Non-zeros	Solution time (s)
1	151	72	373	0.21
2	151	72	373	0.18
3	153	74	503	0.22
4	155	75	507	0.32

Table 8
Number of coordination constraints

Coordination between protective devices	Number of constraints
Relay versus fuse on category three tap points	α
Recloser versus recloser on main feeder	$\frac{(n_1 - 1)(n_1 - 2)}{2}$
Reclosers between main feeder versus category three tap points	$\alpha(n_1\text{-}1)$
Recloser on main feeder versus fuse on category three tap points	$\alpha(n_1\text{-}1)$
Recloser versus recloser on the category three laterals	$\sum_{i=1}^{\alpha} \frac{n_i(n_i - 1)}{2}$
Fuse versus fuse on the category three laterals	$\sum_{i=1}^{\alpha} \frac{n_i(n_i - 1)}{2}$

Table 9
Number of variables and constraints for a given circuit

Sections	No. of variables	No. of constraints
Main section	$\frac{1}{2} n_1(n_1 - 1)$	$\frac{3(n_1 - 1)(n_1 - 2)}{2}$
Category three laterals	$\sum_{i=1}^{\alpha} n_i(n_i + 1)$	$\sum_{i=1}^{\alpha} 3n_i(n_i - 1)$

Table 10
Maximum number of variables and constraints for a fictitious large feeder

	No. of variables	No. of constraints
24 possible locations on the main feeder	276	759
6 possible locations for each category three laterals	210	450
Relay versus fuse on the category three tap points		5
Recloser versus recloser on the main feeder		253
Recloser on the main feeder versus recloser on the category three tap points		115
Recloser on the main feeder versus fuse on the category three tap points		115
Recloser versus recloser on the category three laterals		75
Fuse versus fuse on the category three laterals		75
Location limitation on the category three laterals		30
Fuse before recloser limitation		75
Available 3-phase and 1-phase protective devices		2

that some researchers dismissal of binary programming for a distribution design optimization arises from a failure to carefully formulate the objectives and practical constraints.

6. Conclusions

This paper has shown the significant improvements in distribution system reliability that can be achieved through proper placement and selection of protection devices. Trade-offs among the various reliability indices are found to allow for full consideration of design objectives. Further, this paper has shown that binary programming can be utilized to identify the optimum design efficiently. Both a theoretic and analytic evaluation of the computational efficiency was performed. Theoretic analysis showed that computations were expected to be polynomial in the number of category three laterals and possible locations on these laterals. Experimental studies on example problems verified this result. Finally, a worst case circuit was considered for analysis with an unusually complicated coordination problem and an excessive number of possible protection locations for consideration. Using the developed computational analysis, a projected solution time of 26.45 s on a Pentium 100 Mhz processor was forecast. This is an inconsequential length of time when solving a planning problem of this nature.

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Appendix A

Two distinct indices have been defined for distribution reliability: load based and customer based indices. For the customer based index, the number of customers per outage and duration of outage is calculated. The connected load unit per outage and duration of outage is calculated for a load based index. A few of the more common indices are defined in the following:

A.1. Customer based indices

System average interruption frequency index

$$\mathrm{SAIFI} = \frac{\text{Total Number of Customers Interrupted}}{\text{Total Number of Customers Served}} \tag{A1}$$

System average interruption duration index

$$\mathrm{SAIDI} = \frac{\sum \text{ of Customer Interruption Duration}}{\text{Total Number of Customers Served}} \tag{A2}$$

Customer average interruption duration index

$$\mathrm{CAIDI} = \frac{\sum \text{ of Customer Interruption Duration}}{\text{Total Number of Customers Interrupted}} \tag{A3}$$

Average service availability index

$$\mathrm{ASAI} = \frac{\text{Customer Hours Service Availablility}}{\text{Customer Hours Service Demand}} \tag{A4}$$

Momentary average interruption frequency index

$$\mathrm{MAIFI} = \frac{\text{Total Number of Customer Momentary Interrutions}}{\text{Total Number of Customers Served}} \tag{A5}$$

A.2. Load based indices

Average system interruption duration index

$$\mathrm{ASIDI} = \frac{\text{Connected KVA Minute Interruted}}{\text{Total Connected KVA Served}} \tag{A6}$$

Average system interruption frequency index

$$\mathrm{ASIFI} = \frac{\text{Connected KVA Interrupted}}{\text{Total Connected KM Served}} \tag{A7}$$

Appendix B

This appendix summarizes the equations for calculating the SAIFI and ASIFI indices and the formulation of the optimization problem. More details can be found in [2]. The SAIFI index for a distribution feeder is defined as

$$\mathrm{SAIFI} = \frac{\sum \lambda_i N_i}{N_\mathrm{T}} \tag{B1}$$

where N_i is the number of customers in section i, N_T is the total number of customers on the feeder and λ_i is the net failure rate for section i (the sum of all individual failure rates between the substation and the section). The numerator of Eq. (A1) is written here as

$$\sum \lambda_i N_i = \sum_{q=1}^{\alpha+\beta+1} A_q \tag{B2}$$

where α is the number of category three laterals, β is the number of category two laterals and the first term is the contributions from the main feeder, $q = 1$. A_q for each main feeder, $q = 1$. A_q for each main feeder or lateral q is

$$A_q = \sum_{i=1}^{q_n} (\lambda_{qi} + \gamma_{qi}) \sum_{j=1}^{q_n} N_{qj} - \sum_{i=1}^{q_n} \gamma_{qi} x_{qi2} \sum_{j=1}^{q_n} N_{qj} + \sum_{i=2}^{q_n} \lambda_{qi} \sum_{j=1}^{i=1} N_{qj} \prod_{k=j+1}^{i} x_{qk1} x_{qk2} + \sum_{i=2}^{q_n} \gamma_{qi} \sum_{j=1}^{i-1} (1 - x_{qj2}) \sum_{k=j}^{q_n} N_{qk} \prod_{l=j+1}^{i} x_{ql1} x_{ql2} \tag{B3}$$

where q_n is the number of possible locations on the main feeder or lateral; λ_{qi} is the permanent failure rate; and γ_{qi} is the temporary failure rate for section i of q, respectively, and N_{qj} is the number of customers for section j of q. Note, if there is a three phase device at location qk, then the variable $x_{qkl} = 0$, and otherwise $x_{qkl} = 1$. The subscript 1 is used to represent a three-phase device, and the subscript 2 represents a fuse. For the main feeder Eq. (A3) reduces to

$$A_q = \sum_{i=1}^{q_n} \lambda_{qi} \sum_{j=1}^{q_n} N_{qj} + \sum_{i=2}^{q_n} \lambda_{qi} \sum_{j=1}^{i-1} N_{qj} \prod_{k=j+1}^{i} x_{qk1}, \quad q = 1 \tag{B4}$$

Since a fuse will be installed at the tap and no other protective device will be installed on category two laterals

$$A_q = \sum_{i=1}^{q_n} (\lambda_{qi} + \gamma_{qi}) \sum_{i=1}^{q_n} N_{qi} \in \alpha + 2 \ldots \alpha + \beta + 1 \tag{B5}$$

Since the contribution of the category two lateral is constant, then minimizing

$$z = \sum_{q=1}^{\alpha+1} A_q \tag{B6}$$

is equivalent to minimizing the SAIFI index.

Note, when load based indices are used, the formulation will be similar to the SAIFI index, except that the number of the customer will be replaced with the connected load. Finally, the constraits for this problem include coordination, design and application limitations, cost, and those due to the reduction of integer programming problem to zero–one linear programming problem. These are detailed in Ref. [2].

References

[1] IEEE Working Group on System Design. Trial use guide for electric power distribution reliability indices. Report. 1366, Draft no. 14.

[2] Soudi F, Tomsovic K. Optimized distribution protection using binary programming. IEEE Transactions on Power Delivery 1998;13(1):218–224.

[3] Soudi F, Tomsovic K. Optimal trade-offs in distribution protection design. IEEE Transactions on Power Delivery (in press).

[4] Ravindran A, Phillips DT, Solberg JJ. Operations research-principle and practice. New York: Wiley. pp. 184–209.

[5] Bazaraa MS, Jarvis JJ, Sherali HD, Linear programming and network flows. New York: Wiley, 1990. pp. 371–416.

[6] IBM General Information Manual. An introduction to modeling using mixed integer programming. Amsterdam, 1972.

[7] Nemhauser GL, Wolsey LA. Integer and combinatorial optimization. New York: Wiley, 1988. pp. 114–143.

[8] Using the CPLEX Callable Library, (version 4.0). 1989–1995, CPLEX Optimization, Inc.

[9] Cormen TH, Leiserson CE, Rivest RL, Introduction to Algorithms. New York: McGraw-Hill, 1990. pp. 42–52.

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