Distribution Expansion Problem:
Formulation and Practicality for a Multistage Globally Optimal Solution

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Abstract: Despite numerous research efforts of the past 40 years in the area of distribution expansion, a clear definition of the problem, and a truly multistage formulation that addresses practical concerns is yet to be developed. In this paper, the problem is clearly defined and analyzed from a practical point of view. A directed graph minimum edge cost network flow modeling of the problem for a truly multistage formulation using mathematical programming that guarantees global optimality and addresses the noted deficiencies is proposed. The proposed formulation is implemented on small case studies under varying assumptions. Comparative analysis indicates the importance of improved expansion planning.

Keywords: Power distribution planning, distribution expansion, multistage planning, mathematical programming.

I. INTRODUCTION

The distribution expansion is a nonlinear, NP-Complete practical problem, with a 40-year history of continued efforts and contributions for improved solutions. The first publication in this area has been attributed to [1] in 1960. Various heuristic and mathematical techniques have been proposed for this problem. Detailed categorical analyses of the previous contributions identifying the significant shortcomings, have been presented in [2-3].

While growth of electric demand has recently slowed through efforts in the area of energy management, the need for a continued expansion seems inevitable in the foreseeable future. This general description of the expansion problems is somewhat independent of many other issues facing the both the suppliers and the consumers of electrical energy. For example, the deregulation concept, as an attempt to promote competition by giving more choices to the consumers, although will impact the supplies' planning strategies, it cannot however, contain or limit the growth in demand or the system expansion in the global sense.

The overall approach of previous research to distribution system expansion may be divided into two distinct categories, single stage, and multistage. Single stage refers to the case where the full expansion requirements for the area are determined in one period. Multistage on the other hand refers to expansion of the system in successive plans over several stages, representing the natural course of progression.

Multistage approach, due to the interdependency between stages, is challenging to formulate but the solution offers a more useful result. Still, the majority of the development has addressed the problem by a single stage approach. Here, the primary focus will be the relatively few multistage approaches to the expansion problem [4-15].

Adams and Laughton [12] first proposed a decomposition method for a multistage formulation, but this was not fully implemented. Gönen and Foot [5] published the first fully implemented multistage formulation. The approach was limited to small size problems due to the inclusion of many decision variables that have in fact been standardized by the utilities. A more general MILP formulation of [5] was later developed by Gönen and Remirez to include a complex set of voltage drop constraints, but not reliability [7].

Sun [6], and later Remirez and Gönen [8], introduced a technique that can be best described as a pseudo dynamic algorithm. In this type of algorithm, the horizon stage is considered first using a static model identifying the set of expansion elements to be used for the entire planing period. Then a series of concatenated single stage algorithms are employed for the intermediate stages. The algorithms for the intermediate stages are so formulated to only select elements form the solution set provided by the initial horizon stage algorithm. No voltage drop or reliability concerns were addressed. Neither offers a true multistage solution and more importantly, the dynamic change process of the objective function within the linear program, was not presented in [5, 10, or 7]. Similarly in [4, 9, and 10], the approaches comprised a set of static algorithms used by different parent algorithms to achieve the multistage objectives.

El-Kady, presented a sparsity based MILP formulation in [11]. Explicit voltage drop constraints require lineariztion and use of a Step-Wise feeder Flow-Impedance characteristic requiring a significant number of additional integer variables. Blanchard et al. in [15] proposed a multistage heuristic method in five phases. The model is quadratic mixed integer programming, and uses a solution strategy based on the pseudo dynamic algorithms used in [6], and [8]. No existing facilities, voltage drop, or reliability was considered. The authors emphasized acceptability of a near optimal compromise solution in the interest of the CPU calculation time similar to the arguments found in the evolutionary technique approaches analyzed in [2].

A fundamental question may be whether or not these multistage approaches can find the global optimum. The answer for the heuristic techniques is uncertain as discussed in [2]. As for the decomposition techniques, the question can only be answered with certainty if an approach that guarantees global optimality is developed and the solutions compared. This is rarely practical.

Due to the recent increased demand for reliable service, efforts have been made to include reliability in formulations. Several studies addressed the impacts of unreliable service, and attempted to quantify the cost of reliability as seen by the suppliers as well as their customers [4, 16, 17].

Although the efforts have better illuminated the significance of reliability and have quantified some of the penalties incurred, the cost of reliability still remains a crude estimate. Furthermore, comparison of this approximate cost against the fairly precise installation and maintenance costs of the switching and the protection apparatus is far from accurate for the following specific reasons.

Firstly, the reliability cost is not only an estimate, but also, only considers one .component of the service reliability, namely the service continuity. Even at that, as noted by some others [18], more accurate determination requires more accurate data from both the supplier and the customer, which is presently unavailable.

Secondly, there is more than one component to the service reliability than say the service continuity. At least one other component of reliability can easily be attributed to service quality. For example, an extended low voltage condition could bear much higher costs for both the suppliers and the consumers. Again, the cost assignments for the service quality facet of reliability, similar to the one for service continuity can only be a rough approximation at this time although there may be more ongoing sophisticated analyses such as the one suggested by [19].

Based on the above rational, inclusion of any component of reliability, such as service continuity, should be formulated as a separate objective. Except for [10, 18] whom first proposed inclusion of reliability into a multi-objective problem, other research similar to [4, 20] have integrated the costs of reliability along with the fixed and variable costs of the expansion alternatives into a single criterion optimization model.

A. Summary of the significant shortcomings

1) The literature reveals some confusion about the practical distribution expansion objectives.
2) Voltage constraints are completely ignored in some cases and inappropriately applied in some others.
3) Reliability is either ignored, or incorrectly quantified and integrated with other costs.
4) Budgetary constraints, which are faced by all utilities, are missing from most formulations.
5) Variable routing and conductor size options between nodes have not been properly addressed.
6) A true multistage approach that guarantees global optimality has not yet been developed.
7) Commonly considered upgrade possibilities have been completely ignored.

II. PROBLEM DEFINITION

The primary goal of the expansion problem is to timely serve the load growth safely, reliably, and economically. Here, it is assumed that safety considerations have already been translated into a set of operational standards in the design stage. Reliability and economics on the other hand, may be formulated as objectives for optimization programs. First, a single criterion optimization is developed to minimize the total fixed and variable costs at all stages ensuring that;

- every demand center j is served for all stages,
- voltages are within guidelines at every node j for all stages,
- all elements operate within their capabilities and operational constraints,
- all expenditure is within the budget for every stage.

A general mathematical representation of the above is

$$Min\ C=\sum_{t=1}^{T}\left\{\sum_{S\in Stations}Cf_{S,t}+\sum_{S\in Stations}Cv_{S,t}+\sum_{F\in feeders}Cf_{F,t}+\sum_{F\in feeders}Cv_{F,t}\right\} \tag{1}$$

$$Subject\ to:\ \sum Xij,t-\sum Xjk,t=Pj,t\quad \forall\, j\in \text{Load Centers, } ij \text{ and } jk\in \text{Feeders} \tag{2}$$

$$V^{Min}\le Vj,t\le V^{Max}\quad \forall\, j\in \text{Load Centers} \tag{3}$$

$$S_{i,t}\le S_i^{Max}\quad \forall i\in \text{Stations} \tag{4a}$$

$$X_{ij,t}\le X_{ij,t}^{Max}\ \forall\, ij\in \text{Feeder Links} \tag{4b}$$

$$\sum_{S\in Stations}Cf_{S,t}+\sum_{S\in Stations}Cv_{S,t}+\sum_{F\in Feeders}Cf_{F,t}+\sum_{F\in Feeders}Cv_{F,t}\le Bt.\quad \forall\, t=1,2,\ldots,T \tag{5}$$

where T is the number of stages to full expansion
t is each stage of the T stage process
$X_{ij,t}$ is the directional complex power flow from node i to node j at stage t
$X_{jk,t}$ is the directional complex power flow from node j to node k at stage t
$P_{j,t}$ is the diversified peak demand of load center (node) j at stage t
$Cf_{S,t}$ is the fixed cost of substation S to be installed at stage t
$Cv_{S,t}$ is the variable cost of substation S to be incured at stage t
$Cf_{F,t}$ is the fixed cost of feeder F to be installed at stage t
$Cv_{F,t}$ is the variable cost of feeder F to be incured at stage t
$V_{j,t}$ is the voltage at node j at stage t
V^{Min}, V^{Max} are the lower & upper bounds of acceptable voltage
$S_{i,t}$, S_i^{Max} are loading of substation S at stage t and Maximum Capability respectivdy
$X_{ij,t}$, $X_{ij,t}^{Max}$ are the flow in the link ij at stage t and and Maximum Capability respectivdy
B_t is the expansion budget amount for stage t

The value of C to be minimized in (1) is the total cost for the expansion over all the stages. Constraints (2-5) include both physical and performance conditions. Constraint (2) is the well known Kirichof's Current Law (KCL) applied to every node. Constraint (3) sets explicit voltage limits for all the load centers. Constraints (4a) and (4b) ensure that all substation transformers and feeders are loaded within their capabilities, and all other operational conditions are within limits. Finally, constraint (5) is a budgetary constraint so that the expansion costs at each stage are within the budgeted amount that has been generally neglected in all previous formulations.

Introducing the necessary decision variables, separating the linear and the nonlinear terms, and assuming for now (this will be shown later) that all variable costs may be modeled as quadratic functions of power flows, a matrix form representation of the problem may be formulated as shown below.

$$Min\ C = \sum_{t=1}^{T} \left\{ C_{fS,t}{}^{T} \delta_{S,t} + C_{fF,t}{}^{T} \delta_{F,t} + \tfrac{1}{2}\left[X_{S,t}^{T} Q_S X_{S,t} + X_{F,t}^{T} Q_F X_{F,t} \right] \right\} \quad (6)$$

$s.t.:$

$$A_j X_t = P_{j,t} \qquad X_t = [X_{S,t}\ X_{F,t}]^T \quad (7)$$

$$V^{Max} \le V_{j,t} \le V^{Max} \quad (8)$$

$$X_t \le b_t \quad (9)$$

$$C_{fS,t}^{T} \delta_{S,t} + C_{fF,t}^{T} \delta_{F,t} + \tfrac{1}{2}\left[X_{S,t}^{T} Q_S X_{S,t} + X_{F,t}^{T} Q_F X_{F,t} \right] \le B_t \qquad \forall t \quad (10)$$

where

$C \in R$ is the total cost for the ultimate system expansion
$t \in Z$ is the stage number of the multistage study
$m \in Z$ is the total number of nodes
$n \in Z$ is the total number of feeders and the substations
$n_S \in Z$ is the number of Substations
$B_t \in R$ is the expansion budget for stage t
$X_{F,t} \in R^{(n-ns)}$ is the vector of feeder power flows
$X_{S,t} \in R^{ns}$ is the vector of substation loads
$X_t \in R^{n}$ is the feeder/substation loading vector
$\delta_{F,t} \in \{0,1\}^{(n-ns)}$ is the vector of feeder decision variables
$\delta_{S,t} \in \{0,1\}^{ns}$ is the vector of substation decision variables
$V_{j,t} \in R^{m}$ is the vector of node voltages
$V^{Min}, V^{Max} \in R^{m}$ are the voltage bound vectors
$C_{fS,t} \in R^{ns}$ are the vectors of fixed substation costs
$P_{j,t} \in R^{m}$ is the vector of Load Center demands
$C_{fF,t} \in R^{(n-ns)}$ is the vectors of fixed feeder costs
$b_{,t} \in R^{n}$ is the capacity bound vector at stage t
$Q_S \in R^{ns \times ns}$ is the loss cost matrix for the substations
$Q_F \in R^{(n-ns)\times(n-ns)}$ is the loss cost matrix for the feeder links
$A_j \in R^{m \times n}$ is the system node to branch incidence matrix
R, Z are sets of real and integer numbers respectively.

All other variables are as defined earlier. Note, the variable costs $C_{vF,t}$ and $C_{vS,t}$ have been mapped in to elements of Q_F and Q_S respectively. The nature of this mapping will be clarified later.

The above problem is a nonlinear and mixed integer optimization problem. Mixed integer problems in general, and specifically this problem, computationally belong to the class NP complete. NP completeness refers to a class of problems for which algorithmically, the computational complexity of the solution searches grows exponentially (non polynomially) with some parameter [21].

III. DESIGN CRITERIA AND ASSUMPTIONS

Detailed analysis and the rational for the following stated design criteria and assumptions have been provided in [3]

1) Candidate Substation locations, all or a subset of which are to be developed, are assumed known. (Fig. 1)
2) Location and loading data for existing and future load centers are available, and the load is distributed on main laterals known as local loops.(Fig. 2)
3) All local loops are self contained in their protective and power factor correction equipment and viewed as a unity power factor load at the inter connection point to the main feeder.
4) Any feeder link between two nodes may have a multiple routing options, and multiple size options may be considered for each routing. (Fig.3)
5) Unit costs for every option, and the characteristic data for all equipment are known.
6) Inflation adjusted present worth costs as defined in [3] will be used for both fixed and variable costs.

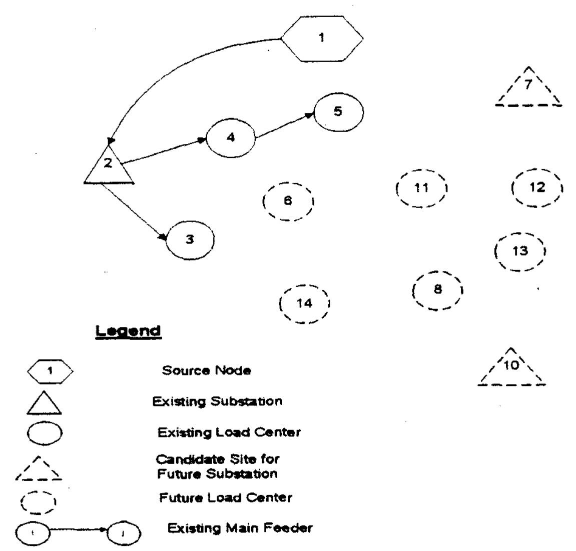

Fig.1 Existing and future substations and load centers

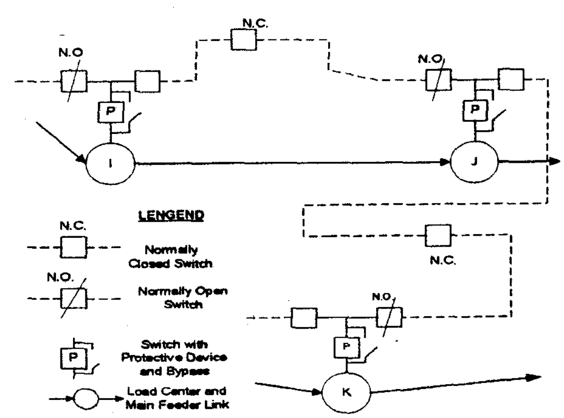

Fig.2_Local Loops supplied by nodes i and j

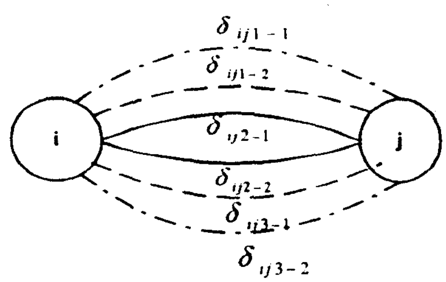

Fig 3 Routing and size options

IV. ENERGY LOSSES

A quantitative measure of losses was presented in [3]. Here, to better understand the effects of the losses as compared to the fixed costs, in a parallel study, the upper and the lower bounds for the ratio of the quadratic to linear terms of the distribution expansion MIQP by considering the following quadratic mixed integer objective function:

$$MinZ = c^T \delta + 1/2 X^T QX \qquad (11)$$

where $X = [X_1, X_2, ..., X_n]^T \in R^n$ is a vector of continuous variables and $\delta = [\delta_1, \delta_2, ..., \delta_n]^T \in \{0,1\}^n$, $i=1,2,...,n$ is the vector of decision variables. Matrix Q is positive definite and diagonal when the expansion is modeled as a minimum edge cost network flow problem on a directed graph. Therefore, all eigen values q_i of Q are known positive real numbers.

Assume that X and δ are coupled such that $\forall X \in R^n \; \exists$ a corresponding $\delta \in \{0,1\}^n$. Also note that, if $Q \Rightarrow 0$, then Quadratic Program (QP) $\Rightarrow$ Linear Program (LP). Now, defining r as the quadratic dominance ratio of the objective function, we have:

$$r = \frac{\frac{1}{2} X^T QX}{c^T \delta} \qquad (12)$$

By *Rayleigh-Ritz* inequality from Linear System Theory[ ], for any symmetric positive definite Q,

$$\lambda_{Min} X^T X \leq X^T QX \leq \lambda_{Max} X^T X \qquad \forall X \qquad (13)$$

where, λ_{Min} and λ_{Max} are the smallest and the largest eigen values of Q respectively. Therefore, r is bounded above and below by:

$$\frac{\frac{1}{2}\lambda_{Min} X^T X}{c^T \delta} \leq r \leq \frac{\frac{1}{2}\lambda_{Max} X^T X}{c^T \delta} \qquad (14)$$

Now, for a conservative bounding (worst case), one can use maximum limit values of X (conductor capability limits in the distribution expansion problem) for the upper bound and minimum values of X (or zero) for the lower bound.

The study [22] showed that even under conservative assumptions, the dominance ratio evaluated at the QP minimizer is consistently below 10% for the distribution example. The study further showed that for a QP in which the dominance ratio is below 25%, a single approximating LP can be found with the same minimizer as the QP. Therefore the expansion problem may be accurately modeled as a single MILP, without the need for the conventional linearization techniques that proliferate the number of variables.

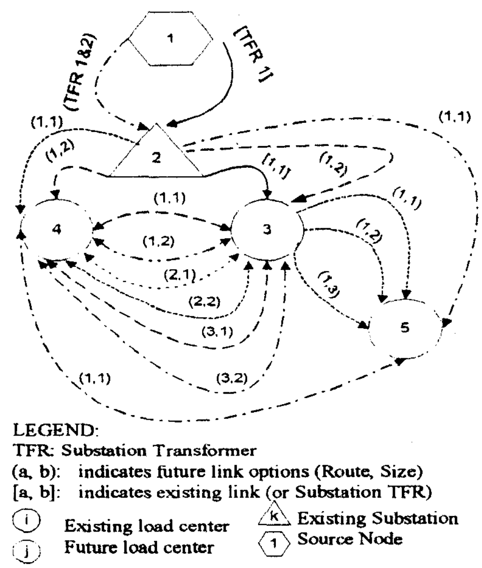

Fig. 4 Test system configuration

V. MODELING AND FORMULATION

Directed graph minimum edge cost network flow modeling is proposed for this problem. The directionality choice, reduces the number of flow variables in the objective function (1) to:

$$Min\ C = \sum_{t=1}^{n} \{ C_{f,t}^T \delta_t + \tfrac{1}{2}[X_t^T Q X_t] \} \qquad (15)$$

where

$C_{f,t} \in R^n$ is the vector of fixed costs in stage t

$\delta_{S,t} \in \{0,1\}^n$ is the vector of decisions at stage t

Note further with a diagonal Q, a completely decoupled expansion of (15) is

$$Min: C = \sum_{t=1}^{T} \left\{ \sum_{ij \in Lposs} C_{f,ij,t} \delta_{ij,t} + \sum_{ij \in Lposs} C_{v,ij,t} X_{ij,t}^2 \right\} \qquad (16)$$

where

$C_{f,ij,t}$ is the fixed cost of link ij at stage t.
$C_{v,ij,t}$ is the variable cost coefficient of link ij at stage t
$Lposs$ is the set of all link possibilities
$X_{,ij,t}$ is the diversified power flow in the link ij at stage t

All other variables are as defined earlier. The MIQP represented in (16) may be accurately formulated as a single MILP details of which has been deferred to future publications.

VI. NUMERICAL RESULTS

A simple test case of Fig. 4 consisting of one existing substation node 2, and one existing load center, node 3, and two future load centers 4 and 5, was studied based on the a linear version of the above formulation. Table 1 provides the load grow data and the link characteristics; unit cost data is given by Table 2.

MW Loads at Center .	stage 1 Pj,1	stage 2 Pj,2	stage 3 Pj,3
1	-6	-13	-20
2	0	0	0
3	6	6	9
4	0	4	6
5	0	3	5

Table 1. Load growth data

i	j	TC	rt	ss	link type/size	r	l	CAP	FC
1	2	TFR#1	1	1	3 Phase 12/14 MVA	0.2	-	14	60
1	2	TFR#2	1	2	3 Phase 12/14 MVA	0.2	-	14	60
2	3	OH	1	1	715.5AL	0.1468	9	23.5	40
2	4	OH	1	1	715.5AL	0.0966	4	29.8	42
2	4	UG	2	1	1000EPRPVC(AL)	0.1019	6	25.5	185
2	5	OH	1	1	715.5AL	0.0966	10	29.8	42
3	4	STR	1	1	4/0 ACSR	0.445	6.7	11	50
3	4	STR	1	2	666ACSR	0.142	7	21	52
3	4	STR	1	3	954ACSR	0.0982	7	27	54
3	4	UG	2	1	4/0 XLPPVC	0.472	7.5	10.8	175
3	4	UG	2	2	700XLPPVC	0.1457	7.5	21	180
3	4	UG	2	3	1000EPRPVC(AL)	0.1019	7.5	25.5	185
3	5	OH	1	1	4/0 AL	0.4873	4	11	38
3	5	OH	1	2	715.5AL	0.1468	4	23.5	40
3	5	OH	1	3	715.5AL	0.0966	4	29.8	42
4	3				SAME AS 3-4				
4	5	OH	1	1	715.5AL	0.0966	12	29.8	42

Key :
i :source terminal
j :load terminal
l : Length in Miles
r : Line Resistance in Ohms/Mile
rt :Routing option designation
ss :Link/TFR size designation
CIC : Cable in conduit
OH :Overhead construction
OTH :Other construction
UG :Underground construction
STR :Streamline construction
TFR :Substation transformer
TC :Type construction
FC : Fixed Cost \$ /ft or \$/KVA for TFR
CAP :Link/TFR Capacity in MVA

Table 2. Link characteristic and cost data

Several, 10 year, three stage, optimization algorithms were implemented using commercial software. In all studies, two and eight year periods were considered between the first two stages, and a 30 year period for cumulative loss calculation beyond stage 3. Further, uniform load growths were considered for the first two stages, and it was assumed that the system is fully developed beyond stage 3 with no load growth.

	From	To	Select. Option	Flow(MW)	Volts @ end
Stage 1	1	2	1-2	6	126
	2	3	1-1	6	123.6
Stage 2	1	2	1-2	13	126
	2	3	1-1	9	122.4
	2	4	1-1	4	125.52
	3	5	1-2	3	121.86
Stage 3	1	2	1-2	20	126
	2	3	1-1	14	120.4
	2	4	1-1	6	125.28
	3	5	1-2	5	119.5

Table 3. Optimization solution

The optimal solution for test case 1 is shown in table 3. Note that the optimization program has selected the link 2-4 to supply load center 4 and the link 3-5 to supply load center 5 as the optimal solutions.

To verify the importance of having multiple size gradation in conjunction with existence of explicit voltage constraints, two additional studies were conducted in which only a single size option (4/O AL) was considered for the link 3-5. It was noted that for the case where the explicit voltage constraints were included, the program rendered the sub optimal solution of choosing the longer link (2-5) to serve load center 5. In the case without the explicit constraints, the solution remained (3-5) at a reduced node 5 voltage.

It should also be noted for this case, that the existing transformer (designated as 1-1 for the link 1-2) could adequately serve the load until stage 3. This shows the importance of developing an upgrade capability in the program, as temporary, economical solutions that are upgraded in future are commonly practiced by the utilities.

VI. CONCLUDING REMARKS

A categorical analysis of the past 40 years of research indicates that even though many advances have been made towards the solution of the distribution expansion problem, there still remain many areas for future research. Implementation of multiple routing and size options, inclusion of upgrade possibilities, and treatment of reliability and other objectives are major areas in need of future development regardless of the techniques.

A directed graph, minimum edge cost network flow modeling in a three stage formulation including multiple routing and size options has been proposed and implemented. It has been shown that inclusion of multiple size gradation is a significant factor when implementing explicit voltage constraints. Investigations of a simple test case indicate that inclusion of voltage constraints without consideration of multiple routing and size options render the solution sub optimal. It is also proposed that upgrade possibilities need be considered for the problem to be practical. This is vital for maximum asset utilization, optimality of the solution, and it is inherently aligned with industry practices and training.

The only variable costs that can influence the solution and need be considered are the energy losses. Although neglecting the losses as done by [9], or piecewise / stepwise linearization techniques can provide adequate solutions, inclusion allows for more accurate modeling. It is proposed that reliability, social/environmental impacts, and other objectives be considered as separate objectives and not integrated into a single objective formulations.

REFERENCES

[1] U.G. Knight, "The Logical Design of Electrical Networks Using Linear Programming Methods," *IEE Proceedings*, Vol. 33, No. A, 1960, pp. 306-314.

[2] M. Vaziri, K. Tomsovic and T. Gönen, "Distribution Expansion Problem Revisited PART 1: Categorical Analysis and Future Directions," *IASTED Power and Energy Systems Proceedings*, Paper No. 319-155, September 2000, Marbella, Spain pp. 283-290.

[3] M. Vaziri, K. Tomsovic and T. Gönen, "Distribution Expansion Problem Revisited PART 2: Proposed Modeling and Formulation", *IASTED Power and Energy Systems Proceedings*, Paper No. 319-215, September 2000, Marbella Spain, pp. 291-298.

[4] Y. Tang, "Power Distribution System Planning with Reliability Modeling and Optimization," *IEEE Transactions on Power System*, Vol. 11, No. 1, February 1996, pp. 181-189.

[5] T. Gönen and B. L. Foote, "Mathematical Dynamic Optimization Model for Electrical Distribution System Planning," *Electrical Power & Energy Systems*, Vol. 4, No. 2, April 1982, pp. 129-136.

[6] D. I. Sun, D. R. Farris, P. J. Cote, R. R. Shoults, and M. S. Chen, "Optimal Distribution Substation and Primary Feeder Planning Via the Fixed Charge Network Formulation," *IEEE Transactions on Power Apparatus and Systems*, Vol. 101, No. 3, March 1982, pp. 602-609.

[7] T. Gönen and I. J. Ramirez-Rosado, "Optimal Multi-Stage Planning of Distribution Systems," *IEEE Transactions on Power Delivery*, Vol. 2 No. 2, April 1987, pp. 512-519.

[8] I. J. Ramirez-Rosado and T. Gönen, "Psedodynamic Planning for Expansion of Power Distribution Systems," *IEEE Transactions on Power Systems*, Vol. 6, No. 1, February 1991, pp. 245-254.

[9] K. Nara, T. Satch, K. Aoki, and M. Kitagawa, "Multi-Year Expansion Planning For Distribution Systems," *Transactions on Power Systems*, Vol. 6, No. 3, August 1991, pp. 952-958.

[10] N. Kagan and R. N. Adams, "A Benders' Decomposition Approach To The Multi-Objective Distribution Planning Problem," *International Journal of Electrical Power & Energy Systems*, Vol. 15, No. 5, October 1993, pp. 259-271.

[11] M. A. El-Kady, "Computer-Aided Planning of Distribution Substation and Primary Feeders," *IEEE Transactions on Power Apparatus and Systems*, Vol. 103, No. 6, June 1984, pp. 1183-1189.

[12] H. K. Youssef and R. Hackam, "Dynamic Solution of Distribution Planning in Intermediate Time Range," *IEEE Transactions on Power Delivery*, Vol. 3, No. 1, January 1988, pp. 341-348.

[13] R. M. Adams and M. A. Laughton, "Optimal Planning of Power Networks Using Mixed Integer Programming," *IEE Proceedings*, Vol. 121, February 1974, pp. 139-148.

[14] O. M. Mikic, "Mathematical Dynamic Model for Long-Term Distribution System Planning," *IEEE Transactions on Power Systems*, Vol. 1, No. 1, February 1986, pp. 34-40.

[15] M. Blanchard, L. Delorme, C. Simard, and Y. Nadeau, "Experience With Optimization Software For Distribution System Planning," *IEEE Transactions on Power Systems*, Vol. 11, No. 4, November 1996, pp. 1891-1898.

[16] S. Jonnavithula and R. Billinton, "Minimum Cost Analysis of Feeder Routing in Distribution System Planning," *IEEE Transactions on Power Delivery*, Vol. 11, No. 4, October 1996, pp. 1935-1940.

[17] R. E. Brown, S. Gupta, R. D. Christie, S. S. Venkata, and R. Flecher, "Automated Primary Distribution System Design: Reliability and Cost Optimization", *IEEE Proceedings*, August 1996, pp. 1-6.

[18] I. J. Ramirez-Rosado, R. N. Adams, and T. Gönen, "Computer Aided Design of Power Distribution Systems: Multiobjective Mathematical Simulations", *Proceedings of The International Society for Mini and Microcomputers (ISMM) International Symposium*, March 1991, pp. 35-38.

[19] D. L. Brooks, R. C. Dugan, M. Waclawiak, and A. Sundaram, "Indices for Assessing Utility Distribution System RMS Variation Performance", *IEEE Transactions on Power Delivery*, Vol. 13, No. 1, January 1998, pp. 254-259.

[20] G. J. Peponis, M. P. Papadopoulos, "New Dynamic, Branch Exchange Method for Optimal Distribution System Planning", *IEE Proceedings on Generation, Transmission and Distribution*, Vol. 144, No. 3, May 1997, pp. 333-339.

[21] M. R. Garey and D. S. Johnson, "Computers and Intractability: A Guide to the Theory of NP-Completeness," *W. H. Freeman and Company*, New York, 1979.

[22] M. Vaziri, K. A. Ariyawansa, K. Tomsovic, A.Bose "On the accuracy of approximating a class of NP-complete quadratic programs (QP) by a class of polynomially bounded linear programs (LP) ," *IEEE Transactions on Circuits And Systems I*, In review.

[23] W. J. Rough , "Linear System Theory " *Prentice.Hall*, New Jersey, 1979.