Measuring the volatility of wholesale electricity prices caused by wind power uncertainty with a correlation model

Y. Wei F. Li K. Tomsovic

Electrical Engineering and Computer Science Department, University of Tennessee, Knoxville, USA
E-mail: fli6@utk.edu

Abstract: There is an increasing need to understand the impact of high-penetration wind power on various aspects of power system operation. This study presents a methodology to evaluate the impact of market price volatility on intermittent wind power. The proposed methodology first considers the uncertainty of wind power using a probabilistic distribution of wind speed in combination with the rated speed–MW curve. The correlation among different wind power plants is also modelled. With these statistical models, a Monte Carlo simulation (MCS) approach can be used to assess the probabilistic distribution of the price signals, that is, the probabilistic locational marginal pricing (LMP) distribution. Since the computational effort of MCS is intensive, a lookup table is proposed as preprocessing to greatly simplify the MCS. The proposed approach is tested with the PJM 5-bus system and the IEEE 118-bus system. Rules of thumb are drawn regarding the probabilistically calculated LMP and the correlation coefficients.

1 Introduction

Wind power is a promising clean generation resource at the utility scale. The impact of wind power on power system operation has been extensively studied [1–4]. Some of the assumptions and methodologies for power system study are no longer valid with increasing amount of wind penetration and new methodologies have been devised [5]. Still, intermittency or uncertainty remains a major challenge for wind power integration from a system viewpoint owing to the difficulty of accurate forecasts of MW output. Although results depend on many factors, a typical wind power forecast error is $\approx 10\%$ for hour-ahead, 15% for 12-h-ahead and over 20% for 24-h-ahead forecasting, and even higher for a longer term scheduling [6–8]. Although many new wind speed forecasting techniques have been applied to improve the forecast accuracy [9–12], it is probably that the forecast errors will remain high or show only marginal improvement, as the wind power projection error is closely related to weather forecasts, which have been subject to the similar accuracy problems for decades.

In practice, uncertainty of wind power has already been a concern in several areas in the USA, for example, California and Texas, for competitive market operation [13]. Since the day-ahead energy market trading clears approximately 80–90% of the load in real-time, the uncertainty in wind forecasting may cause a large mismatch in real-time operations. Wind energy is projected or least desired, for up to 20% of the peak load by 2030 in the USA [14]. Also, the electricity price risk would be exposed to rise with increasing wind penetration [15]. At this level, a computationally efficient methodology to assess the impact on price volatility will be of greater interest for market participants and short-term system planners. With this motivation, this paper attempts to address the following basic research questions:

- What is the appropriate and fast approach to model the probabilistic distribution of the market price corresponding to the wind speed uncertainty?
- What is a good ‘rule of thumb’ for impact on market price from wind speed uncertainty?
- What is the appropriate correlation model of the MW output among different wind plants?

Although other market structures exist, the locational marginal pricing (LMP) methodology has been adopted by many wholesale electricity markets in the USA. Thus, the LMP model will be taken as the basis for this paper with assumed data for case studies. The previous research work in [16] presents a concept called probabilistic LMP forecasting considering load uncertainty. The complexity of probabilistic LMP is owing to the observation of critical load levels (CLLs) [17–19]. A CLL indicates a sudden price change as a new binding transmission or generation constraint is reached. This is depicted in Fig. 1 for a modified PJM 5-bus system, in which A–E stand for five different buses [16].

Although the available output of a wind farm is treated simply as a negative load in some analyses, this assumption may not be valid since transmission congestion may prevent wind power from being fully dispatched. Wind power must
be subject to dispatch rules (i.e. optimal power flow with constraints modelled), whereas loads are commonly treated as ‘must-serve’ and grow proportionally in planning studies.

To address this challenge, this paper proposes a computationally efficient approach to model the impact on price volatility from a probabilistic viewpoint. Other contributions include the observation of the impact on market price uncertainty owing to wind forecast error and the impact of wind correlation across different wind farms.

Zhang et al. [20] show the normal quantile of wind speed forecasting error to prove that the wind speed forecasting error is in line with normal distribution. Zeineldin et al. [21] conclude that wind power prediction has an impact on electricity market prices and inaccurate power prediction can either result in underestimated or overestimated market prices that would lead to either savings for customers or additional revenue for generator suppliers. However, these authors did not provide any relations between the market price and the wind power forecasting error. These drive us to perform the proposed research.

This paper is organised as follows: Section 2 discusses the uncertainty model of wind power and correlation model of wind power plants. Section 3 presents a simulation methodology combining Monte Carlo simulation (MCS) and a lookup table (LT) of LMP to perform the study. The proposed methodology proves to be much more computationally efficient than brute-force (BF) MCS. Section 4 demonstrates the consistency of the proposed approach by applying a similar simulation to the IEEE 118-bus system. Finally, concluding remarks and future work are given in Section 5.

2 Uncertainties affecting LMP

2.1 Uncertainty model of wind power

To characterise the uncertainty that wind power might bring into market operation, the stochastic nature of wind speed should be analysed. It is a characteristic of general wind speed series that variation can be modelled well using a Weibull distribution [22]. There are mainly two approaches, persistence model and numerical weather prediction (NWP) model, to perform the wind speed forecast. The NWP method has proved to be more precise than the traditional persistence approach at the cost of more sophisticated input variables and significant computational intensity. Still, even for a short-term forecast, say from a few hours to 1 day ahead, the error for wind speed forecast remains at least 5% [6–8]. Although many new forecasting methodologies [23, 24] such as radial basis function network, adaptive neuro-fuzzy inference system model, neural logic network and so on, have been adopted for better prediction, the results have shown only relatively marginal improvement. Hence, wind forecast errors must be considered when its impact to market price signals is studied. This work will apply a normal distribution to wind speed forecast error, which has been shown in [25–28].

Wind power output from a wind turbine is strongly related to wind speed. This can be expressed using a classic wind–power curve as shown in Fig. 2.

It should be noted that wind power cannot be generated under any wind speed. Usually, wind turbines are not only designed to start running at a cut-in speed somewhere around 3–5 m/s but also programmed to stop at a high cut-out speed, say 25 m/s, in order to avoid damaging the turbine or its surroundings.

The power curve shows strong non-linearity of power output against wind speed. It is always difficult to make exact forecasts of the wind speed. For example as shown in Fig. 2, this paper will use Vestas V90-3.0 MW wind turbine as the prototype of power curve model to conduct a case study, of which the cut-in speed, cut-out speed, rated speed and rated wind power are 3.5 m/s, 25 m/s, 15 m/s and 3 MW, respectively. The power curve is obtained by applying an LT with the general specification of wind turbine in [29]. In the power curve, where the wind speed falls between 5 and 15 m/s, initial errors in wind speed forecast will be amplified according to the slope of the power curve. If one has a 10% error in forecast around 10 m/s wind speed, then the wind power output may be as much as 33% higher or lower. This could dramatically increase the volatility of electricity price for a given wind speed forecast error. Furthermore, the power curve will also influence the forecast error of the power prediction by transforming the symmetric normal distribution of wind speed forecast error into a beta distribution owing to the non-linear wind speed against power output curve [8]. In Fig. 3, it shows a group of sample data of forecasted wind speed and the projected wind power. The x-axis represents the simulated wind speeds for a given forecast around 10 m/s that follow normal distribution with constant mean (10 m/s)
and standard deviation (1.5 m/s), whereas the y-axis is the conditional distribution of wind power obtained from the simulated values of wind speed using the aforementioned LT.

2.2 Correlation model among wind farms

In [17], it is rigorously proved that for a lossless direct current optimal power flow (DCOPF) simulation model, generations of all the marginal units follow a linear pattern with respect to load.

To further address the uncertainty of wind power, correlation analysis is needed to model the relationship between each wind farm and its impact on LMP when there are more than one wind plants in the system. The prediction error of the wind speed at a single location tends to follow normal distribution. It is likely that for two different locations, the errors for each wind speed forecast will be jointly normal distributed. We have

\[ W_s \sim N(\mu, \sigma) \] (1)

\[ \phi(w_{s1}, w_{s2}) = \frac{1}{2\pi \sigma_{w_s1} \sigma_{w_s2} \sqrt{1 - r_{w_{s1}w_{s2}}}} \exp \left[ -\frac{z_{w_{s1}w_{s2}}}{2(1 - r_{w_{s1}w_{s2}})} \right] \] (2)

\[ z_{w_{s1}w_{s2}} = \frac{(W_{s1} - \mu_{w_{s1}})^2}{\sigma_{w_{s1}}^2} - 2\rho_{w_{s1}w_{s2}}(W_{s1} - \mu_{w_{s1}})(W_{s2} - \mu_{w_{s2}}) \frac{\sigma_{w_{s1}} \sigma_{w_{s2}}}{\sigma_{w_{s2}}^2} \] (3)

\[ \mu = (\mu_{w_{s1}}, \mu_{w_{s2}}) \] (4)

\[ \Sigma = \left[ \begin{array}{cc} \sigma_{w_{s1}}^2 & \rho_{w_{s1}w_{s2}} \sigma_{w_{s1}} \sigma_{w_{s2}} \\ \rho_{w_{s1}w_{s2}} \sigma_{w_{s1}} \sigma_{w_{s2}} & \sigma_{w_{s2}}^2 \end{array} \right] \] (5)

\[ \rho_{w_{s1}w_{s2}} = \frac{\text{cov}(W_{s1}, W_{s2})}{\sigma_{w_{s1}} \sigma_{w_{s2}}} = \frac{E[(W_{s1} - \mu_{w_{s1}})(W_{s2} - \mu_{w_{s2}})]}{\sigma_{w_{s1}} \sigma_{w_{s2}}} \] (6)

where \( W_s \) is the wind speed forecast error in m/s; \( N \) denotes normal distribution; \( \mu \) the mean of wind speed forecast in m/s; \( \sigma \) the standard deviation of wind speed forecast in m/s.
3 Simulation methodology to identify the impact on market operation

3.1 Market simulation model

In market operation, LMP is the dominant approach to clear the electricity market. The LMP model will be used in this paper as the energy market model to assess market price. It should be noted that here we investigate the economic dispatch model considering a particular time point, while ignoring the unit commitment (UC) problem. The reason is that the goal of this study is to investigate the impact on LMP, which is typically cleared every 15 or 5 min depending on the market, whereas UC is updated less frequently, typically an hour. Therefore within that hour, UC can be assumed to be fixed (unless there is a generation outage requiring new units to be brought online). Hence, this study aims to analyse static snapshots of the LMPs considering wind output uncertainty.

Here, we assume that there exists a conventional generator, a wind generator and a load at each bus for simplicity of the formulation. This model can be briefly described as follows

\[
\min \sum_{i=1}^{N} C_{c,i} \cdot G_{c,i} + \sum_{i=1}^{N} C_{w,i} \cdot G_{w,i} + \sum_{i=1}^{N} D_{i} = 0
\]

s.t.

\[
\sum_{i=1}^{N} G_{c,i} + \sum_{i=1}^{N} G_{w,i} - \sum_{i=1}^{N} D_{i} = 0
\]

\[
\sum_{i=1}^{N} \text{GSF}_{k,i} \cdot (G_{c,i} + G_{w,i} - D_{i}) \leq F_{k}^{\max},
\]

for \( k \in \{\text{all lines considering both directions}\} \) (9)

\[
C_{c,i}^{\min} \leq G_{c,i} \leq C_{c,i}^{\max},
\]

for \( i \in \{\text{all conventional generators}\} \) (10)

\[
C_{w,i}^{\min} \leq G_{w,i} \leq C_{w,i}^{\max}, \quad \text{for } i \in \{\text{all wind generators}\} \]

(11)

where \( N \) is the number of buses; \( C_{c,i}, C_{w,i} \), the conventional and wind generation costs at bus \( i \) in S/MWh, respectively; \( G_{c,i}, G_{w,i} \) the conventional and wind generation dispatches at bus \( i \) in MWh, respectively; \( D_{i} \), the demand at bus \( i \) in MWh; \( \text{GSF}_{k,i} \) the generation shift factor to line \( k \) from bus \( i \) and \( F_{k}^{\max} \) the transmission limit of line \( k \) in MWh.

The general formulation of LMP at bus \( i \) can be written as follows

\[
\text{LMP}_{i} = \text{LMP}_{i}^{\text{energy}} + \text{LMP}_{i}^{\text{cong}} + \text{LMP}_{i}^{\text{loss}}
\]

(12)

\[
\text{LMP}_{i}^{\text{energy}} = \lambda
\]

(13)

\[
\text{LMP}_{i}^{\text{cong}} = \sum_{k=1}^{M} \text{GSF}_{k,i} \cdot \mu_{k}
\]

(14)

\[
\text{LMP}_{i}^{\text{loss}} = \lambda \cdot (\text{DF}_{i} - 1)
\]

(15)

where \( M \) is the number of lines; \( \lambda \) the Lagrangian multiplier of the equality constraint that is, system energy balance in (8); \( \mu_{k} \) the Lagrangian multiplier of the \( k \)th transmission constraint; and \( \text{DF}_{i} \) the delivery factor at bus \( i \). It should be noted that the model in (7)–(11) ignores losses for easy illustration.

3.2 Brute-force MCS model for uncertainty of LMP considering wind uncertainty

Owing to the intermittency of wind power, \( G_{w} \) in the above model (7)–(11) should not be treated as a deterministic value. However, it is a common practice in market simulation or forecast to use a single forecasted value for each input variable, say, load or wind power, to perform a deterministic market simulation to forecast LMPs and congestion. Some recently improved practices use discretised model such as using 50–100 probability–weighted scenarios to forecast market trends. However, owing to the step-change nature of LMP at CLLs [16], the discretised model may lead to some inaccuracy. Therefore in this paper, MCS is employed to handle the probabilistic nature of these stochastic inputs while executing each simulation deterministically. The MCS process can be illustrated as follows:

1. Wind speeds (m/s) at various wind plants are generated in accordance with multivariate normal distribution. This is to consider the randomness and correlation among wind power plants.
2. Wind power MW outputs are obtained using power curve model and the wind speed data obtained in step 1.
3. Apply (7)–(15) to obtain market signals.
4. Repeat 1–3 to generate more samples.
5. Perform statistical analysis, such as the probability distribution of LMP at each bus corresponding to wind speed forecasts.

MCS can be highly computationally intensive and may be impractical for large systems. The above mentioned MCS for calculating LMP distributions is a BF and time-consuming approach. A simplified MCS approach based on an LT to take advantage of the step-change characteristic of LMP is presented next.

3.3 Speeded MCS: using a lookup table

Computational efficiency study of LMP behaviour exists because of its step-change characteristic. Fig. 1 shows a typical LMP against load curve for a sample system slightly modified from the original PJM 5-bus system defined in [16]. The load level at which a step change occurs is termed a CLL. At each CLL, there will be changing...
binding and unbinding limits as well as marginal and non-marginal units.

The step-change features also hold when both the system load level and the wind power output are considered. For instance, for a specific wind power level a staircase curve such as Fig. 1 of system load against LMP can be obtained. On the other hand, for a specific system load level, a similar staircase curve of LMP against the wind power output within an area can be obtained. A wind power output level where a step change of LMP occurs is termed a critical wind level (CWL). Thus, a three-dimensional diagram with step changes can be obtained if both the system load level and the wind power output are considered as variables, as shown in Fig. 4.

When more than two variables are considered (e.g. load and/or wind in different areas are subject to its own variation pattern), extension to high dimension is possible although difficult to visualise. This is also the reason that MCS is necessary rather than the analytical approach based on integral, which was used in [16] for the single variable case. When multiple stochastic variables are involved, multiple integral is too complicated to be manageable.

For computational purpose, an LT can be built as a pre-processing step. Once we know the range of possible wind power outputs, the following steps can be performed:

1. A corresponding LT can be built by performing a number of DCOPF runs to identify CLLs and critical wind levels where the step changes occur. Prices signals will be stored at the CLLs and CWLs with the model in (7)–(15).
2. A number of MCS samples need to be generated by ‘throwing the dice’, which takes \( t_{\text{trial}} \) to obtain the random output of wind power plants.
3. The wind-plant output will be used to find the corresponding values in the LT.

The computational time of the above three steps is briefly discussed next. The computational time for step 1 is \( n_{\text{sc}} \cdot t_{\text{DCOPF}} \), where \( t_{\text{DCOPF}} \) is the time taken to perform a DCOPF run and \( n_{\text{sc}} \) is the number of step-changes in the LT, that is, number of CLLs. The time for step 2 is \( t_{\text{trial}} \), which is insignificant (i.e. \( t_{\text{trial}} \simeq 0 \)), if compared to \( t_{\text{DCOPF}} \). The time for step 3 is \( t_{\text{lookup}} \) is also insignificant for \( t_{\text{DCOPF}} \) (i.e. \( t_{\text{lookup}} \simeq 0 \)), since there are only a limited number of stairs in the LT. Therefore the overall running time with this LT approach is given by

\[
T_{\text{LT}} = n_{\text{sc}} \cdot t_{\text{DCOPF}} + t_{\text{trial}} + t_{\text{lookup}} \simeq n_{\text{sc}} \cdot t_{\text{DCOPF}} \tag{16}
\]

As a comparison, if the number of sample trials is \( n_{\text{sample}} \), the computational time for the basic, BF MCS approach is

\[
T_{\text{BF}} = n_{\text{sample}} \cdot t_{\text{DCOPF}} \tag{17}
\]

Since the number of step-changes, \( n_{\text{sc}} \), is much less than \( n_{\text{sample}} \) (e.g. dozens against tens of thousands), the LT gives a great speedup.

Several cases with different systems and different number of trials are run, and Table 1 shows the average running times using the BF MCS approach and the LT MCS approach for three different systems for illustrative purpose. The promising performance improvement is also illustrated as a bar chart in Fig. 5, where the speedup ratio is defined as \( \frac{T_{\text{BF}}}{T_{\text{LT}}} \).

It should be noted that performance improvement depends on the range of forecasted wind power output but similar improved performance is expected. In general, a larger forecasted range will benefit more from the proposed LT-based MCS. It should also be noted that \( T_{\text{LT}} \) in the test is obtained by using a quick search technique such as binary partition in a range of loads and/or wind power outputs. Further improvement to reduce \( T_{\text{LT}} \) is possible by employing a systematic, fast approach to find the CLLs [16].

The speedup of a single simulation is not the only advantage. Each LT of LMP w.r.t. different load levels and wind levels can be saved and reused when there is a similar load level or wind level in future papers.

### 4 Case studies with the IEEE 118-bus system

In this section, a study on a large system is performed. Although it would be desirable to perform a case study using an actual ISO’s data, this is difficult as the data are proprietary. Therefore the IEEE 118-bus system [30] is used to perform the case study, which demonstrates the applicability of the proposed concepts and methods to
larger systems. The 118-bus system consists of 118 buses, 54 generators and 186 branches. The total system load is 4242 MW with 9966 MW total generation capacity. A detailed system data and diagram can be found in [30].

In the original IEEE 118-bus system, there is no generator bidding data and branch thermal limit data that are indispensable to perform the economic study. Therefore generator bidding data are assumed as follows for illustrative purpose: 20 low-cost generators with bids from $10 to 19.5 with $0.5 increment; 20 expensive generators with bids from $30 to 49 with $1 increment; and 14 most expensive generators with bidding from $70 to 83 with $1 increment. Five thermal limits are applied to the transmission system: 345 MW for line 69–77, 630 MW for line 68–81, 106 MW for lines 83–85 and 94–100 and 230 MW for line 80–98.

The deterministic LMP against load curve for the IEEE 118-bus system is shown in Fig. 6. The LMP against load curve are drawn on some selected buses rather than all buses simply for better illustration.

4.1 LMP and wind plant locations

In this case study, three wind power plants are connected to bus 85 (wind plant no.1 or WP1), bus 22 (wind plant no. 2 or WP2) and bus 38 (wind plant no. 3 or WP3). Assume that the wind speed forecast errors for WP1, WP2 and WP3, that is $W_{s1}$, $W_{s2}$ and $W_{s3}$, are mutually independent. For a forecast of 7 m/s, they are all following the same normal distribution, that is, $W_{s1}$, $W_{s2}$, $W_{s3}$ ~ $N(0,1)$, in which the mean value and standard deviation for the three forecasted wind speeds are 7 and 1 m/s. The power curves are obtained by applying an LT from the technical specifications of Vestas V90-3 MW wind turbines with a tentative scaling parameter $k$ in order to scale the actual capacity of the wind power plant to 80 MW. Then, the projected wind power from forecasted wind speed is nearly 47.36 MW.

Fig. 7 shows the correlation between four variables, $W_{s1}$, $W_{s2}$ and $W_{s3}$ (i.e. the wind speed at three wind-plant locations at buses 85, 38 and 22, respectively) and the LMP at bus 73. The off-diagonal items in the upper left $3 \times 3$ block show the wind speed correlations which are the input variables, whereas the last row and column show the correlation between the wind speeds (the input variables) and the LMP at bus 73 (the output variable). The colour of each off-diagonal entry shows the relative strength of the correlations. Deeper colour represents higher negative correlation. Although Fig. 7 shows only one scenario with no correlation, the same colour pattern that LMP is related to wind-plant location can be observed under many other scenarios with different wind speed correlations. This demonstrates that LMP is closely related to the wind-plant location, as wind generation affects the output of marginal units. In the deterministic average case (i.e. 7 m/s of wind speed at each wind plant, 47.36 MW projected wind power at each wind plant and the base load level at 4242 MW), one of the marginal units is located at bus 85. Since, bus 73 is very close to bus 85, it is not surprising that the LMP at bus 73 is more closely related to $W_{s1}$ than $W_{s2}$ and $W_{s3}$.

Wind power plants integrated to the marginal unit buses will always have a greater impact on the LMP than those connected to the non-marginal buses under given forecasted load and forecasted wind power output. This observation and the quantitative MCS approach will help market participants make reasonable judgment on whether the integrated wind power will affect the LMP market and how much of an impact it would be, considering the uncertainty and correlation of wind speed forecast.

4.2 LMP variation owing to different wind speed correlations

In real-practice, it is more likely that wind power plants are located in geographically adjacent areas such that they are more or less correlated in terms of wind speeds. Here, the wind speed forecast errors for three wind power plants are jointly normally distributed and follow $W_s \sim N(\mu, \sigma)$, given by forecasted wind speeds of $W_{s1}, W_{s2}, W_{s3}$, with $\mu = (0, 0, 0)$ and

$$
\sigma = \begin{bmatrix}
\rho_{12} & \rho_{13} & \rho_{13} \\
\rho_{12} & 1^2 & \rho_{23} \\
\rho_{13} & \rho_{23} & 1^2
\end{bmatrix}
$$

in which the unity variance for all normal distributions participating in the multivariate model is assumed. We can summarise the correlation coefficients of $\rho_{12}$, $\rho_{13}$ and $\rho_{23}$ into three patterns:

Fig. 6 Deterministic LMP curve at selected buses with respect to different system loads for the IEEE 118-bus system

Fig. 7 Colour map for correlation between the forecasted wind speed and LMP at bus 73
pattern 1: two of the three wind plants are correlated, whereas the third one is independent. Although there can be numerous cases under this category, two cases, case 1A and case 1B, are tested at two different load levels. In both cases, \( r_{13} = r_{23} = 0 \), whereas \( r_{12} \) is varied from \(-1\) to \(1\).

pattern 2: two of the three wind plants are positively correlated, whereas the third one is independent. Although there can be numerous cases under this category, two cases, case 1A and case 1B, are tested at two different load levels. In both cases, \( r_{13} = r_{23} = 0 \), whereas \( r_{12} \) is varied from \(-1\) to \(1\).

pattern 3: the wind speeds at all three plants are positively correlated. Two cases, case 3A and case 3B, are tested at two different load levels. In both cases, \( r_{12} = r_{13} = r_{23} = \rho \) which is varied from \(-1\) to \(0\) since plant 3 is negatively correlated with plants 1 and 2, respectively. Here, \( r_{13} \) and \( r_{23} \) are not independently varied because a three-dimensional plot is needed otherwise.

Cases 'A': The expected LMP at bus 1 decreases when the correlation coefficient among the three wind speed forecasts increases. This is shown in the top three diagrams in Fig. 8. It is apparent that LMP at certain buses can either increase or decrease under different correlation coefficients among wind speed forecasts of the three wind power plants.

Cases 'B': Different from the decreasing pattern shown in the 'A' cases, the expected LMP at bus 1 increases when the correlation coefficient increases. This pattern is shown in the bottom three diagrams in Fig. 8.

Fig. 8 shows the results of all six test cases (1A, 1B, 2A, 2B, 3A and 3B) corresponding to the above three patterns. It is apparent that LMP at certain buses can either increase or decrease under different correlation coefficients among wind speed forecasts of the three wind power plants.

The above observation of the difference between cases ‘A’ and ‘B’ is related to the position of the ‘modified’ operating point, which is equal to the system load level minus the mean wind output, in the staircase LMP against the system load curve. The cause of the two different patterns will be explained later in Section 4.3.

It should be noted that although there may be many wind plants in a system, many of them can be grouped into an area following the same wind forecast. Correlations will be modelled among these areas. This is similar to area–load modelling. Hence, the correlation matrix should be of a manageable size.

4.3 Observation and summary

First of all, under the deterministic mean values, if the modified operating point, that is the system load level minus the expected (mean) wind-plant output, is closer to a CLL, the price volatility owing to the intermittent wind power is higher. Here, price volatility is referred to as the probability of a price step-change with respect to the change of wind power output. The increasing or decreasing trend in the LMP against correlation coefficient curve is determined by whether the modified operating point is near the beginning or the end of a staircase segment, which is essentially determined by the CLLs.

When we consider the probabilistic correlation of the wind speed forecasts at three wind plants, we have the following observation:

1. negative correlation coefficient means that the wind power plants tend to compensate each other’s output variation. Thus, this keeps the LMP at its value when the mean wind power output is deterministically considered. For instance, in case
1. LMP at bus 1 is exactly $30.995/\text{MWh}$ (i.e. the same as the deterministic LMP value) when the correlation coefficient $\rho_{\text{ws}} = -1$ and in case 1B, the LMP at bus 1 is exactly $19.750/\text{MWh}$, also the same as the deterministic LMP value, when $\rho_{\text{ws}} = -1$.

2. A similar analysis can be applied to the ‘B’ cases, where the mean modified load level is close to the end of a staircase segment (such as point A where $P_B = 3450 \text{ MW}$ in Fig. 8). In these cases, the simultaneous increase of two wind-plant outputs will lead to a possible jump of price. This is why the probabilistic LMP is higher when $\rho_{\text{ws}} = 1$ than $\rho_{\text{ws}} = -1$ as shown in case 1A. In general, when the correlation $\rho_{\text{ws}}$ increases, the probabilistic LMP should decrease in the ‘A’ cases as shown in Fig. 8.

5. **Concluding remarks**

Uncertainty of high wind penetration and the step change characteristic of LMP–load and LMP–wind curves are the main reasons for LMP uncertainty. The contribution of this paper can be summarised as follows:

1. A systematic methodology using MCS is presented to investigate the impact on market price volatility considering the intermittent nature of wind power generation.

2. LT is proposed to speedup the basic, BF MCS approach. Test results verify the computational efficiency of the LT-based MCS.

3. It can be concluded that different patterns of LMP against wind correlation can be predicted given the forecasted wind generation and load. It is the positive correlation that decides how the expected LMP will vary considering wind uncertainty. Also, it is the modified operating point that decides whether the probabilistic LMP will actually increase or decrease.

4. Test results based on the IEEE 118-bus systems verify the expectations and conclusions.

Note that this is a methodological paper, which presents a mathematical tool to combine the wind speed error distribution model into an LT-based MSC model. This approach allows a quantitative assessment of the impact of wind uncertainty on electricity price. Simulation tests are based on assumed data for wind uncertainty with a typical LMP-based USA market structure. In practice, market participants are often interested in price volatility and wind/load uncertainty. The proposed mathematical approach can always be employed by a market participant using their own, actual wind error model with correlations and the LT-based MCS. In addition, the conclusions in this paper, serve as a broad guideline to predict the LMP pattern at a certain location given a known wind profile. This can be important information for strategic bidding and other decision-making.

Future work is needed to address some other practical concerns. To begin with, the current effort investigates the relationship of price and wind forecast error at a particular point in time using the economic dispatch model. State-of-the-art wind forecasting tools suggest that the wind forecast uncertainty shows considerable time dependency. This should be included in future work along with changes in the UC, as well as other market rules such as forward contracts, ancillary service and so on. Also, consideration of the system power losses and equipment (generation and transmission) outages would be valuable. In addition, investigation of the applicability of other market models with different regulatory environment such as in European countries can be carried out in the future.

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7. **References**


