1. Introduction

The primary objective of power system operation is to deliver power to its customers meeting strict tolerances on voltage magnitude and frequency. Accordingly, the operation control problems naturally divide into control of voltage magnitudes or the voltage control issues, and the control of system frequency or the frequency control problems. Power system being an interconnected large system spread over a geographically wide network, operation of the large system is complex. The controls are built to exploit the inherent time-scale and structural properties of the system. In this chapter, we focus on the frequency control problem as an example of power system controls. In fact, Automatic Frequency Control in the North American electric power grid was the first instance of a successful implementation of a large-scale network based control scheme.

The frequency control includes two subproblems. First, we need to determine optimal values of generations that minimize the total generation costs while meeting the load demands. This problem is denoted the economic dispatch problem and is discussed in Section 2.

Dynamically, it can be shown that differences between total active power that is generated and the total active power that is consumed, lead to frequency drifting. Since the load fluctuations themselves are random, it is not possible to exactly match the total generation with the power consumption at all times. Therefore, the system frequency will tend to drift. In North American grids, there exists a central closed-loop controller that samples system wide power-flows and frequency in order to maintain the system frequency within tight tolerances while also maintaining economically dispatched generations. A brief introduction to the control called the Automatic Generation Control or the Load Frequency Control is presented in Section 3.

In addition to the topics in Sections 2 and 3, the voltage control problem in itself is a complex problem. It is mostly done by distributed automatic local controls in the North American grid. However, some European countries such as France do have automatic coordinated voltage control schemes for the large network.

The operation of the power system also has to meet regulations on security and reliability. Roughly speaking, the system is required to continue normal operation even with the loss of any one component. These studies are grouped under the framework of power system security which is a broad topic in itself.
2. Generation dispatch

This power system must generate sufficient power at all times to meet the load demand from the grid. The amount of load connected to the system varies significantly based on seasonal and time-of-day considerations. Also, the cost of producing power at different generators varies from plant to plant, depending on the efficiency of plant design and fuel costs at the generator location. Therefore, it is not economical to divide the required generation capacity arbitrarily among the available generators. The problem of determining how the total load requirement is to be divided among the generators in service is denoted the *generation dispatch* problem. This problem clearly optimizes the total generation costs in producing the required amount of active power. We discuss this problem first in Section 2.a under a classical formulation. Moreover, there are also active power losses involved in transmitting real power from generators to loads. Some generators such as those near coal mines with low fuel costs may incur large transmission losses in transferring the generated power to load centers. An optimization formulation that includes some simple consideration of transmission losses together with generation costs is presented in Section 2.b. Section 2.c discusses a general framework for posing detailed optimization problems in the form of optimal power-flow formulation.

2.a. Classical lossless generation dispatch:

The cost $C$ of generating power $P$ at a thermal plant can be roughly stated by the nonlinear function

$$C(P) = \alpha + \beta P + \gamma P^2$$

Let us assume that there are $N$ thermal generators in the system which should share the total load demand say $P_D$. The economic dispatch problem tries to minimize the total generation costs for generating power at the $N$ generators while meeting the load demand $P_D$. In the classical lossless formulation, we also assume that there are no transmission losses involved which simplifies the optimization considerably. For the lossless case, the power conservation equation or the power balance equation is simply stated as

$$\sum_{i=1}^{N} P_i = P_D$$

where $P_i$ denotes the power generated at plant $i$. Then, the economic dispatch reduces to the constrained optimization problem for minimizing the total generation cost $C_T$

$$\text{Min} \quad C_T = \sum_{i=1}^{N} C(P_i)$$

subject to

$$\sum_{i=1}^{N} P_i = P_D$$

The problem can be easily solved using the Lagrangian formulation by defining the Lagrangian $L(P_1, P_2, ..., P_N, \lambda)$ as

$$L(P_1, P_2, ..., P_N, \lambda) = \sum_{i=1}^{N} C_i(P_i) + \lambda \left( P_D - \sum_{i=1}^{N} P_i \right)$$
By setting the partial derivatives of the Lagrangian $L$ with respect to $P_i$ and $\lambda$ equal to zero, the optimal solution can be described by the conditions

$$IC_i = \frac{dC_i}{dP_i} = \lambda \text{ for } i = 1,2,...,N$$

(2.1)

and

$$\sum_{i=1}^{N} P_i = P_D$$

(2.2)

In other words, the generation dispatch becomes optimal when the incremental generation costs $IC_i$ become equal at all the generators. Since the generation cost $C_i$ is a quadratic function of $P_i$, the incremental cost $IC_i$ is a linear function of $P_i$. Therefore, the optimality problem reduce to solving N+1 linear equations (1) and (2) stated above in terms of N+1 unknown variables $P_1$ through $P_N$ and $\lambda$. The unique solution is easily solved as

$$\lambda = \frac{1}{\sum_{i=1}^{N} 2\gamma_i} \text{ and } P_i = \frac{\lambda - \beta_i}{2\gamma_i}.$$  

Thus far, we have considered no limits on the generation capacity at individual plants. In reality, there are lower and upper limits say $P_{\min,i}$ and $P_{\max,i}$ on the generation output $P_i$. These limits can be easily incorporated into the optimization problem above by appending the inequality constraints

$$P_{\min,i} \leq P_i \leq P_{\max,i}$$

(2.3)

with the conditions (1) and (2). Since these equations are linear, the optimal solution can be computed by simply freezing $P_i$ at either $P_{\min,i}$ or $P_{\max,i}$ whenever the limit is reached while finding the optimal solution. Details can be found in any standard text book on power system analysis.

The discussion in this section has thus far been focused on thermal generation plants. The operation costs of a hydroelectric plant are fundamentally different since there are no fuel costs involved. Here, the concern is to maintain adequate level of water storage while maintaining required water flow through the generator turbines. Clearly the stored water capacity depends on water in-flow and out-flow rates, and these issues are typically studied over longer time horizons as compared to the dispatch of thermal power plants. The problem of coordinating the generation outputs of hydro generators and thermal plants for meeting load demands while minimizing generation costs is called “hydrothermal coordination”. Again, the formulation and solution details can be found in standard power system text books.

2.b. Lossy economic dispatch:

In the real system, there will always be transmission losses associated with sending power from the generation facilities to the load buses. In the previous section, these losses were ignored which resulted in simple solutions for the generation dispatch problem. In this section, we modify the power balance equation (2) to include the power losses $P_L$. For a given set of generations $P_i$, the line losses have to be computed from the line currents which in turn requires solving the corresponding power-flow equations.
Such a problem of optimizing the total generation costs while solving nonlinear power-flow equations is called the *optimal power-flow formulation*, and it will be discussed in the next section. In this section, we approximate the total line losses $P_L$ as a direct function of the generations $P_i$ by the equation,

$$
P_L = \sum_{i=1}^{N} \sum_{j=1}^{N} B_{ij} P_i P_j + \sum_{i=1}^{N} B_{i0} P_i + B_{00}
$$

(2.4)

Here the term $B_{00}$ is constant while the coefficients $B_{ij}$ and $B_{i0}$ summarize the quadratic and linear dependence of line losses on the generations. Equation (2.4) is called the *$B$ matrix loss formula*, and the $B$ coefficients in (2.4) are called the *loss factors*. There exist a rich history of literature on the computation of loss factors for a given economic dispatch problem.

In this section, let us address the economic dispatch problem when the losses are represented by (2.4). Then, the optimization changes to

$$
\text{Min } \sum_{i=1}^{N} C_i(P_i)
$$

subject to $\sum_{i=1}^{N} P_i = P_D + P_L(P_1, P_2, \ldots, P_N)$

We redefine the Lagrangian $L(P_1, P_2, \ldots, P_N, \lambda)$ as

$$
L(P_1, P_2, \ldots, P_N, \lambda) = \sum_{i=1}^{N} C_i(P_i) + \lambda \left( P_D + P_L(P_1, P_2, \ldots, P_N) - \sum_{i=1}^{N} P_i \right)
$$

The optimal solution can be determined by solving

$$
IC_i = \frac{dC_i}{dP_i} = \lambda \left( 1 - \frac{\partial P_i}{\partial P_i} \right) \text{ for } i = 1, 2, \ldots, N
$$

(2.5)

and

$$
\sum_{i=1}^{N} P_i = P_D + P_L(P_1, P_2, \ldots, P_N)
$$

(2.6)

While equations (2.1) and (2.2) for the lossless case were linear in the variables $P_i$ and $\lambda$, the equations (2.5) and (2.6) for the lossy case are quadratic. The solution from the lossless formulation provides an excellent initial condition and the lossy economic dispatch solution can be computed by iterative solution strategies such as the Newton-Raphson algorithm discussed in the power system analysis chapter.

Equations (2.5) provide the relationship between the power generations $P_i$ and the Lagrangian multiplier $\lambda$. That is, we can restate (2.5) as $\lambda = \frac{IC_i}{1 - \frac{\partial P_i}{\partial P_i}}$. The product
term \( \frac{1}{1 - \frac{\partial P_L}{\partial P_i}} \) denotes the penalty term that is being multiplied to the incremental cost \( IC_i \) from the contribution of generation \( P_i \) to the line losses \( P_L \).

Moreover, if the multiplier \( \lambda \) is specified, the power generations \( P_i \) can be uniquely determined from equations (2.5) since the equations (2.5) are linear in \( P_i \). Therefore, it follows that the optimal solution of the equations (2.5) and (2.6) essentially reduces to the problem of finding the Lagrangian multiplier \( \lambda \). There exist excellent iterative techniques in the literature for finding the optimal solution by iterating on \( \lambda \).

2.c. Optimal power-flow formulation:

In the previous subsection, we had simplified the network power balance equations into a single power conservation equation by either ignoring the losses (equation (2.2) in section 2.a) or by approximating the line losses (equation (2.6) in subsection 2.b). However, in both earlier approaches, the network nature of the power transmission was completely ignored. In this section, we treat the power transmission in earnest by stating the power balance equations in full, which relate the transfer of power from the generators to the loads through the transmission network. The optimization itself has the same objective, that of minimizing the total cost of generation.

Let us assume that the system has \( N \) generators like before, and the total number of buses including generator and load buses is \( M \). For simplifying the notation, bus number 1 is assumed to be the slack bus. Buses 2 to \( N \) are the PV buses and buses numbered \( N+1 \) through \( M \) are PQ buses. The slack bus generation \( P_{G1} \) becomes a dependent variable while the remaining generations \( P_{G2} \) through \( P_{GN} \) are the control variables for the minimization. Real and reactive loads are denoted by \( P_{Di} \) and \( Q_{Di} \) respectively. Suppose the \((i,j)\)-th entry \( Y_{ij} \) of the \( Y_{bus} \) matrix has the magnitude \( Y_{ij} \) and the phase \( \gamma_{ij} \).

The basic optimal power-flow problem can then be stated as

\[
\text{Min}_{P_{Gi}} \quad C_T = \sum_{i=1}^{N} C_i(P_{Gi})
\]  

subject to the power-flow equations

\[
P_i = P_{G_i} - P_{D_i} = \sum_j Y_{ij} V_j V_i \cos(\delta_i - \delta_j - \gamma_{ij}) \]  \( \quad \) (2.8)

\[
Q_i = Q_{G_i} - Q_{D_i} = \sum_j Y_{ij} V_j V_i \sin(\delta_i - \delta_j - \gamma_{ij}) \]  \( \quad \) (2.9)

for \( i = 2, \ldots, N \). The power-flow variables, namely the bus voltages \( V_2, \ldots, V_N \) and phase angles \( \delta_2, \ldots, \delta_N \) become the dependent variables in the optimization procedure. In general, there may exist inequality constraints on the control variables as well the state variables in the optimal power-flow formulation.
The optimal power-flow problem can then be formally stated as

\[
\text{Min } u \quad f(x,u) \tag{2.10}
\]

subject to

\[
g(x,u) = 0 \tag{2.11}
\]

and

\[
h_i(x,u) \leq 0 \quad \text{for } i = 1,2,...,k. \tag{2.12}
\]

Here \(u\) denotes the optimization control variables; \(x\) denotes the network state variables which are dependent on \(u\) through (2.11). The function \(f(x,u)\) is the objective function to be minimized. The equations (2.11) denote the equality constraints, and the number of equations say \(n\) in (2.11) matches the dimension of \(x\). The equations (2.12) represent \(k\) different inequality constraints.

A solution to the general constrained optimization problem in (2.10)-(2.12) can be found by using the celebrated Kuhn-Tucker Theorem. We first define a generalized Lagrangian \(L(x,u,\lambda,\mu)\) for the problem as

\[
L(x,u,\lambda,\mu) = f(x,u) + \lambda^T g(x,u) + \mu^T h(x,u) \tag{2.13}
\]

where \(\lambda \in \mathbb{R}^n\) and \(\mu \in \mathbb{R}^k\). The Kuhn-Tucker Theorem provides the conditions that must be satisfied at the optimal solution to be

\[
\frac{\partial f}{\partial x} + \lambda^T \frac{\partial g}{\partial x} + \mu^T \frac{\partial h}{\partial x} = 0 \tag{2.14}
\]

\[
\frac{\partial f}{\partial u} + \lambda^T \frac{\partial g}{\partial u} + \mu^T \frac{\partial h}{\partial u} = 0 \tag{2.15}
\]

\[
g(x,u) = 0 \tag{2.16}
\]

\[
h_i(x,u) \mu_i = 0 \quad \text{with either } \mu_i = 0 \text{ and } h_i(x,u) < 0 \quad \text{or } \mu_i > 0 \text{ and } h_i(x,u) = 0 \tag{2.17}
\]

Therefore, the optimal power-flow problem becomes the solution of equations (2.14)-(2.17). In large-scale power system formulations, it is quite difficult to find an exact solution to the Kuhn-Tucker conditions. It is quite often sufficient to find a suboptimal solution using heuristic optimization algorithms. In recent history, excellent progress has been made in the development of such heuristic algorithms.

3. Frequency control

The power system load is continually undergoing change as individual loads fluctuate while others are energized or de-energized. Generation must precisely match these changes to maintain system frequency, a function called load frequency control (LFC), and at the same time follow an appropriate economic dispatch of the units as discussed in the previous section. Together these functions are referred to as Automatic Generation Control (AGC). Accordingly, all modern power systems have centralized control centers that run software called the Energy Management Systems (EMS) that, in addition to other functions, monitor frequency and generator outputs. Units that are on AGC will receive raise and lower signals to adjust their set points.
3.a AGC

Ensuring the power balance is commonly referred to as regulation and can be sensed by changes in the system frequency. If load (including losses) exceeds the generation input, then energy must be leaving the system over time. This energy will be drawn from the kinetic energy stored in the rotating masses of the generators. Hence, the generators will begin to rotate more slowly and the system frequency will decrease. Conversely, if generation input exceeds the load, then frequency will increase. It is the responsibility of the governor on a generator to sense these speed deviations and adjust the power input (say through the opening or closing of valves on a steam unit) as appropriate.

Specifically, the governor will change the power input in proportion to the speed deviation. This is referred to as the droop or speed regulation, $R$, and can be expressed as

$$\Delta P = -\frac{1}{R} \Delta f$$  \hspace{1cm} (2.18)

where $\Delta f$ is the change in frequency and $\Delta P$ is the resulting change in power input. $R$ is measured in Hz/MW or alternatively as a percentage of the rated capacity of a unit. Typical droops in practice are on the order of 5-10%. If no further action is taken, the system will then operate at this new frequency.

Fig 1a illustrates a standard scenario. Assume a simple system with a nominal frequency of 60 Hz and a 100 MW unit operating with 5% droop. The regulation constant is calculated to be $R = 0.05 \cdot \frac{60}{100} = 0.03$ Hz/MW. If the generator unit senses a drop in frequency of say 0.6 Hz, then this corresponds to a 20 MW increased power output. In addition to governor actions, many loads are frequency sensitive (e.g., motors). For these loads, the load will decrease as the frequency drops so that the needed increase from the generators is less. This can be expressed as

$$\Delta P = -\left(\frac{1}{R} + D\right) \Delta f$$  \hspace{1cm} (2.19)
where $D$ is the damping. Thus, the effective droop is slightly less, i.e., the frequency drop will be less, when load damping is considered. This load damping is often approximated as a 1% decrease in load for every 1% decrease in frequency.

Now, as would normally be true, if there are several generators interconnected and on regulation, then each will see the same frequency change (after any initial transients die out) and respond. Assuming the same percentage droop on each unit, the load will be picked up according to the relative capacities. This is depicted in Fig. 1b with units of
100 MW, 150 MW and 200 MW, respectively, all operating with 5% droop. Analytically with $R$ on a per unit basis, we can express this for $n$ units as

$$\Delta P = -\left( \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} + D \right) \Delta f$$  \hspace{1cm} (2.20)

The control system as described above suffers from a serious drawback. Frequency will never return to the nominal, i.e., desired, point. A supplemental control is needed to make the appropriate adjustments. One may think of this as adjusting for the initial loss of rotating kinetic energy when the load change first occurred by modifying the generator set points. This action must be coordinated among the units.

An additional issue arises in the coordination for this supplemental control. With relatively few exceptions (e.g., some islands), utilities are interconnected with their neighboring systems. Not only does this provide additional security and reliability in the case of outages but allows more economic operation by judicious energy trades. Still, the utilities must coordinate their operation to maintain energy schedules and meet demand. If a load change occurs in a neighboring system, both systems will see the frequency change and respond. There is no difference from the viewpoint of the generator between the loads in the different areas. Clearly, each utility wishes only to supply loads for which it is responsible and will receive compensation.

Generally then systems are broken into separate control areas reflecting the different responsibilities of the utilities as shown in Fig. 2. The flow on tie lines between these control areas is monitored. The generator set point adjustments are made to maintain these scheduled tie flows. Note while the first function of AGC is load following, over time these adjustments to generator set points may lead the units away from the most economic dispatch. Thus, the supplemental control may include additional adjustments for economic operation of the units. As summarized in the block diagram of Fig. 3, the supplemental control serves several functions, including:

- restoration of the nominal frequency,
- maintenance of the scheduled interchanges, and
- provision for the economic dispatch of units.

The coordination among areas is achieved by defining the so-called Area Control Error (ACE) as a measure of the needed adjustments to the generator set points. Let

$$ACE = \Delta P_{tie} - 10 \beta_f \Delta f$$  \hspace{1cm} (2.21)

where $\Delta P_{tie}$ is the deviation in the tie line from the scheduled exchange $\beta_f$ f is called the frequency bias and by tradition is negative and measured in MW/0.1 Hz (thus, the multiplier 10 in (4) above). If the $ACE$ for an area is negative, then this means that the area generation is too small and the unit set points should be increased.
Considering the system in Fig. 2, notice the effect of a sudden load increase in Area A on the $ACE$ in both areas A and B. First, the frequency in both areas will decrease and accordingly the power output for all regulated units will increase according to their respective droop settings. Since the load change occurs in Area A, there will be an additional unscheduled inflow from B. $ACE$ in area A will have two negative terms: schedule error $\Delta P_{tie}$ and the frequency bias, or control, error. Conversely, $ACE$ in Area B will have a positive schedule error but a negative frequency bias.

It may seem that ideally the $ACE$ in B would be zero and that in A it would precisely match the load change but this is neither practical nor particularly necessary. Instead, the $ACE$ is integrated over time and this signal is used to determine the generator set points. Integrating the error ensures that the actual energy exchange between areas is precisely maintained over time. Since the control center must calculate the $ACE$ after gathering data from the system, the set point controls are discrete. In North America, the AGC signals are fed to the units typically about once every 2-4 seconds. In many parts of the world, such frequent adjustment is considered excessive and supplementary control signals are sent over much less frequent time intervals. In each country, regulating agencies determine the required performance of the AGC so that utilities don’t “lean” too hard on their neighbors. For example, the traditional performance criteria in North America is that the $ACE$ has to return to zero or its pre-disturbance level within 15 minutes following the start of the disturbance.
3.b Contemporary Issues

The supplementary control system as described above reflects AGC operation more or less as it has existed since the advent of computer controls in the late 1960s. Recent moves to deregulate the power system and open the system to competition greatly complicate the traditional control philosophy. In an open market, the load responsibilities are not so clearly defined nor are they geographically restricted. For example, an independent generator may have a contract to serve a load in a neighboring control area and this will impact the scheduled flows on the intertie. A number of methods have been proposed to facilitate this control, including the establishment of a separate market for a “load-following” service. Customers would pay for this service along with their energy fee. No regions in the world have fully opened AGC control to market competition but incremental steps have been taken. Broadly speaking, while the supply of energy has proven to be amenable to economic competition, closed loop controls are proving more difficult to relegate to an open transparent market.

Fig. 3 Block diagram of AGC system