Seasonal Planning for Best Fuel Mix under Multiple Objectives using Fuzzy Mathematics

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Abstract - This research develops a framework for establishing a “best” fuel mix for a hydro-thermal system under multiple objectives using a fuzzy mathematical approach. Fuzzy logic is used to resolve conflicting objectives. An important contribution of this work is an investigation of appropriate forms for fuzzy membership functions and fuzzy logical operators. It is seen that the form of membership function and the logical operator are linked. It appears that fuzzy mathematics can provide a good model of expert judgment, particularly, where subjective considerations are involved.

Keywords - Fuzzy sets, fuzzy programming, multi-objective programming, seasonal planning, trade-off analysis.

1. INTRODUCTION

The aim of seasonal planning of a hydro-thermal production system is to find a fuel mix which serves as a best trade-off between several objectives. Important objectives include total production cost, environmental impact, security of supply, and volatility (i.e., sensitivity to price changes). Traditionally, total production cost has been the most important objective and included in all operation planning models. During recent years, the concern for environmentally sound solutions to the fuel mix problem has grown more important as the issues of, for example, acid rain and global warming have received more attention. Furthermore, in order to maintain a stable supply of electricity, the fuel supply must be secure. The security of fuel supply depends on many factors, including: the means of transportation, political and economic stability of the country supplying the fuel, and possible storing options. The last objective called volatility, is an attempt to account for the uncertainty in fuel prices. In general, utilities are risk averse and would tend to avoid fuels whose price fluctuations would greatly influence total cost. In other words, the volatility objective is a way of incorporating the decision makers subjective assessment of the relative stability of fuel prices.

A traditional approach to multi-objective decisions is to weight the objectives [1] thereby creating a single objective function. This approach has the advantage of simplicity, however, weights are often difficult to assign. Furthermore, they assume that the relative importance of an objective does not change with the situation. Finally, a general criticism of weighting is that they may obscure the subjective nature of a decision. A trade-off analysis method [2, 3, 4] has been developed which finds sets of “best” solutions that clarifies trade-offs among objectives. This method only analyzes sets of pre-specified plans. In general, a final solution is not obtained but rather a set of reasonable solutions. The significance of describing and including uncertainties in the planning process is becoming increasingly important [5, 6]. Recently, fuzzy mathematics have been suggested as a method of managing multiple objectives and uncertainties [7, 8, 9]. Fuzzy membership functions are constructed for each objective and constraint in order to represent the degree to which the objectives and constraints are satisfied. The membership objectives are aggregated with an intersection operator.

In this work, fuzzy mathematics are applied to clarify trade-offs and subjective assessments in the fuel mix problem. The various objectives are represented as functions of the amount of electric energy produced in the differential thermal units, i.e., nuclear, coal, oil and natural gas. In the case of total cost, a production costing program for hydro-thermal systems is used in order to obtain a functional relationship between total cost and amount of each fuel used. The obtained cost function is therefore an approximation of the true function but guarantees that constraints on hydro discharge and load coverage are fulfilled. The environmental objective is approximated by a linear relationship between emissions and energy produced by the different fuels. The function used as a security objective is constructed from the decision makers subjective assessment of three stages of the fuel import: supply, transportation, and domestic storage. Supply reflects the relative availability of the fuel, for example, the political stability of the country supplying the fuel. Transportation concerns issues such as land/sea transports versus pipelines. Domestic storage reflects the ability to increase security by storing large amounts of the fuel. The impact of uncertainty in fuel prices is approximated in the model by including a volatility objective. This is accomplished
by including an objective where the decision makers have
defined volatility parameters for each fuel reflecting their
belief of possible price swings.

A number of practical problems arise in the application
of fuzzy mathematics. Foremost among these problems
is establishing acceptable membership functions. In this
work, single objective optimizations are used to establish
levels of expectation for objectives. These expectations
are combined with some standard functional shapes to
investigate such representations for ease of application
and computational considerations. In particular, linear
(or trapezoidal) and Dombi [10] forms are investigated.
Another problem that arises in fuzzy applications is defining
appropriate logical operators. A trade-off is normally
modeled as an intersection operation. While the mini-
num function is the most commonly used other opera-
tors may be more reasonable [11]. It is seen that selection
of an operator must concur with the form of the mem-
bership function for meaningful results to be obtained.
Several numerical examples are used to highlight compro-
mises among individual objectives, and show the effect of
different membership functions and T-norms.

This paper is organized as follows. The fuel mix prob-
lem is defined precisely. A simple introduction to fuzzy
mathematics is given. The fuel mix problem is then re-
formulated as a fuzzy optimization problem. Numerical
examples illustrate the proposed technique. Discussions
and comments follow.

2. FUEL MIX PROBLEM

The fuel mix problem is solved in order to determine a
best combination of fuels for a given production system
during a specific planning period. In this paper, a plan-
ing period of one year divided into weeks is being studied
with an existing production system composed of hydro,
nuclear, natural gas, oil and coal fired units.

For an electric utility with a significant amount of thermal
generation, fuel expenses constitute a large part of the
overall supply cost. A natural incentive is therefore to minimize the fuel costs, hence using as much as possible
of the cheapest fuel. Assuming this is the only objective
of the utility and that future fuel prices were known with
a high degree of certainty, the solution of the planning
problem is straightforward, simply loading the units in
merit order. The single objective becomes:

\[ \min C_T(k, W_i(k)) \] 

(1)

where \( C_T() \) is the thermal variable production cost, \( W_i() \)
is the electric energy generated by fuel \( i \), and \( k \) is the time
period.

During recent years, the concern for environmentally
sound solutions to the fuel mix problem has grown. This
is reflected by the public interest in, for example, acid
rain and global warming. As the solution to the fuel mix
problem with the cost objective (1) does not necessarily
guarantee an environmentally acceptable solution, it is
necessary to either constrain the use of various fuels or
introduce a new objective. A general form for the envi-
ronmental objective is:

\[ \min E(W_i) \] 

(2)

where \( E() \) is the environmental impact function, and \( W_i \)
is the total yearly energy from fuel \( i \).

The function \( E() \) in (2) will take different forms de-
dpending on which environmental problem the objective is to
address. For example, if the aim is to minimize emissions of sulphur, \( E() \) becomes the functional relationship be-
 tween emission of sulphur and energy generated by each
fuel \( i \).

The cost and environmental objectives above applies to
most utilities. For utilities dependent on imports of fu-
els, a third objective is introduced accounting for the se-
curity of fuel supply. The security of supply of a fuel is
dependent on a multitude of factors including: political
and economic stability of the country supplying the fuel,
the means of transportation, and possible storing options
open to the utility. The objective function estimating the
security of a fuel mix is highly subjective, measuring the
decision makers beliefs on the relative security of the
different fuels. The fuel supply security objective is:

\[ \max S(W_i) \] 

(3)

where \( S() \) is a security function. In section 4, a simple
security function is developed.

Fuel prices are obviously important variables in the so-
lution of the fuel mix problem. When the problem is
solved for a future reference year, the uncertainty in the
fuel price variables must be accounted for in some way. In
general, utilities are risk averse and will avoid fuels whose
price fluctuations would greatly influence total variable
cost. In order to represent the utility's aim to minimize
the use of such fuels a fourth objective, called volatility,
is introduced. As was the case with the security of supply
objective (3), the volatility objective is highly subjective,
modeling the decision makers views on the different fuel
prices relative stability. The total volatility objective is:

\[ \min V(W_i) \] 

(4)

where \( V() \) is a volatility function.

The multiple objective fuel mix problem is solved subject
to a number of constraints pertaining to the specific pro-
duction system. In this paper, a hydro thermal system
has been modeled with the following constraints.

Energy balance requirement

\[ W_D(k) = W_H(k) + W_T(k) \] 

(5)

where \( W_D(k) \) is the energy demand of week \( k \), \( W_H(k) \) is
the total hydro energy, and \( W_T(k) \) is the total thermal
energy.

Water balance

\[ x_j(k + 1) = x_j(k) - u_j(k) + w_j(k) \] 

(6)
where \( x_j() \) is the hydro reservoir contents plant \( j \), \( u_j() \) is the reservoir discharge, and \( w_j() \) is the effective inflow.

**Bounds on reservoir storage**

\[
x_j^m(k) \leq x_j(k) \leq x_j^M(k)
\]

where \( x_j^m() \) is the minimum hydro reservoir contents, and \( x_j^M() \) is the maximum hydro reservoir contents.

**Bounds on discharge**

\[
u_j^m(k) \leq u_j(k) \leq u_j^M(k)
\]

where \( u_j^m() \) is the minimum discharge, and \( u_j^M() \) is maximum discharge.

**Hydraulic production**

\[ W_{H,j}(k) = f_j(u_j(k), x_j(k)) \]

where \( f_j() \) is the hydro production function.

**Bounds on thermal generation**

\[
W_{T,j}^m(k) \leq W_{T,j}(k) \leq W_{T,j}^M(k)
\]

where \( W_{T,j}^m() \) is the minimum thermal production at plant \( j \), and \( W_{T,j}^M() \) is the maximum thermal production.

### 3. FUZZY FORMULATION

In this section, fuzzy logic is reviewed. More complete developments are widely available, e.g., [12].

Fuzzy sets represent uncertainties associated with the structure of a class or set of objects. An element of a fuzzy set is an ordered pair containing a set element and the degree of membership in the fuzzy set. A membership function is a mapping, typically:

\[ \mu : X \rightarrow [0, 1] \]

For fuzzy set \( A \):

\[ A = \{ (x, \mu_A(x)) | x \in X \} \]

where \( X \) is the universe and \( \mu_A(x) \) represents the degree of uncertainty, or, the degree to which \( x \) fits the characteristic feature of the set \( A \). A higher value of \( \mu_A(x) \) indicates a greater degree of membership. The following definitions of fuzzy set operations are commonly used. If \( C = A \cap B \),

\[ \mu_C(x) = \min(\mu_A(x), \mu_B(x)) \]

and if \( C = A \cup B \),

\[ \mu_C(x) = \max(\mu_A(x), \mu_B(x)) \]

and if \( C = A \),

\[ \mu_C(x) = 1 - \mu_A(x) \]

As mentioned previously, the intersection operator acts as a compromise or trade-off function in the fuzzy set formulation. However, the minimum function may not be the appropriate operator for a given decision problem. It is generally accepted that the intersection operator should belong to the T-norm class [11]. This class includes the minimum operator. Two other T-norm operators from [11] are investigated here. The product operator where if \( C = A \cap B \):

\[ \mu_C(x) = \mu_A(x) \mu_B(x) \]

and the operator proposed by Dombi where:

\[ \mu_C(x) = \frac{1}{1 + \left( \frac{1}{\mu_A(x)} - 1 \right)^\lambda + \left( \frac{1}{\mu_B(x)} - 1 \right)^\lambda} \]

and if \( \lambda > 0 \) then \( C = A \cap B \). The larger the magnitude of \( \lambda \), the greater the “strictness” of the intersection. Notice as \( \lambda \rightarrow \infty \), (16) approaches (12).

The development of the fuzzy programming problem can be found in [7, 13]. For the purpose of this discussion, the decision problem consists of:

- Formulation of the objective or fuzzy constraint as a membership function which represents the degree to which the objective is satisfied on a scale of [0,1].
- Computation of the overall satisfaction with a decision by applying an appropriate intersection operator to all of the objectives.
- Maximization of the overall satisfaction such that no constraints are violated.

This framework is a bit misleading in its simplicity. Obviously, there are computational efficiency and numerical stability problems which must be considered in forming the decision problem. For example, linear membership functions with a minimum operator for intersection operator results in a linear programming problem [7]. On the other hand with more complex T-norms and membership functions, typical optimization problems of local minima and poor convergence may arise. While analysis of such computational considerations is beyond the scope of this paper, they certainly play an important part in evaluating the capability of fuzzy methods for reaching compromises.

### 4. DEVELOPED MODEL

In this section, it is described how the objectives stated in section 2 are converted into linear functions of the electric energy produced from the different fuels. Furthermore, the process of transforming these functions into membership functions for the fuzzy mathematics is set forth.
Linear objectives

The economical objective (1) measures the fuel mix in terms of its total fuel cost. The resulting fuel cost for a hydro-thermal production system is a function of a multitude of variables, including: total demand, installed capacities, hydro inflow, fuel prices, and outage rates. In order to arrive at an accurate estimate of the cost, a production costing simulation must be performed, solving the problem formulated by objective (1) and constraints (5)-(10). The solution to this problem yields the total cost and electric energy produced in the different units (from the different fuels). Furthermore, an average cost per generated MW/h can easily be found. The obtained solution guarantees that as much as possible of a cheaper fuel is used before a more expensive is utilized in each time period.

When the trade-off between cost, environmental impact, security, and volatility objectives are performed, the total electric energy generated from each fuel $W_i$ is determined so that a compromise in these objectives is obtained. It is therefore necessary to create a cost objective function which is a function of generated electric energy by each fuel. In this project, an approximate cost function is created in the following way. A production costing simulation according to objective (1) and constraints (5)-(10) is performed. A total cost $TC_0$ is obtained as well as total energy by fuel $W_{i,0}$ and the average variable cost $\overline{v}_c$ as:

$$\overline{v}_c = \frac{TC_0}{\sum_i W_{i,0}} \quad (17)$$

The linear relationship between total cost and change in electric energy from the different fuels is approximated by:

$$TC(\Delta W_i) = TC_0 + \sum_i \delta c_i \Delta W_i \quad (18)$$

where $\Delta W_i$ is the change in energy from fuel $i$, and $\delta c_i$ is the variable cost of fuel $i$, and

$$\delta c_i - v c_i - \overline{v}_c. \quad (19)$$

For the environmental objective (2) we want to find a linear relationship between environmental impact and energy from different fuels. If the environmental objective is to minimize the total emissions of certain pollutants, then the objective takes the following form:

$$TE(\Delta W_i) = TE_0 + \sum_i e_i \Delta W_i \quad (20)$$

where $TE()$ is total emissions of pollutant, $TE_0$ is total emissions for original solution, and $e_i$ is emissions of pollutant in kg/GWh for fuel $i$.

$TE_0$ is obtained from:

$$TE_0 = \sum_i e_i W_{i,0} \quad (21)$$

The subjective function measuring the security of the fuel mix is constructed from the decision makers’ assessment of three stages of the fuel import: supply, transportation, and storage. The aim is to arrive at a linear objective function similar to (18) and (20) above. The approach taken for the subjective assessment is to ask the decision maker to rank the fuels relative merit in each stage. Hence, for each stage, the decision maker assigns a weight to each fuel reflecting the fuels relative benefit in that stage. Then, the three stages are assigned relative weights reflecting their importance. The overall weight factor to include in the security objective for each fuel is thus obtained according to:

$$s_i = \sum_{t=1}^{3} s_{i,t} \quad (22)$$

where $s_i$ is overall weighting for fuel $i$, $s_{i,t}$ is weighting of stage $t$, and $s_{i,t}$ is relative weighting of fuel $i$ in stage $t$.

The resulting security objective function is:

$$TS(\Delta W_i) = TS_0 + \sum_i s_i \Delta W_i \quad (23)$$

where $TS()$ is total security, and $TS_0$ is security from the original solution.

The linear objective function measuring the volatility of a fuel mix is obtained in the same way as the security objective above. The decision maker is asked to rank the fuels in increased order of likelihood of price swings. The more stable the fuel price the lower the weighting assigned. The resulting objective for volatility becomes:

$$TV(\Delta W_i) = TV_0 + \sum_i v_i \Delta W_i \quad (24)$$

where $TV()$ is total volatility, $TV_0$ is volatility for the original solution, and $v_i$ is volatility weighting of fuel $i$.

Constraints

As the solution to the production costing simulation optimized the use of fuels in cost order, the cheapest fuel has been used to a maximum. This means that in order to maintain feasibility, the use of a cheaper fuel cannot be increased at the expense of a more expensive fuel. Assuming the fuels $i$ are numbered in increasing cost order the following becomes the bounds on energy changes for the cheapest fuel:

$$-W_{i,0} \leq \Delta W_i \leq 0 \quad (25)$$

and for all other fuels:

$$-W_{i,0} \leq \Delta W_{i,0} \leq \min(W_i^M - W_{i,0}, \sum_{l=1}^{i-1} W_{l,0}) \quad (26)$$

where $W_i^M$ is maximum possible energy from fuel $i$. 


In order to make sure that no less costly fuel is exchanged for a more expensive fuel:

$$\sum_{i=1}^{n_f} \Delta W_i \leq \sum_{m=i+1}^{n_f} \Delta W_m \quad i = 1, \ldots, n_f - 1 \quad (27)$$

where $n_f$ is the number of fuels. Also, in order to maintain total thermal production constant, we have:

$$\sum_{i} \Delta W_i = 0 \quad (28)$$

Establishing membership functions

Linear membership functions for the four objectives (18), (20), (23), and (24) are established by solving four linear programs composed of the constraints (25) - (28) and each of the objectives. The four solutions obtained are then used for calculating outcomes in terms of each objective. From these four outcomes, it is possible to observe a best and a worst case.

The linear membership functions for the objectives are then constructed according to the following. Assume one wants to minimize the objective and denote the best and worst outcomes $z_b$ and $z_w$, respectively. Solutions equal to $z_b$ have a membership value of 1, and solutions equal to $z_w$ a membership value of 0. In between the two boundaries, the membership value decreases linearly. Thus:

$$\mu_z(x) = 1 - \frac{x - z_b}{z_w - z_b} \quad \text{for } z_b \leq x \leq z_w \quad (29)$$

Another membership function proposed in [10] is also investigated. Specifically, for $x \in [a, b]$:

$$\mu_z(x) = \frac{(1 - \nu)^{\lambda - 1}(z_b - x)^\lambda}{(1 - \nu)^{\lambda - 1}(z_b - a)^\lambda + \nu^{\lambda - 1}(x - z_w)^\lambda} \quad (30)$$

where four parameters characterize each transition from 0 to 1: the lower limit $z_b$, the upper limit $z_w$, which are the best and worst case values, respectively, the transition rate $\lambda$, and the inflection point $\nu$. Increasing $\lambda$ quickens the transition and increasing $\nu$ shifts the inflection point to the right. Notice the similarity in form with (16). It is felt such consistency in structure provides a more coherent foundation for reaching decisions and appears to alleviate some numerical problems.

5. CASE STUDY AND NUMERICAL RESULTS

A case study has been performed with the methodology described in the sections above. For this purpose, a system with the following data was created.

**Demand data**
- Yearly energy: 25000 GWh
- Maximum power: 4560 MW

**Production data**

The production system consists of hydro, nuclear, natural gas, coal, and oil fired units. Data for these different units are given in Table 1. In addition to the thermal units described in Table 1, a hydro unit with the following data is included in the production set.

- Yearly energy: 6500 GWh
- Maximum power: 1500 MW

A production costing simulation with this data was performed utilizing the water value technique. Results of this run in terms of total cost, electric energy generated from hydro and the different fuels, and average variable cost are given in Table 2. The data from Table 2 are used to create the total cost objective function (18) and the constraints (25) to (28). The cost objective becomes:

$$TC(\Delta W_i) = 442 - 0.008 \Delta W_N + 0.025 \Delta W_G + 0.006 \Delta W_C + 0.026 \Delta W_O \quad 10^6 S \quad (31)$$

Results from the production costing simulation together with data from Table 1 gives the following emission objective:

$$TE(\Delta W_i) = 2246 * 10^3 + 8.8 \Delta W_G + 307.6 \Delta W_C + 447.2 \Delta W_O \quad \text{kg} S \quad (32)$$

The subjective ranking in terms of security of the fuels as described above gave the following objective for maximizing security:

$$TS(\Delta W_i) = 16539 + 1.2 \Delta W_N + 0.4 \Delta W_G + 0.8 \Delta W_C + 0.6 \Delta W_O \quad (33)$$

For this example, a volatility objective has not been included.

Change in fuel use for obtaining a best solution in each of the modeled objectives is given in Table 3. Also included in the table are change in objective values for each objective at the best and worst case respectively.

In tables 4 and 5 results of intersection operation with the modeled membership functions and T-norms are given.

6. DISCUSSION

The case studies presented in this work are intended to highlight some general observations about reaching decisions in a subjective environment. The following observations were noted for the fuel mix problem appear to be general observations about using the fuzzy techniques. Specifically:

- In order to simplify defining membership functions for various objectives, it is important to breakdown objectives in terms of fundamental considerations. For the problem under consideration, a fully developed system would probably need to break down each of the objectives even further, e.g., isolating a particular aspect of security in one objective.
<table>
<thead>
<tr>
<th></th>
<th>Nuclear</th>
<th>Natural gas</th>
<th>Coal</th>
<th>Oil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity [MW]</td>
<td>1100</td>
<td>220</td>
<td>500</td>
<td>220</td>
</tr>
<tr>
<td># of units in study</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>F.O.R. [%]</td>
<td>5.9</td>
<td>4.9</td>
<td>12</td>
<td>4.9</td>
</tr>
<tr>
<td>Efficiency [%]</td>
<td>32.4</td>
<td>40.7</td>
<td>35.1</td>
<td>40.2</td>
</tr>
<tr>
<td>Variable cost [$/MWh]</td>
<td>10</td>
<td>43</td>
<td>24</td>
<td>44</td>
</tr>
<tr>
<td>Emissions of S [kg S/GWh]</td>
<td>0</td>
<td>8.8</td>
<td>307.6</td>
<td>447.2</td>
</tr>
</tbody>
</table>

Table 1: Production unit data

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Total Cost [10^8$]</td>
<td>442</td>
</tr>
<tr>
<td>Hydro energy [GWh]</td>
<td>6439</td>
</tr>
<tr>
<td>Nuclear energy [GWh]</td>
<td>8049</td>
</tr>
<tr>
<td>Coal energy [GWh]</td>
<td>6542</td>
</tr>
<tr>
<td>Gas energy [GWh]</td>
<td>3435</td>
</tr>
<tr>
<td>Oil energy [GWh]</td>
<td>455</td>
</tr>
<tr>
<td>Average variable cost [$/MWh]</td>
<td>18</td>
</tr>
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Table 2: Result of production costing

<table>
<thead>
<tr>
<th>Objective</th>
<th>TC</th>
<th>TE</th>
<th>TS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta W_C$ GWh (best case)</td>
<td>0</td>
<td>3880</td>
<td>-3435</td>
</tr>
<tr>
<td>$\Delta W_C$ GWh (best case)</td>
<td>0</td>
<td>-3880</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta W_O$ GWh (best case)</td>
<td>0</td>
<td>0</td>
<td>3435</td>
</tr>
<tr>
<td>$\Delta W_N$ GWh (best case)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta$ Objective (best case)</td>
<td>0</td>
<td>-1.16*10^6</td>
<td>687</td>
</tr>
<tr>
<td>$\Delta$ Objective (worst case)</td>
<td>74</td>
<td>1.51*10^6</td>
<td>-1552</td>
</tr>
</tbody>
</table>

Table 3: Best and worst case values

<table>
<thead>
<tr>
<th>Case</th>
<th>$\mu$ form</th>
<th>T-norm</th>
<th>$\Delta W_G$ [GWh]</th>
<th>$\Delta W_C$ [GWh]</th>
<th>$\Delta W_O$ [GWh]</th>
<th>$\Delta W_N$ [GWh]</th>
<th>Satisfaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Linear</td>
<td>Minimum</td>
<td>443</td>
<td>-443</td>
<td>0</td>
<td>0</td>
<td>0.61</td>
</tr>
<tr>
<td>2</td>
<td>Linear</td>
<td>Product</td>
<td>412</td>
<td>-424</td>
<td>12</td>
<td>0</td>
<td>0.34</td>
</tr>
<tr>
<td>3</td>
<td>Linear</td>
<td>Dombi ($\lambda=2$)</td>
<td>376</td>
<td>-376</td>
<td>0</td>
<td>0</td>
<td>0.53</td>
</tr>
<tr>
<td>4</td>
<td>Linear</td>
<td>Dombi ($\lambda=4$)</td>
<td>376</td>
<td>-376</td>
<td>0</td>
<td>0</td>
<td>0.57</td>
</tr>
<tr>
<td>5</td>
<td>Dombi ($\nu=0.25$)</td>
<td>Dombi ($\lambda=2$)</td>
<td>397</td>
<td>-397</td>
<td>0</td>
<td>0</td>
<td>0.72</td>
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Table 4: Case study fuel usage
<table>
<thead>
<tr>
<th>Case</th>
<th>$\mu$</th>
<th>T-norm</th>
<th>$\Delta TC [10^8]$</th>
<th>$\Delta TE [10^8$ kg S]</th>
<th>$\Delta TS$</th>
<th>Satisfaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Linear</td>
<td>Minimum</td>
<td>8.4</td>
<td>-132</td>
<td>-177</td>
<td>0.61</td>
</tr>
<tr>
<td>2</td>
<td>Linear</td>
<td>Product</td>
<td>8.1</td>
<td>-121</td>
<td>-167</td>
<td>0.34</td>
</tr>
<tr>
<td>3</td>
<td>Linear</td>
<td>Dombi ($\lambda=2$)</td>
<td>7.2</td>
<td>-112</td>
<td>-151</td>
<td>0.53</td>
</tr>
<tr>
<td>4</td>
<td>Linear</td>
<td>Dombi ($\lambda=4$)</td>
<td>7.2</td>
<td>-112</td>
<td>-151</td>
<td>0.57</td>
</tr>
<tr>
<td>5</td>
<td>Dombi ($\nu=0.25$)</td>
<td>Dombi ($\lambda=2$)</td>
<td>7.5</td>
<td>-119</td>
<td>-158</td>
<td>0.72</td>
</tr>
</tbody>
</table>

Table 5: Case study objective results

- Simple linear trends over the maximum and minimum for each objective as suggested by [7] and tested here may not adequately address practical objectives.
- The form of membership function and T-norm operator appear to be linked and thus, cannot be selected independently. As shown, linear membership functions yield a similar result for all three of the T-norms. Although not investigated closely here, both the product and Dombi T-norms had numerical problems with local minima and convergence.

While the framework reported here is somewhat simplistic, some interesting results were obtained. The careful modeling of subjective assessments in decision making within the expert system area appears to merit further investigation.

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REFERENCES

References


