DISTRIBUTION EXPANSION PROBLEM REVISITED.

PART 2

PROPOSED MODELING AND FORMULATION

MOHAMMAD VAZIRI
Department of EECS
Washington State University
Pullman WA 99163 USA
mvaziri@eecs.wsu.edu

KEVIN TOMSOVIC
Department of EECS
Washington State University
Pullman WA 99163 USA
tomsovic@eecs.wsu.edu

TURAN GÖNEN
Department of EEE
California State University
Sacramento CA 95826 USA
gonen@ecs.csus.edu

Abstract:
In this part, the problem is clearly defined from a practical point of view. A general multi stage mathematical programming formulation of the problem addressing the shortcomings is presented. The complexity issues of this general formulation is also addressed and discussed. Finally, we propose a directed graph, minimum edge cost network flow modeling of the problem for a truly multi stage formulation that would guarantee global optimality. Results form a simple test case based on the proposed formulation is presented and analyzed.
Keywords: Power system planning, Distribution expansion, NP complexity, Multistage upgrades.

Problem Definition

As mentioned earlier, the primary goal of the expansion problem is to timely serve the load growth safely, reliably, and economically. Here, it is assumed that safety considerations have already been translated into a set of operational standards in the design stage. Reliability and economics on the other hand, may be formulated as objectives for optimization programs. For example, a single criterion optimization program may be one that maximizes the level of reliability while another may be developed to minimize the total cost. So to continue this example the objective could be to minimize the total fixed and variable costs at all stages ensuring that;

- every demand center j is served for all stages,
- voltages are within guidelines at every node j for all stages,
- all elements operate within their capabilities and operational constraints,
- all expenditure is within the budget for every stage.

A general mathematical representation of the above formulation would then be;

$$Min\ C = \sum_{t=1}^{T} \left\{ \sum_{S \in Stations} Cf_{S,t} + \sum_{S \in Stations} Cv_{S,t} + \sum_{F \in feeders} Cf_{F,t} + \sum_{F \in feeders} Cv_{F,t} \right\} \qquad (1)$$

$$Subjectto: \sum X_{ij,t} - \sum X_{jk,t} = P_{j,t} \quad \forall j \in \text{Load Centers}, ij \text{ and } jk \in \text{Feeders} \qquad (2)$$

$$V^{Min} \le V_{j,t} \le V^{Max} \quad \forall j \in \text{Load Centers} \qquad (3)$$

$$S_{i,t} \le S_i^{Max} \quad \forall i \in \text{Stations} \qquad (4a)$$

$$X_{ij,t} \le X_{ij,t}^{Max} \quad \forall ij \in \text{Feeder Links} \qquad (4b)$$

$$\sum_{S \in Stations} Cf_{S,t} + \sum_{S \in Stations} Cv_{S,t} + \sum_{F \in Feeders} Cf_{F,t} + \sum_{F \in Feeders} Cv_{F,t} \le B_t, \quad \forall t = 1,2,\dots,T \qquad (5)$$

where

T	is the number of stages to full expansion
t	is each stage of the T stage process
$X_{ij,t}$	is the directiona l complex power flow from node i to node j at stage t
$X_{jk,t}$	is the directiona l complex power flow from node j to node k at stage t
$P_{j,t}$	is the diversifie d peak demand of load center (node) j at stage t
$Cf_{S,t}$	is the fixed cost of substation S to be installed at stage t
$Cv_{S,t}$	is the variable cost of substation S to be incured at stage t
$Cf_{F,t}$	is the fixed cost of feeder F to be installed at stage t
$Cv_{F,t}$	is the variable cost of feeder F to be incured at stage t
$V_{j,t}$	is the voltage at node j at stage t
V^{Min}, V^{Max}	are the lower & upper bounds of acceptable voltage
$S_{i,t}, S_i^{Max}$	are loading of substation S at stage t and Max. Capability respective ly
$X_{ij,t}, X_{ij,t}^{Max}$	are the flow in the link ij at stage t and and Max. Capability respective ly
B_t	is the expansion budget amount for stage t

The value of C to be minimized in equation (1), is the total cost for the expansion of the system over all the stages. Constraints (2) through (5) include both physical and performance conditions. Constraint (2) is the well known Kirchhoff's Current Law (KCL) applied to every node. This is also known as the flow conservation law in mathematical literature. If there is no local demand at the node, it is usually referred to as the transshipment node. Constraint (3) sets explicit voltage limits for all the load centers. Constraints (4a) and (4b) ensure that all substation transformers and feeders are loaded within their capabilities, and all other operational conditions are within limits. Finally, constraint (5) is a budgetary constraint so that the expansion costs at each stage are within the budgeted amount. As discussed previously, although this is an important constraint to include in all practical planning, it has been generally neglected in all previous formulations.

So far, we have only expressed a basic structure for the problem formulation. It is necessary to introduce a set of decision variables, many of which are discrete. This will require all of the variables shown in the basic formulation (with the exception of $P_{j,t}$ and $V_{j,t}$) to be modified, as well as introduction of some additional variables introduced. Considering first the flow and load variables $X_{ij,t}$ and $S_{i,t}$ are defined as the power flow for the link ij and the substation S loading at stage t respectively. The power flow and loading only exist if the decisions to build the path ij and/or to build the substation S is affirmative at some stage. This requires introduction of a binary decision variable associated with every future feeder link or substation. It is also necessary to establish an association between the continuous flow and loading variables and the binary decision variables. We propose these associative relations be implemented in the form of constraints in the formulation. The objective function includes the fixed installation costs as well as the variable costs associated with the flows and loading of the facilities. Therefore the object is a function of both sets of variables.

Closer attention to the objective function and the constraints reveals that constraints (2) and (4) are linear while constraint (3) is generally nonlinear but may also be rather easily linearized. The Objective function and constraint (5) on the other hand are nonlinear due $C_{vF,t}$ and $C_{vS,t}$ which are defined as the variable costs for the feeders and substations. Although we have yet to define all components of these costs, it is well known that at least one component of each of these costs must be attributed to the facility losses which is a quadratic function of the power flow variable.

Separating the linear and the nonlinear terms, and assuming for now, (this will be shown later) that all variable costs may be modeled as quadratic functions of power flows, a matrix form representation of the problem may be formulated as shown below.

$$Min\ C = \sum_{t=1}^{T} \left\{ C_{fS,t}{}^{T}\delta_{S,t} + C_{fF,t}{}^{T}\delta_{F,t} + \tfrac{1}{2}\left[X_{S,t}^{T}Q_S X_{S,t} + X_{F,t}^{T}Q_F X_{F,t}\right]\right\} \quad \text{(1m)}$$

$s.t.:$

$$A_j X_t = P_{j,t} \quad X_t = [X_{S,t} \ \ X_{F,t}]^T \quad \text{(2m)}$$

$$V^{Max} \le V_{j,t} \le V^{Max} \quad \text{(3m)}$$

$$X_t \le b_t \quad \text{(4m)}$$

$$C_{fS,t}^{T}\delta_{S,t} + C_{fF,t}^{T}\delta_{F,t} + \tfrac{1}{2}\left[X_{S,t}^{T}Q_S X_{S,t} + X_{F,t}^{T}Q_F X_{F,t}\right] \le B_t \qquad \forall t \quad \text{(5m)}$$

Where,

$C \ \in R$ is the total cost for the ultimate system expansion
$t \ \in Z$ is the stage number of the multi stage study
$m \ \in Z$ is the total number of nodes
$n \ \in Z$ is the total number of feeders and the substations
$n_S \ \in Z$ is the number of Substations
$B_t \ \in R$ is the expansion budget for stage t
$X_{F,t} \in R^{(n-ns)}$ is the vector of feeder power flows
$X_{S,t} \in R^{ns}$ is the vector of Substation loads
$X_t \in R^{n}$ is the combined vector of substation loading and feeder power flows
$\delta_{F,t} \in \{0,1\}^{(n-ns)}$ is the vector of binary feeder decision variables
$\delta_{S,t} \in \{0,1\}^{ns}$ is the vector of binary substation decision variables
$V_{j,t} \in R^{m}$ is the vector of node voltages
$V^{Min}, V^{Max} \in R^{m}$ are the vectors of upper and lower bounds on node voltages
$C_{fS,t} \in R^{ns}$ are the vectors of fixed substation costs
$P_{j,t} \in R^{m}$ is the vector of Load Center demands
$C_{fF,t} \in R^{(n-ns)}$ is the vectors of fixed feeder costs
$b_{,t} \in R^{n}$ is the vector of power flow bounds for substations and feeders at stage t
$Q_S \in R^{ns \times ns}$ is the loss cost coefficient matrix for the substations
$Q_F \in R^{(n-ns)\times(n-ns)}$ is the loss cost coefficient matrix for the feeder links
$A_j \in R^{m \times n}$ is the node to branch incidence matrix for the system
R , Z are sets of real and integer numbers respectively.

All other variables are as defined earlier. Note, the variable costs $C_{vF,t}$ and $C_{vS,t}$ have been mapped in to elements of Q_F and Q_S respectively. The nature of this mapping will be clarified later.

The above problem is a nonlinear and mixed integer optimization problem. Mixed integer problems in general, and specifically this problem, computationally belong to the class NP complete. NP completeness (as opposed to Polynomial Boundedness or class P*)*, refers to a class of problems for which algorithmically, the computational complexity of the solution searches grows exponentially (non polynomially) with some parameter [1]. That is, NP complete problems have a computational complexity of the highest degree, and are difficult problems to solve.

While NP complete problems are difficult to solve in general, note the following;
1 – The distribution expansion problem, which by the way resembles many other engineering design problems, despite the complexities, is a very practical problem that awaits a better solution than what's available. In fact, many of the major utilities still rely on experience and rules of thumb when planning expansions. Considering the significance of the unresolved shortcomings discussed previously, any effort for resolution would be a step in the right direction and could conceivably generate significant interest.

2 – Perhaps more importantly, there are many practical NP complete problems which have been efficiently solved by mathematical programming. For example the mixed integer linear programming algorithm was used by Soudi and Tomsovic in [2-3] to efficiently solve the optimal placement of protective equipment on feeders, was previously considered unfeasible due to the perceived complexity. The important point is that practical considerations often limit the number of solutions and render the NP complete problems computationally tractable.

Design Criteria and Assumptions

To aid the formulation, note the following definitions and assumptions:

Source Node

There exists only one source node designated as Node#1 shown in Fig 1. This is viewed as the equivalent infinite bus supply coming from Transmission. System characteristic data behind this point is beyond the scope of the distribution expansion problem and therefore not considered. It is further assumed that this node is fully capable of all loading and voltage requirements for the entire, fully expanded area.

Substation Nodes

These nodes represent substations, usually without any local demands, and are modeled as transshipment nodes. Substation nodes are fed only from node 1 (source node). As discussed previously, the set of candidate locations for the future substation nodes are assumed known. The optimal sub set of this candidate location set and it's time chronology for development however, is determined by the proposed algorithm. (See Fig. 1)

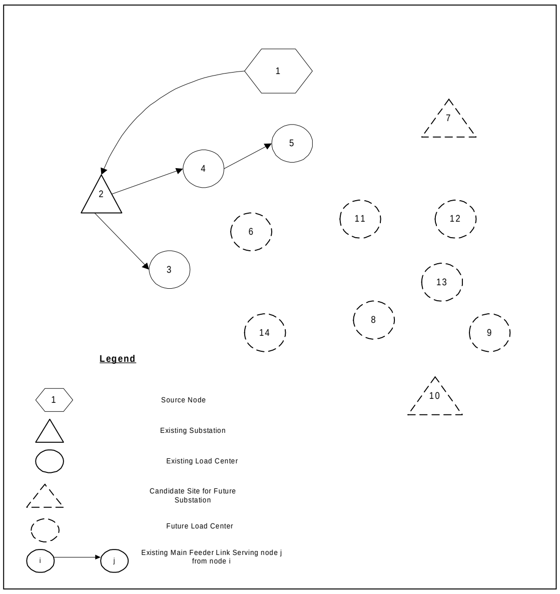

Fig. 1- Existing and future load centers and substations

Local Demand Nodes (Load Center Nodes): These nodes have local demands. The local area demand is usually distributed along one of the main lateral sections in a typical distribution feeder. These laterals are sometimes referred to as the local loops. As shown by Fig. 2, a local loop has the possibility of being fed from two different demand nodes, preferably (for reliability reasons) from different feeders.

Aside from the switches at either end, the local loop usually has a normally closed switch located approximately at it's mid section. The middle switch is operated (either manually or remotely via SCADA) to sectionalize the faulted line section during emergencies. One of the switches, at either end, is normally closed, thus supplying the local loop demand while the other switch is normally open to run the system in a radial configuration. The normally open switch is considered as the alternate feed switch for the loop. The protective equipment at either end should be set to detect faults for the entire loop. Load served by a local loop can be between 80-150 amperes, as the smaller loops are not economical and the protective equipment settings for larger ones will usually have coordination problems with the substation main feeder breaker ground relay.

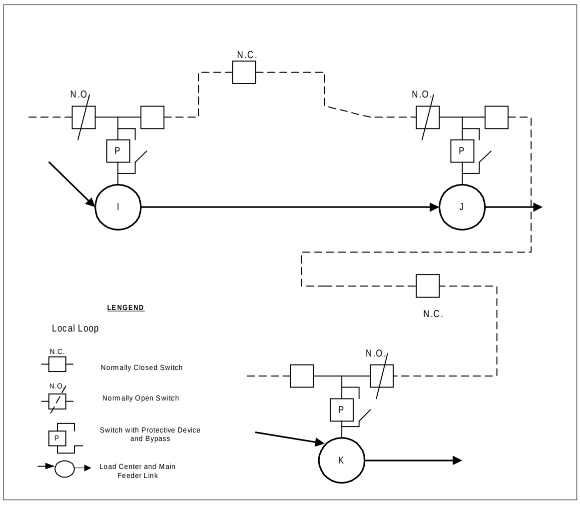

Fig.2_Local Loops supplied by nodes i and j

Unity Power Factor for Substations and Main Feeders:

It is also assumed that power factor corrections (e.g. capacitor placements) for the local area demand is done in the local loop itself. That is, in so far as the main feeder link is concerned, the local demand at each load center node is at unity power factor. This assumption should not only be viewed as a simplifying factor for the formulation, but also as a legitimately imposed design criterion. This follows since the feeder losses are at minimum, or nearly so, under this operating condition. Therefore, a design based on this criterion, inherently minimizes the line losses for any expansion plan and need not directly consider losses.

Load Center Demands:
Locations of the load centers and their diversified peak demands are assumed known to certainty for the first stage. At each subsequent stage, the demands are assumed known but with some degree of uncertainty. The uncertainty in loading information for the middle stages is typically higher than the initial and the final stages. For the first stage (present and the immediate future planning cycle in the proposed formulation), the firm load growth is known. Firm load is a term used by some utilities and refers to a future load for which the customer has already requested service. Horizon stage loading information can also be approximated based on the composition of the types of loads (industrial, commercial, and residential), geographical boundaries, and comparative analyses of other fully developed areas. Of course, there is an uncertainty associated with the time it takes for full development for an area.

Noting the uncertainties, and in an effort to be aligned with the industry practices, a three stage algorithm is proposed and developed for the multi stage formulation. The first stage will include the past (inclusive of all existing facilities) and the immediate highly certain future. The second stage is the stage with highest degree uncertainties and user defined variable time period. The last stage serves as a target plan for the ultimate development. This approach is intended to provide possibilities for inclusion of various uncertainties in future formulations (especially for the middle stage). It is also proposed that the loading information be updated and the algorithm executed every planning cycle as the development progresses similar to the planning practices of the industry

Main Feeder Link
This is a section of the main distribution feeder connecting any two nodes. All feeder links are modeled as two terminal lines without any distributed loading. As discussed earlier all loads are distributed along the paths of the local loops. Feeder links connecting the source node to the substation nodes are fictitious links having zero lengths to represent the substation transformers. A user defined variable number of routing options is considered between any two load center nodes, and for each routing option, a user defined variable number of size options will be considered. The fictitious links representing substation transformers will have one routing option but multiple size options to represent different transformer sizes in graduation of the substation to it's ultimate design capacity. The choice in determination of the link possibilities is left to the discretion of the designer which would normally be the planning engineer or the manager. Fig 3 shows an example in which three routing options (say overhead pole line, underground, and streamline overhead), and two conductor sizes (per routing option) for a total of six alternatives have been considered for the feeder link between nodes *i* and *j*. Flexibility of having user defined choices in routing and size options is necessary because in practice, not all the links will have the same number of alternatives. For example, it is generally considered bad practice to use a small conductor size for the links emanating from the substation.

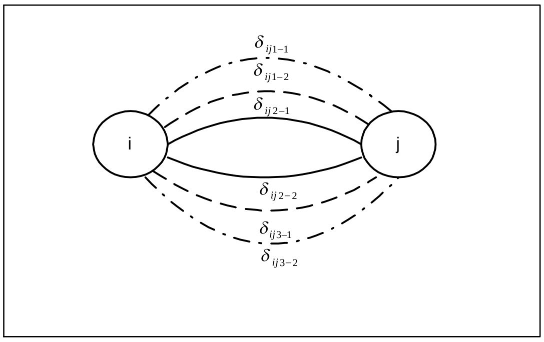

Fig. 3 – Multiple routing and size options of link ij

In this formulation, because of considering multiple link possibilities, the number of feeder link possibilities grow rapidly and become the limiting factor rather than the number of nodes. Before proceeding to a more detailed formulation, it is necessary to further analyze the following:

- The nature of Fixed and Variable Costs.
- Expenditures and Budgets.
- Optimization vs. Engineering Economic analysis.
- Analysis of the Losses.

Fixed Costs

Fixed cost, or the zero order cost as defied by [4], refers to a one time expenditure for installation of any equipment. It contains the cost of material, transportation, and labor for the installation and commissioning of the facility. Fixed costs are independent of loading and therefore should not be modeled as functions of power flows. Fixed costs are also the dominant costs of the expansion project and are usually paid in installments over the future years.

Variable Costs

Variable Costs in general refers to costs that are functions of loading such as the costs associated with production and transport of energy. Although the operating costs are generally considered as variable costs, not all operating costs are functions of loading. For those that may be modeled as functions of loading, they differ radically as functions of types of loading. Many researchers, such as [5 – 7] have modeled the operating and maintenance (O&M) costs as linear or quadratic functions of loading without distinction to load type. Before expressing the types of variable costs that should be considered, it is important to discuss budget and the standard engineering economic studies that are normally performed during planning analyses.

Expenditures and Budgets:

The majority of the distribution systems have historically been owned and operated by regulated utilities. Despite deregulation efforts, local distribution systems are likely to continue to be a monopoly in their local service areas, and therefore subject to some regulations for their rates and operation. Expenditures in regulated utilities, generally fall under two distinct categories: First, expense, and second capital. O&M costs for each year are paid out of the expense budget, which comes out of the revenue as they occur. This expenditure is fully considered by the regulatory commission and taken into account when the utility is applying for a new rate case. The capital budget, which refers to capital investments, on the other hand, is only partially considered in the rate case calculations. The distinction is necessary to ensure that the present customers are not charged for the plants and investments that will be used predominantly by the future customers. All expenses must be paid in the year they occur, investments may be paid over a number of years. Levelized carrying charges for the new investments are usually calculated and paid over the life of the plant (customarily 30 years).

Optimization vs. Engineering Economic analysis

An engineering economic study is usually conducted to help decide the most economical plan for expansion. For this purpose, several equivalent alternatives are considered and studied by the designer. An engineering economic analysis will then determine the levelized carrying charges for each plan. The plan with the lowest carrying charge is then chosen as the most economical alternative. The key point for validity of this analysis is the fact that the alternatives studied must be equivalent. Equivalency may be construed that, for example, all plans will install (or release) the same capacity to the system at a particular future point in time.

An optimization study, with the same objective (as the engineering economics study), on the other hand, seeks the plan with minimum cost among all possible plans. There is no equivalency restriction for this optimization algorithm. As long as the stated constraints are satisfied, the plan is considered a feasible one. The feasible plan with minimum cost is the solution regardless of it's equivalency to other feasible plans. Another fundamental difference is that in an engineering economic study, all constraints are assumed satisfied. Based on the foregoing argument, in contrast to [8], we suggest that the levelized carrying charges should not be used in an optimization study in the same manner as used in an engineering economic study.

Instead, the *inflation adjusted present worth costs* will be used for both fixed and variable costs calculated based on [9] using the following formula

$$P.W.C_{(n)} = \left[\frac{1+i_f}{1+i}\right]^n \qquad (6)$$

where;

$P.W.C_{(n)}$ is the inflation adjusted present worth cost of the facility installed in year n

C is the current cost of the facility

n is the number of years to installation of the facility with current cost C

i_f is the inflation rate (assumed 5% in our study)

i is the fixed charge rate (assumed 14% in our study)

The validity of this approach comes form the fact that the treatment is the same for every expenditure incurred in every stage. Therefore, this eliminates the need to treat the investment costs near the end of the study period differently as indicated by [8]. It should be noted that in general, designers are always searching for alternatives that differ more of the capital costs to the future, as these are usually the more economical plans. Similarly, the foregoing approach in cost modeling, inherently searches for the same alternatives. It is further suggested that the only variable cost that need be considered is the total cost of energy losses in the distribution system alone. This is because all other O&M costs (load type dependent or otherwise), similar to fuel, production, and transport costs, are common among all alternatives. Excluding all other costs except the energy losses, the variable costs become very small compared to the fixed costs.

Analysis of the Losses.

Distribution losses are quadratic functions of the power flows. Therefore the nonlinear term of the objective function which, is solely due to the system energy losses is a quadratic function of the power flows. We contend that the dominant term of the objective function is linear, and neglecting the nonlinear term will not impact the solution accuracy.

Investigation of peak load power losses on distribution feeders of a major California utility revealed that for a typical urban feeder, the peak losses are in the order of 1%- 3% of the peak load, and for a typical rural feeder, it is between 2% –4%. Loss calculations conducted independently by D.I. Sun in [10], were found to be in the same order during peak loading conditions. Sun's report also indicated that during minimum loading conditions, more than 70% of the losses were attributed to transformer core losses. This part of losses is constant and common for all alternatives, which means the nonlinear portion of the objective function could even be reduced further. It should be kept in mind that the few percent energy losses discussed here, are only a measure as compared to the total energy consumption of the distribution system that does not include the fixed capital costs. Obviously, the losses measure even smaller as compared to the combined costs.

Proposed modeling and formulation:

We propose a directed graph minimum edge cost network flow modeling for this problem. The directionality choice, reduces the number of flow variables in the objective function (1) which reduces to:

$$Min\ \ C = \sum_{t=1}^{n} \{\ C_{f,t}^{T}\delta_t + \tfrac{1}{2}[\ X_t^T Q X_t\ \ ]\ \} \qquad (7)$$

where;

$C_{f,t}\ \ \in R^n$ is the vector of fixed costs for each link (feeder or substation) in stage *t*

$\delta_{S,t}\ \in \{0,1\}^n$ is the vector of decisions for each link at stage *t*

Note further that Q is a diagonal matrix in this formulation thus allowing a completely decoupled expansion of the matrix equation (7). The expanded version of (7) will then be;

$$Min : C = \sum_{t=1}^{T} \left\{ \sum_{ij \in Lposs} C_{f,ij,t}\delta_{ij,t} + \sum_{ij \in Lposs} C_{v,ij,t} X_{ij,t}^{2} \right\} \qquad (8)$$

where;

$C_{f,ij,t}$ is the fixed cost of link *ij* at stage *t.*

$C_{v,ij,t}$ is the variable cost coefficient of link *ij* at stage *t*

Lposs is the set of all link possibilities including substation transformers.

$X_{,ij,t}$ is the diversified peak power flow in the link *ij* at stage *t.*

All other variables are as defined earlier.

For calculation purposes, the present worth/unit length costs multiplied by the length for each option was used to find fixed costs $C_{f,ij,t}$. For the variable cost coefficients $C_{v,ij,t}$, the present total cost of energy was used based on the following formula.

$$C_{v,\,ij,\,t} = \frac{(C_{e,\,t})(r_{ij})(l_{ij})(LLf\,)(8760)(10^3)}{KV_{LL}^{2}} \qquad (9)$$

where;

$C_{e,t}$ is the present value of the total cost of energy incurred at stage *t.*

r_{ij} is the resistance of the conductor in ohms/mile for the link *ij*

l_{ij} is the length of the conductor for the link *ij* in miles

LLf is the loss load factor (assumed 15%) [11]

KV_{LL} is the feeder Line-Line operating voltage in KV

8760 is the number of hours at stage *t*, (one year)

Now Q is a diagonal matrix with all positive nonzero elements ($C_{v,ij,t}$), hence positive definite. Therefore, the nonlinear term of the objective function in Equation (7) can be written as a summation of completely decoupled quadratic functions of the link flows as in equation (8) also clarifying the one to one mapping of $C_{v,ij,t}$ to elements of Q . Furthermore, the upper and lower bounds on the influence of the losses can be easily determined because the eigen values of Q are readily available.

Test Case

A simple test case of Fig.4 consisting of one substation and two load-centers, was studied base on the foregoing formulation. Data on routing / size options , and other characteristic data for the system links have been given in table1.The load growth assumptions are shown in table 2. Unit costs for fixed and variable expenditures have been provided in table 3.

Two, five year, three stage, single criterion optimization algorithm details of which are differed for future publications, were implemented using a commercial grade optimization package. Both algorithms have been developed for a single mathematical program. General description of the algorithms and the solutions are discussed in the following.

The first algorithm, although considers multiple routing and size, has no capability for upgrades. That is, once a system link has been installed, it cannot be changed in the future stages. The second algorithm allows upgrades for the system as it advances through the stages. That is, a system link may initially be installed (or most likely it is existing) having a lower grade routing or size, and later upgraded to a higher capability. As mentioned earlier, this is commonly done in the industry, and it is a crucial point for practical system planning. Reconductoring, undergrounding, cut-over to higher voltages are some of the common examples.

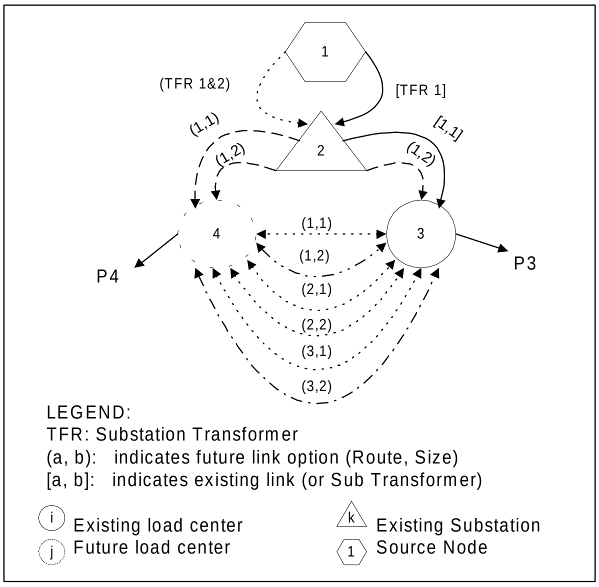

Fig. 4 – Test case configuration

From	To	TYPE	Route	Size	Link Type/Size	r	l	CAP
1	2	TFR 1	1	1	3 Phase 12/14 MVA	0.2	1	14
1	2	TFR 2	1	2	3 Phase 12/14 MVA	0.4	1	28
2	3	STR	1	1	954ACSR	0.0982	5	27
2	3	UG	2	1	1000EPRPVC(AL)	0.1019	7	25.5
2	4	OTH	1	1	1113.5AL	0.0966	4	29.8
2	4	UG	1	2	1000AL	0.1019	6	25.5
3	4	OH	1	1	715.5AL	0.1468	6	23.5
3	4	OH	1	2	1113.5AL	0.0966	6	29.8
3	4	STR	2	1	666ACSR	0.142	6.7	21
3	4	STR	2	2	954ACSR	0.0982	6.7	27
3	4	CIC	3	1	700XLP PVC CONC	0.1457	7.5	21
3	4	UG	3	2	1000AL	0.1019	7.5	25.5

KEY:

From, To	: Link terminals	CIC:	Cable in Conduit
OH	: Overhead	CV :	Variable Cost $ / MVA / mile
UG	: Underground	l :	Length in Miles
STR	: Streamline	r :	Line resistance in Ohms / Mile
OTH	: Other Construction	CAP:	Max. Capacity (MVA)

Table 1- Physical and characteristic data for possible links

Load Center	Stage	T=1	T=2	T=3
		MW	MW	MW
3		6	8	10
4		0	5	8

Table 2- Stage – Demand data

Link From	To	Option Rout-Size	Var. Cost $/MVA	Fixed Cost $ x10E6
1	2	1-1	15	0
1	2	1-2	15	0.84
2	3	1-1	3	0
2	3	2-1	4	6.8376
2	4	1-1	2	0.887
2	4	2-1	4	5.8608
3	4	1-1	3	1.2672
3	4	1-2	3	1.3306
3	4	2-1	3	1.8396
3	4	2-2	3	1.9103
3	4	3-1	4	7.128
3	4	3-2	4	7.326

Table 3- Fixed and variable costs for the links

Table 4 is the solution for the case without upgrades. Table 5 gives the solution for the case that considers upgrades. The present worth of the total costs for the entire planning period in the case without upgrades is $\$2.1934\text{x}10^6$ and the same costs for the other case is $\$1.3208\text{x}10^6$. Note that capacity of the existing link1-2 (Transformer 1) with no additional cost is adequate for stages one and two, but not for stage three. The optimization program in the first case not have the upgrade capability in any stage, reluctantly chooses the higher capacity link in stage one. The program with the upgrade capability on the other hand, correctly utilizes the capability of the existing link for the first two stages, and then calls for an upgrade for this link in stage three when truly needed. This means, exactly as done in practice, the program inherently postpones capital expenditure and maximizes asset utilization of existing facilities as long as possible. Considering the number of existing facilities and what become existing facilities in the future stages, the significance of asset utilization becomes more apparent.

	From	To	Select. Option	Flow (MW)	Volts @ end
Stage 1	1	2	1-2	6	126.0
	2	3	1-1	6	125.1
	2	4			
	3	4			
Stage 2	1	2	1-2	13	126.0
	2	3	1-1	8	124.8
	2	4	1-1	5	125.4
	3	4			
Stage 3	1	2	1-2	18	126.0
	2	3	1-1	10	124.5
	2	4	1-1	8	125.04
	3	4			

Table 4 – Solution without upgrade possibility

Another point aligned with our earlier conjecture was about implementation of explicit voltage constraints without allowing multiple routing and size capability. Knowing the solution, we limited the options for the link 2-4 to only one size with high enough conductor resistance to violate the voltage constraints. It was noted that the program chose the longer, more expensive link 3-4 instead of the optimal route (link 2-4).

	From	To	Select. Option	Flow (MW)	Volts @ end
Stage 1	1	2	1-1	6	126.0
	2	3	1-1	6	125.1
	2	4			
	3	4			
Stage 2	1	2	1-1	13	126.0
	2	3	1-1	8	124.8
	2	4	1-1	5	125.4
	3	4			
Stage 3	1	2	1-2	18	126.0
	2	3	1-1	10	124.5
	2	4	1-1	8	125.04
	3	4			

Table 5 – Solution allowing upgrade possibility

This signifies the point that implementation of voltage constraints without multiple size possibility, indeed render the solution sub optimal.

Concluding remarks:

A directed graph, minimum edge cost network flow modeling in a three stage formulation is proposed for the distribution expansion problem. It is further proposed that a variable, multiple routing and size options need be considered. This is a significant factor when implementing explicit voltage constraints. Investigations of a simple test case indicate that inclusion of voltage constraints without consideration of multiple routing and size options render the solution sub optimal. It is also proposed that, the challenging but crucial upgrade possibility need be considered for the problem to be practical. This issue is also vital for maximum asset utilization of the existing facilities, optimality of the solution, and is inherently aligned with industry practices and training. Design criteria and assumptions should be as closely aligned with the industry practices as well.

The only variable cost that can influence the solution and need be considered, is the energy losses. Although neglecting the losses as done by [12], or piecewise / stepwise linearization techniques can provide adequate solutions, further investigation is needed for a more accurate, more efficient modeling.

It is proposed that reliability, social/environmental impacts, and other objectives be considered as separate objectives and not integrated in single objective formulations. It is proposed that initially a detailed, single objective, truly multistage mathematical programming formulation be developed and tested prior to multi objective formulations.

REFERENCES:

[1] M. R. Garey, D. S. Johnson, “Computers and Intractability: A Guide to the Theory of NP-Completeness”, *W. H. Freeman and Company*, New York, 1979.

[2] F. Soudi, K. Tomsovic, “Optimized Distribution Protection Using Binary Programming”, *IEEE Transactions on Power Delivery*, Vol. 13, No. 1, 1998, pp. 218-224.

[3] F. Soudi, K. Tomsovic, “Optimal Distribution Protection Design: Quality of Solution and Computational Analysis”, *Electrical Power & Energy Systems*, Vol. 21, 1999, pp. 327-335.

[4] Y. Tang, “Power Distribution System Planning with Reliability Modeling and Optimization”, *IEEE Transactions on Power System*, Vol. 11, No. 1, February 1996, pp. 181-189.

[5] S. Jonnavithula, R. Billinton, “Minimum Cost Analysis of Feeder Routing in Distribution System Planning”, *IEEE Transactions on Power Delivery*, Vol. 11, No. 4, October 1996, pp. 1935-1940.

[6] O. M. Mikic, “Mathematical Dynamic Model for Long-Term Distribution System Planning”, *IEEE Transactions on Power Systems*, Vol. 1, No. 1, February 1986, pp. 34-40.

[7] M. A. El-Kady, “Computer-Aided Planning of Distribution Substation and Primary Feeders”, *IEEE Transactions on Power Apparatus and Systems*, Vol. 103, No. 6, June 1984, pp. 1183-1189.

[8] S. E. Shelton, A. A. Mahmoud, “A Direct Optimization Approach to Distribution Substation Expansion”, *IEEE Proceedings*, Paper No. A 78 592-8, Los Angeles, CA, July 1978, pp. 1-9.

[9] T. Gönen, “Engineering Economy For Engineering Managers With Computer Applications”, *J. Wiley and Sons*, 1990, pp. 288.

[10] D. I. Sun, S. Abe, R. R. Shoults, M. S. Chen, P. Kichenberger, and D. Farris, “Calculation of Energy Losses in a Distribution Sytem”, *IEEE Transactions on Power Apparatus and Systems*, Vol. 99, No. 4, July/August 1980, pp. 1347-1356.

[11] T. Gönen, “Electric Power Distribution System Engineering”, *McGraw-Hill*, 1986, pp. 46.

[12] N. Kagan, R. N. Adams, “A Benders’ Decomposition Approach To The Multi-Objective Distribution Planning Problem”, *International Journal of Electrical Power & Energy Systems*, Vol. 15, No. 5, October 1993, pp. 259-271.