

THE STRONG PERFECT GRAPH THEOREM

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Georgia Institute of Technology
and
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joint work with

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PART I

History and relevance of perfect graphs

PART II

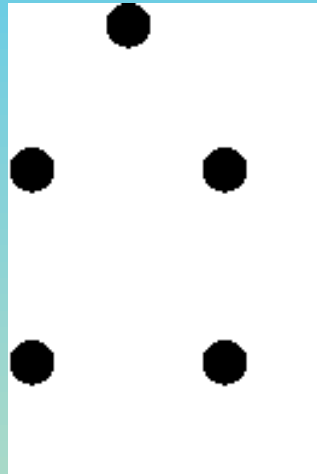
The strong perfect graph conjecture

CLAUDE BERGE

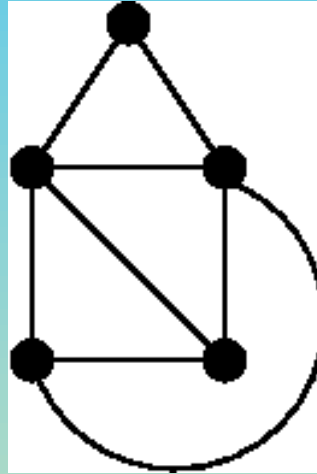


1926–2002

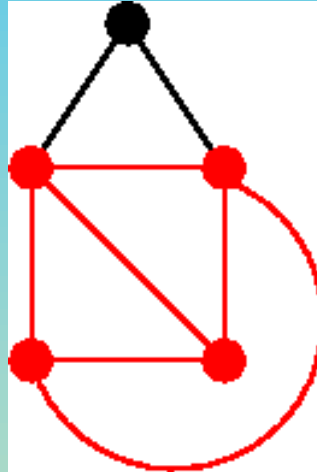
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Graphs have vertices and edges.

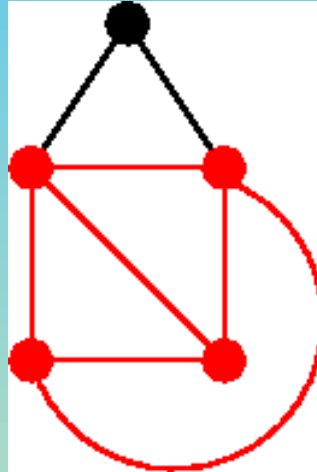


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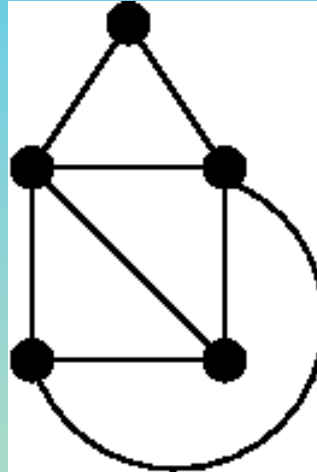
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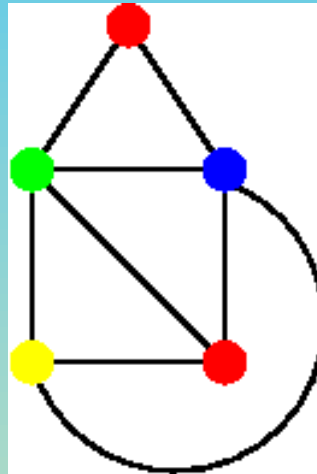


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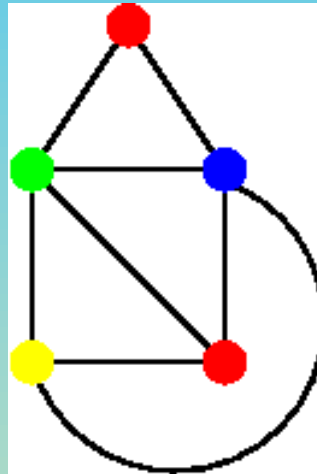


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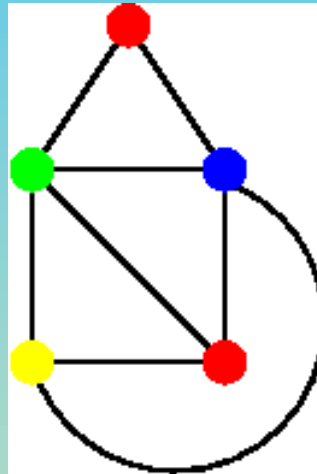
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Graphs that have an odd hole or odd antihole

EXAMPLES OF PERFECT GRAPHS

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Bipartite graphs

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THE STRONG PERFECT GRAPH CONJECTURE (SPGC) (Berge 1960)

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- Fundamental and beautiful open problems

THEOREM (Lovász) Let A be a $0, 1$ -matrix. For every non-negative objective function c the LP

$$\max c^T x \text{ subject to } x \geq 0 \text{ and } Ax \leq 1$$

has integral optimum \Leftrightarrow the undominated rows of A form the vertex versus maximal cliques incidence matrix of some perfect graph.

PART II

The Strong Perfect Graph Conjecture

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MAIN THEOREM Every Berge graph is either basic, or has a certain decomposition.

THE SPGC WAS KNOWN FOR

- planar graphs (Tucker)
- claw-free graphs (Parthasarathy, Ravindra)
- K_4 -free graphs (Tucker)
- diamond-free graphs (Tucker)
- bull-free graphs (Chvátal, Sbihi)
- dart-free graphs (Sun)
- C_4 -free graphs (Conforti, Cornuéjols, Vušković)
- “wheel-and-parachute-free” graphs (Conforti, Cornuéjols)

NOTE All of the above exclude specific graphs.

To prove the SPGC we must show that every Berge graph G satisfies $\chi(G) = \omega(G)$.

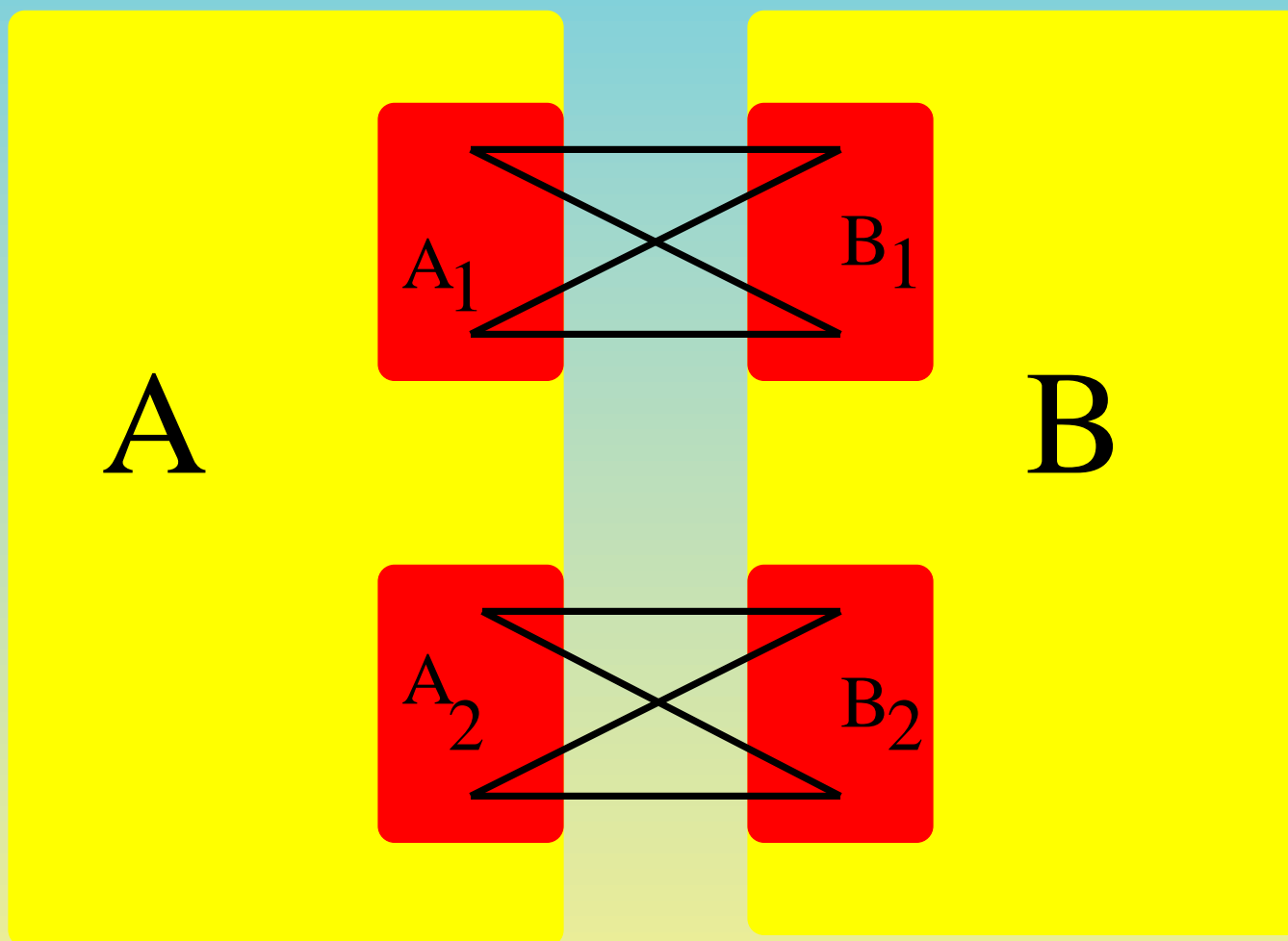
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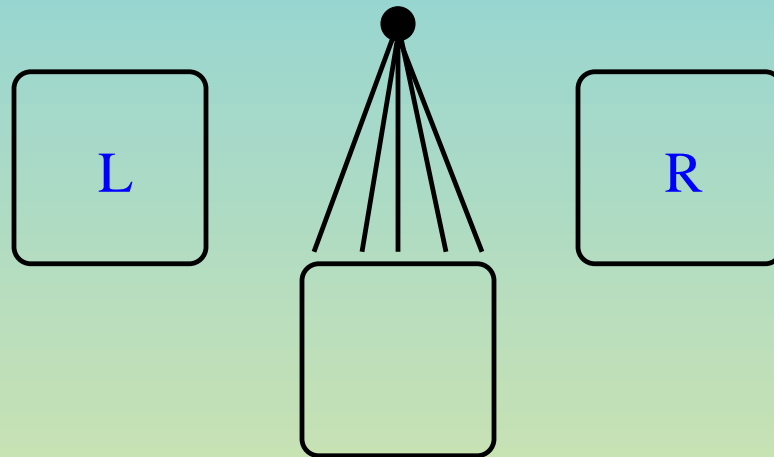
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2-JOIN



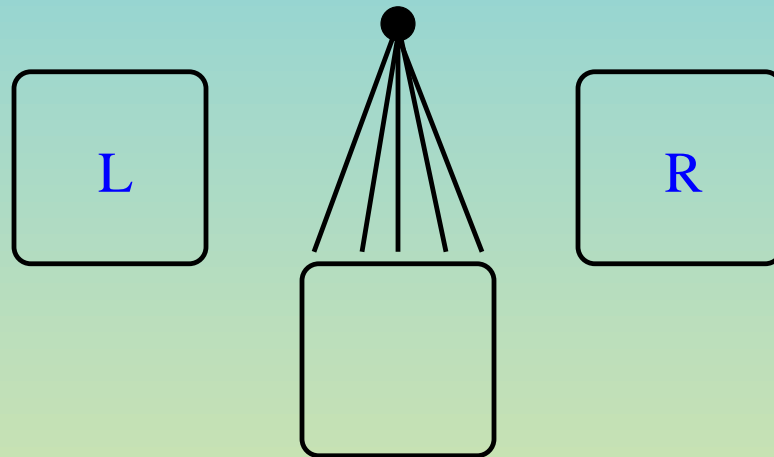
STAR CUTSETS

A vertex cut X is a **star cutset** if some $v \in X$ is adjacent to every other vertex of X .



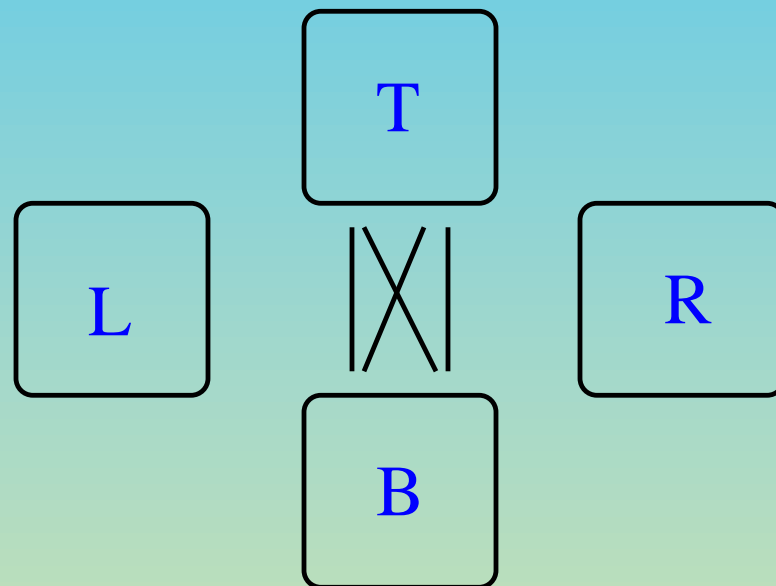
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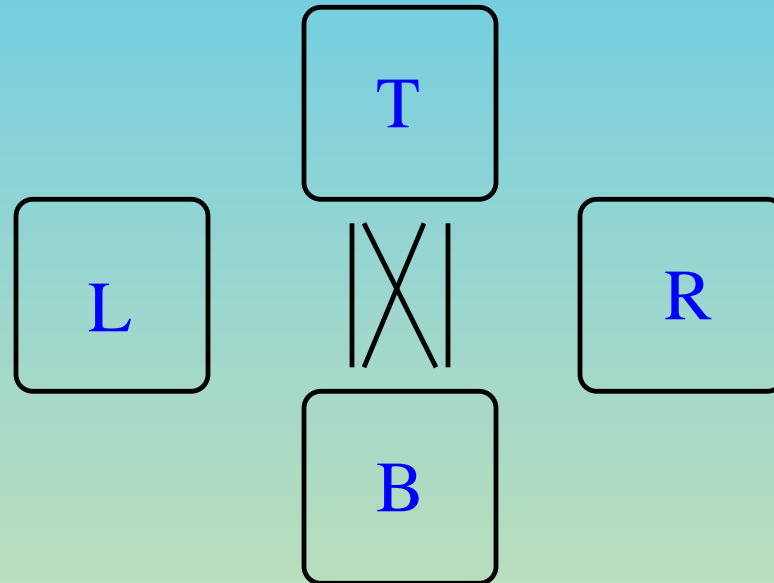


THEOREM (Chvátal) No minimally imperfect graph has a star cutset.

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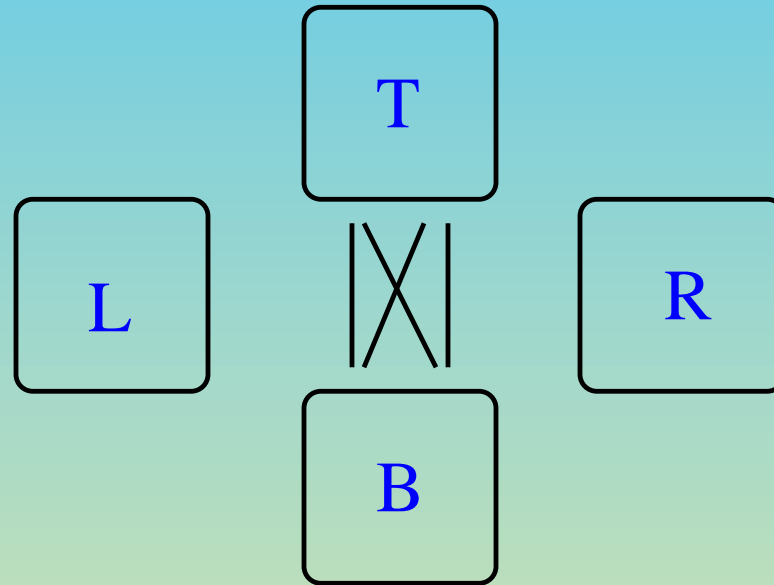


SKEW PARTITIONS



CONJECTURE (Chvátal) No minimally imperfect graph has a skew partition.

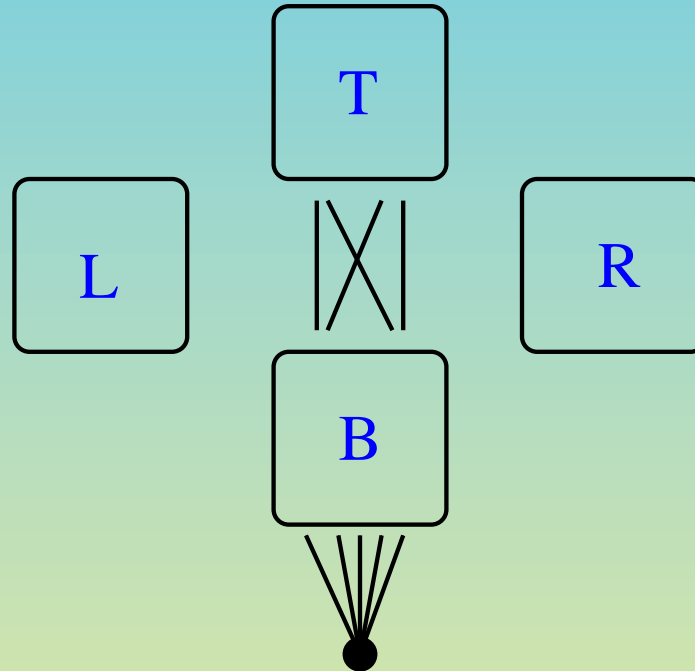
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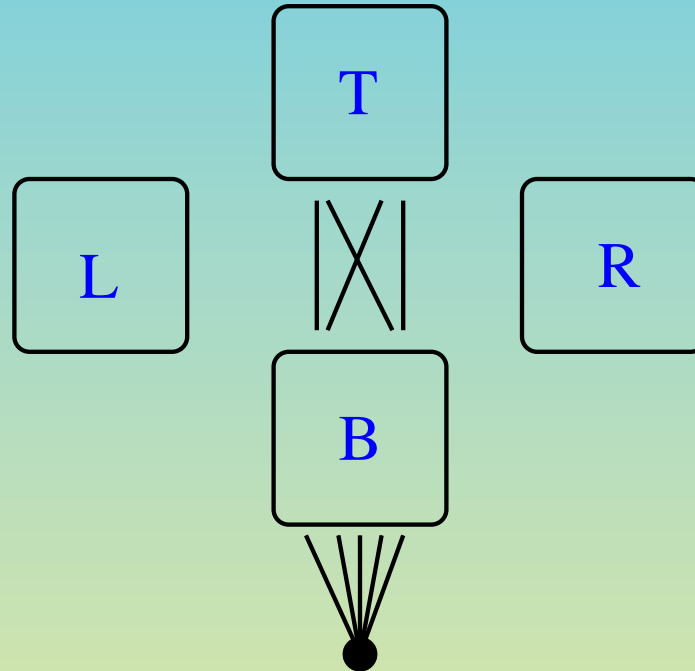
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THM No minimum imperfect graph has an even skew partition.

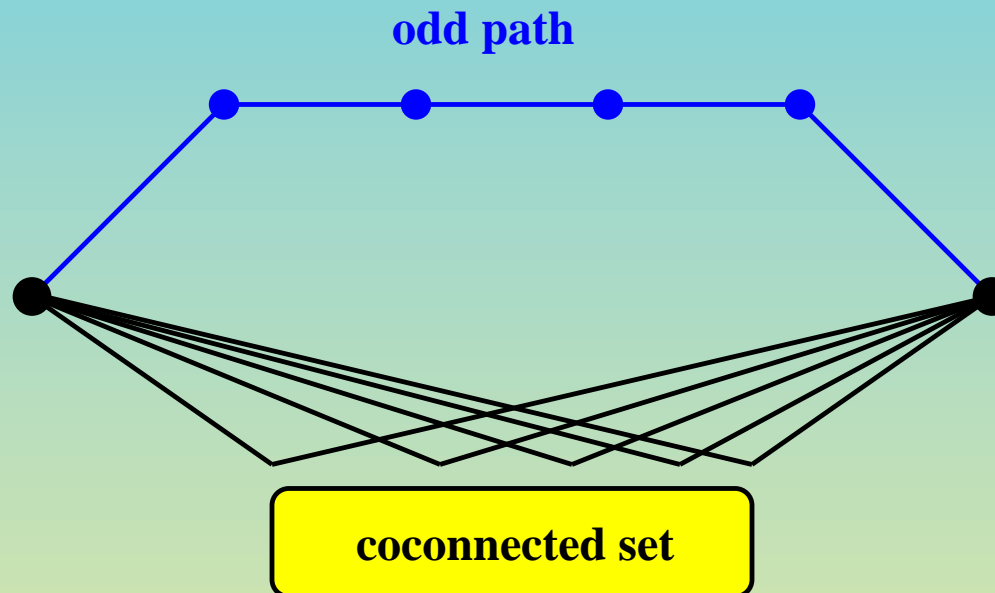
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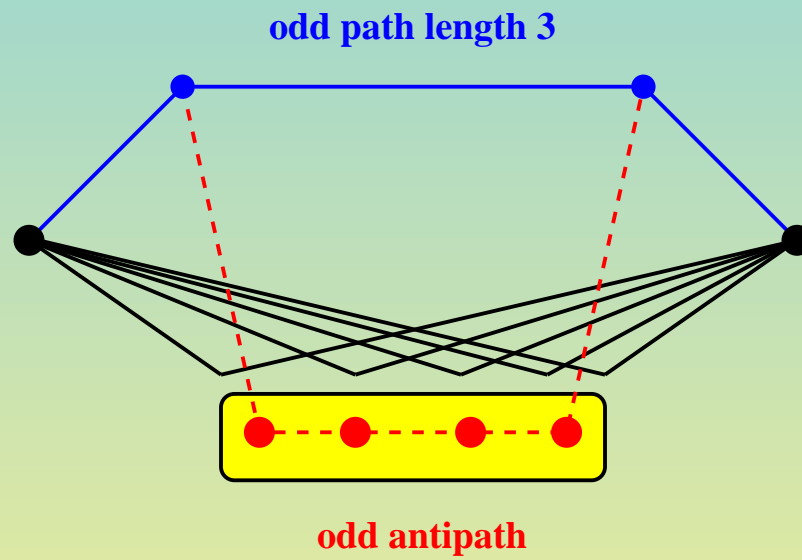
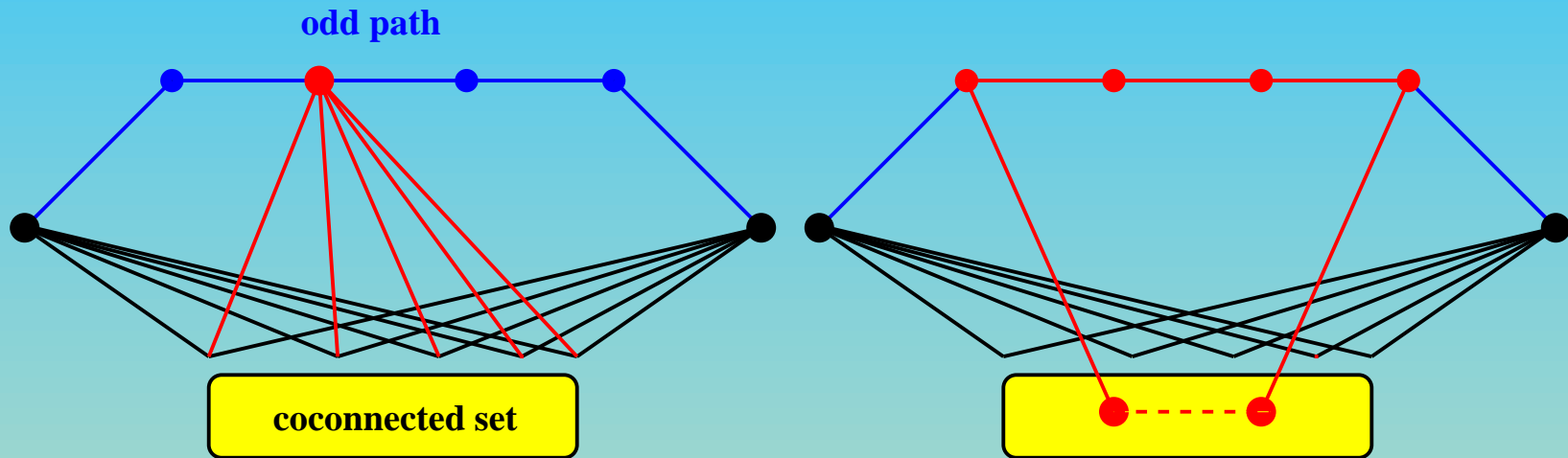
- (1) is bipartite, or
- (2) is a line graph of a bipartite graph, or
- (3) is a bicograph, or
- (4) has an even skew partition, or
- (5) has a 2-join, or
- (6) has an M-join.

A LEMMA ABOUT ODD PATHS

Roussel & Rubio, RST In a Berge graph, if



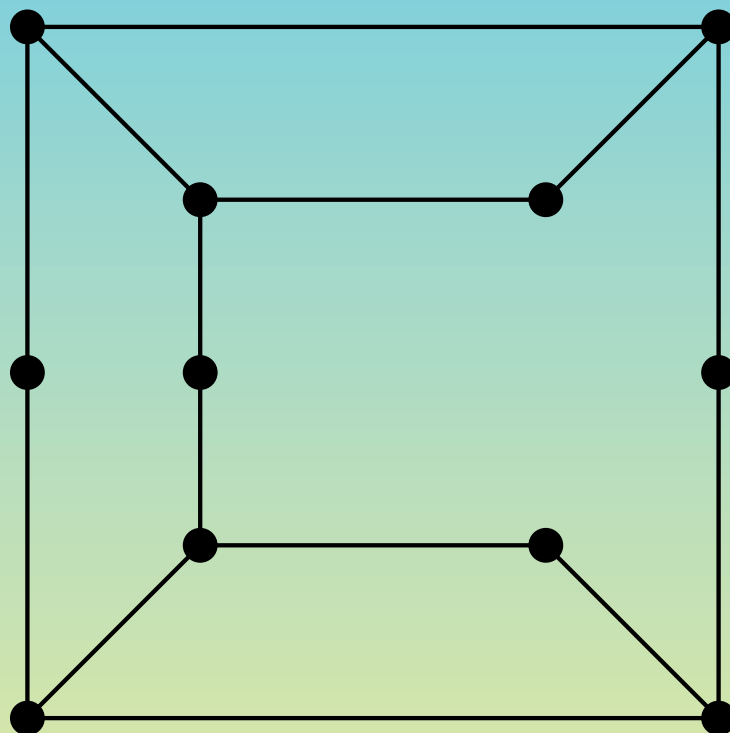
then



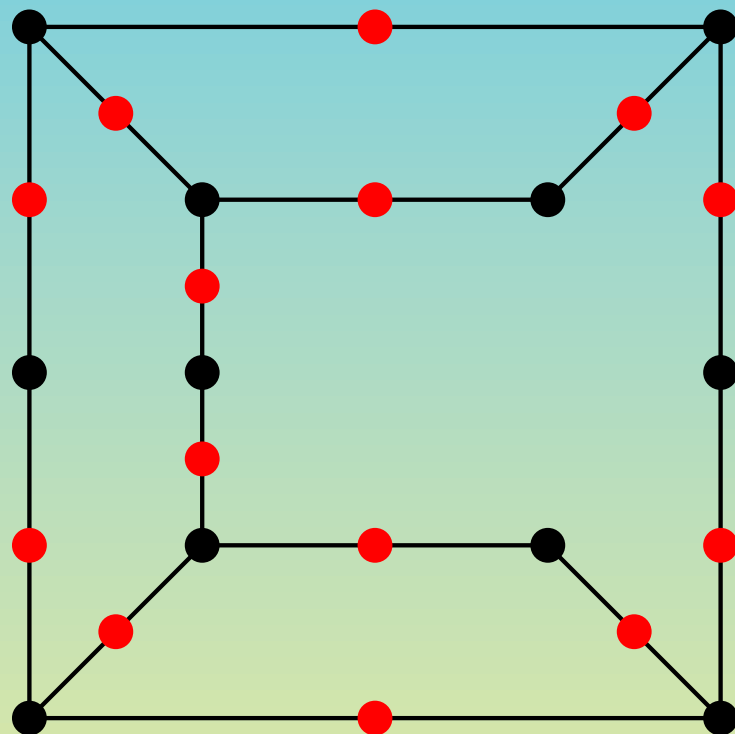
MAIN CASES

- G contains a “large” line graph of a bipartite graph
- G contains a “wheel”
- neither of the above

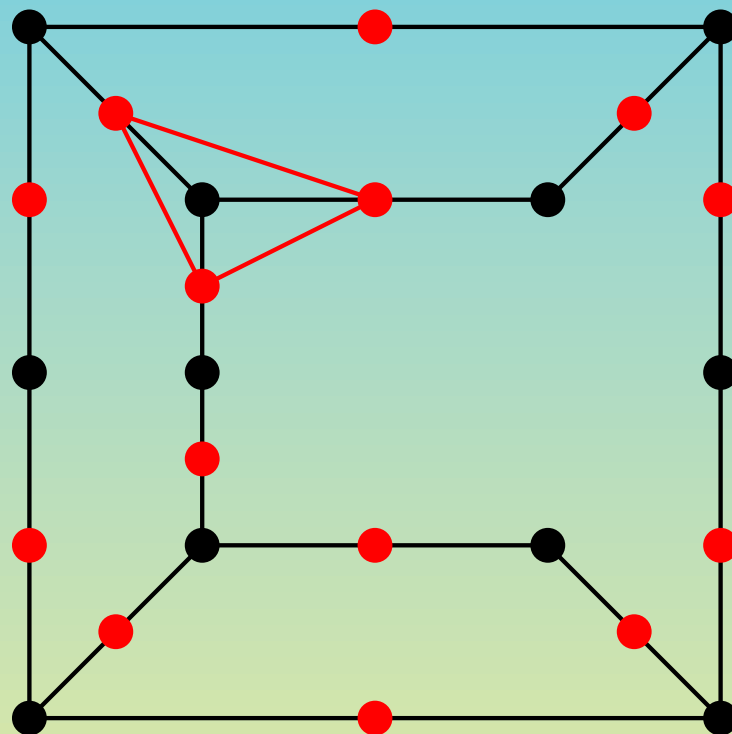
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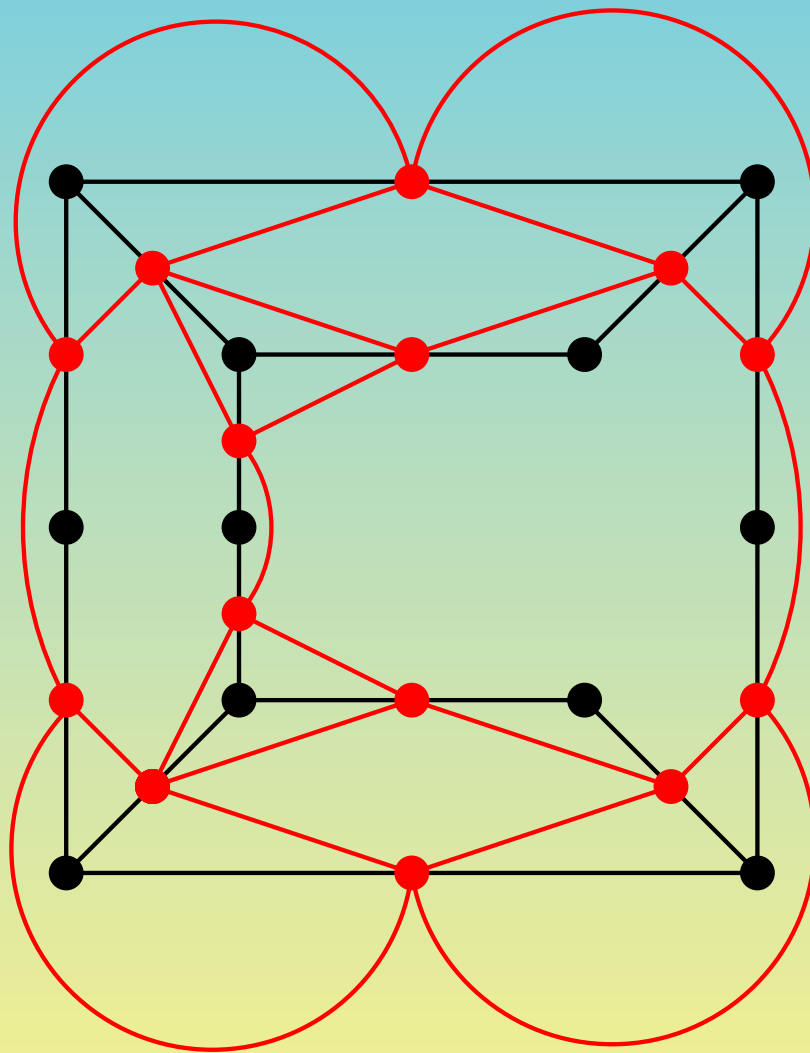
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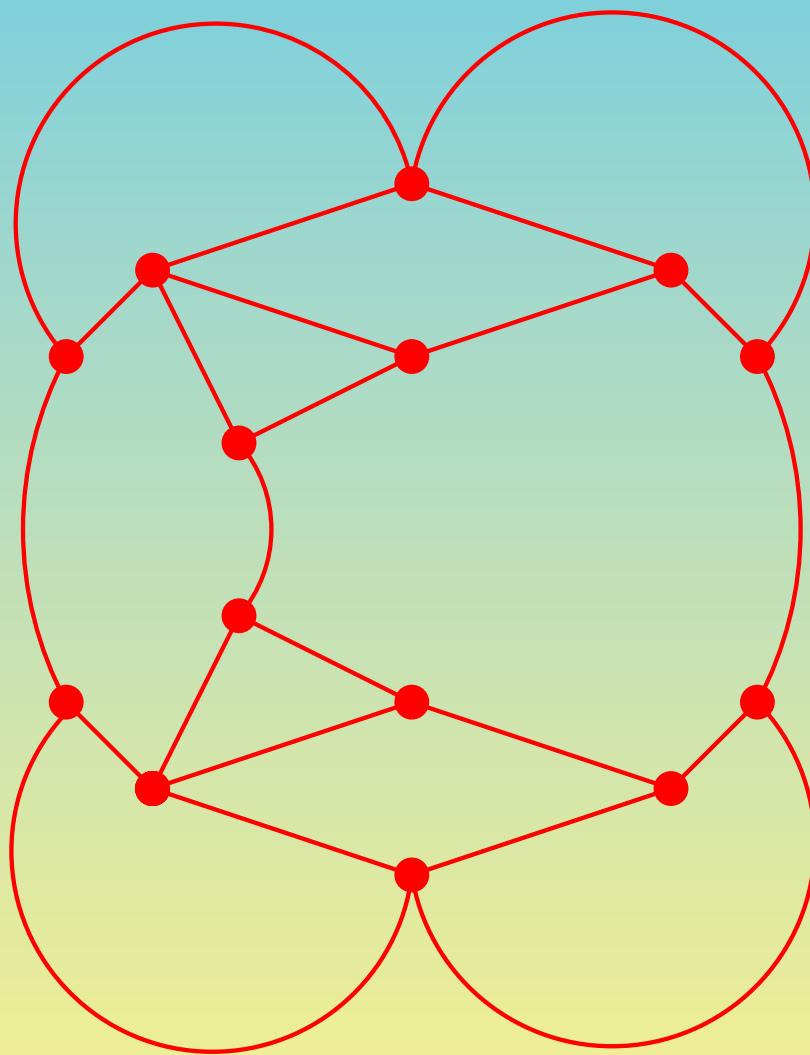
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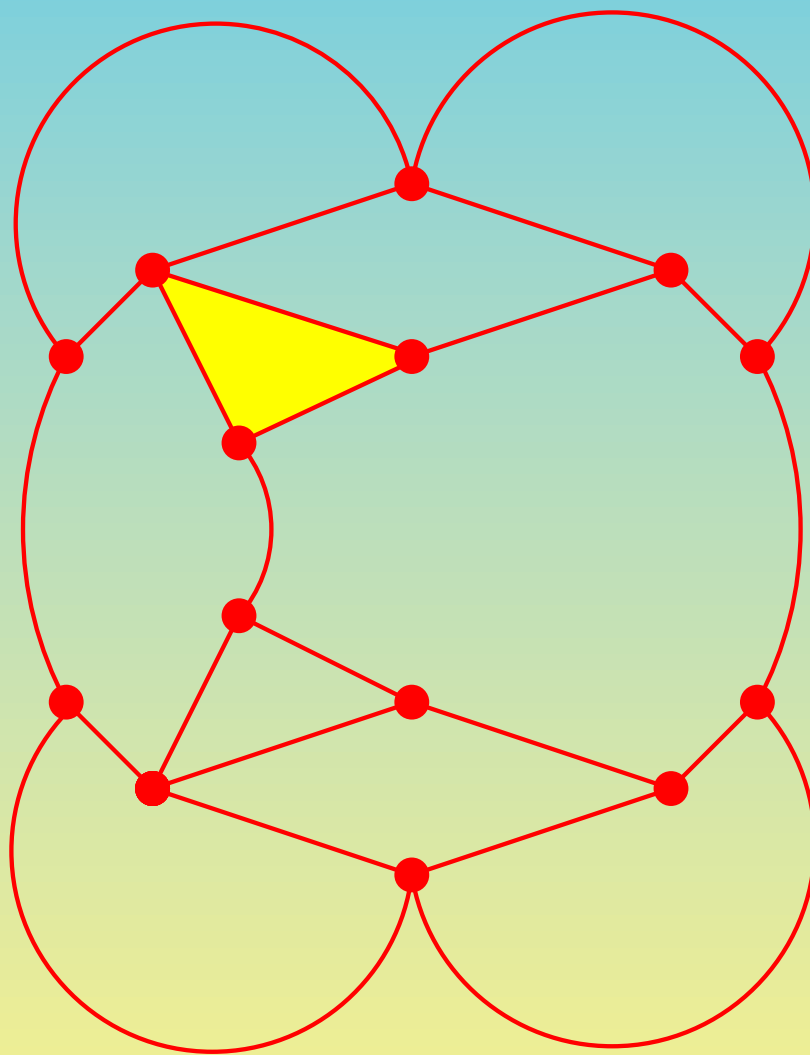
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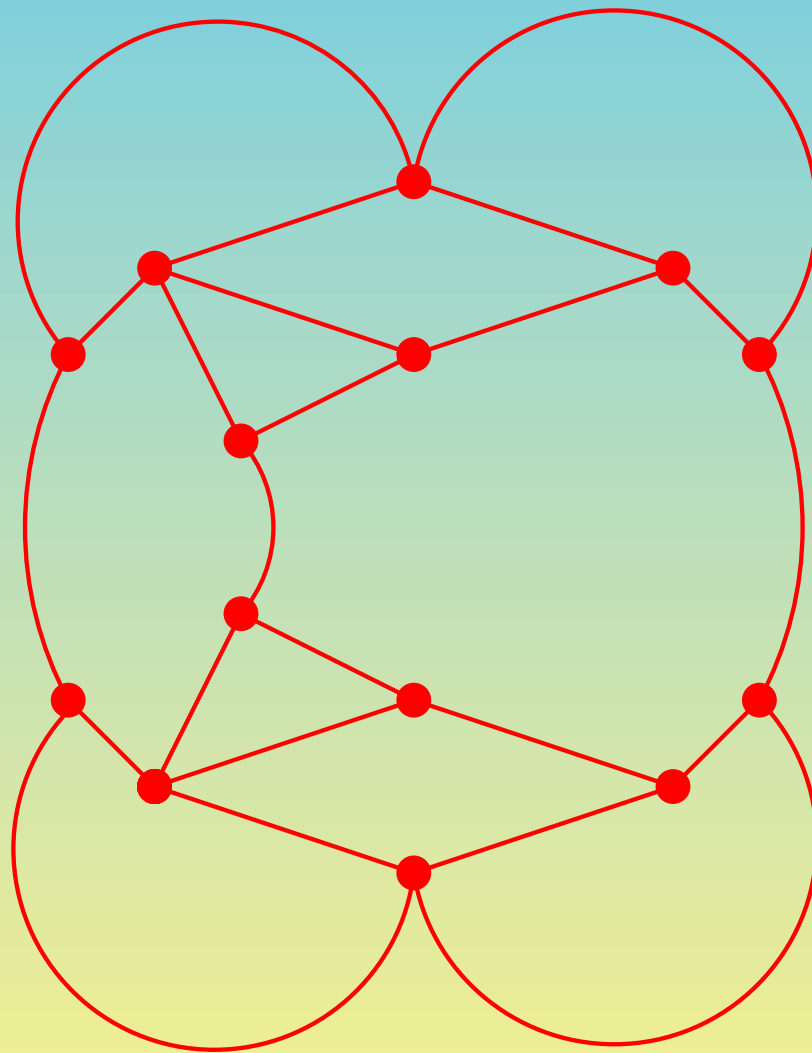
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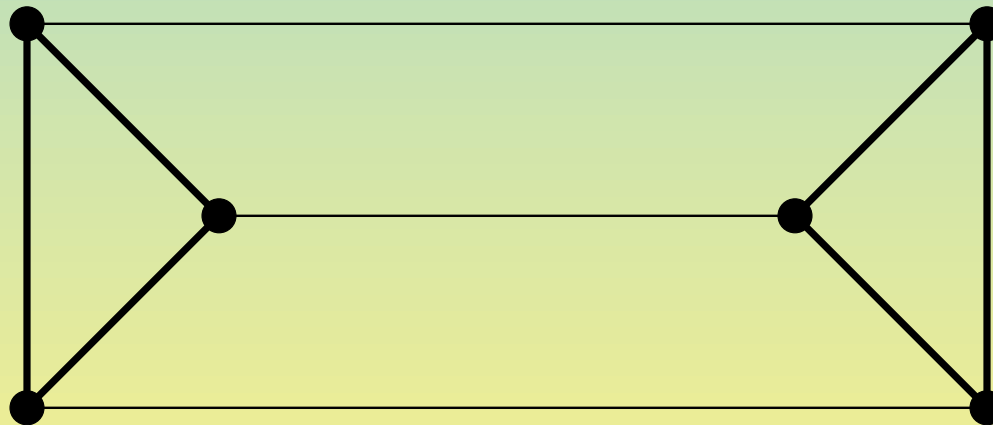


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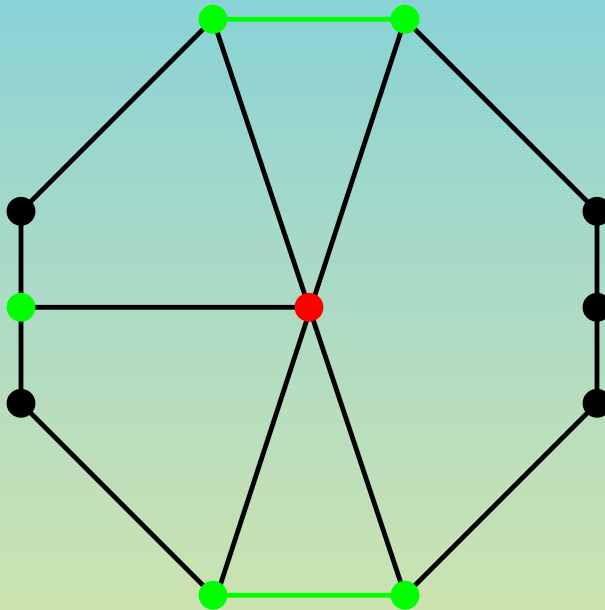
PRISMS

THEOREM If a Berge graph has a prism, then it or its complement is a line graph of a bipartite graph, is a bicograph, has skew partition, has a 2-join or has an M-join (and hence satisfies the conclusion of the main theorem).



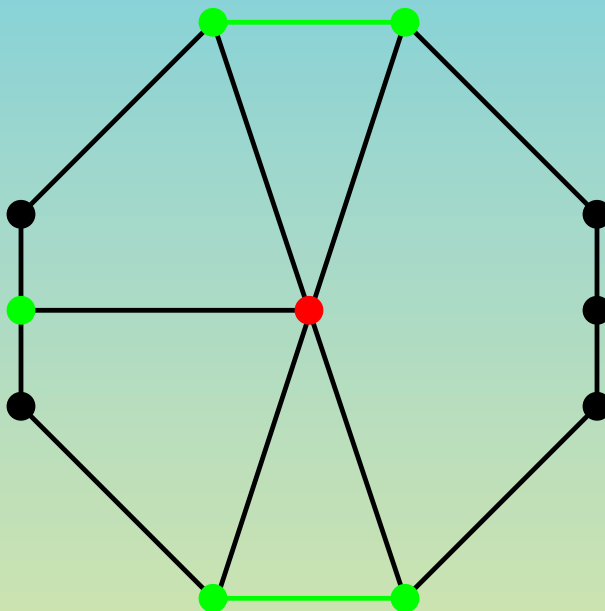
SECOND STEP: WHEELS

A **wheel** consists of a hole of length ≥ 6 (“**rim**”) and a vertex (“**hub**”) forming ≥ 2 triangles.



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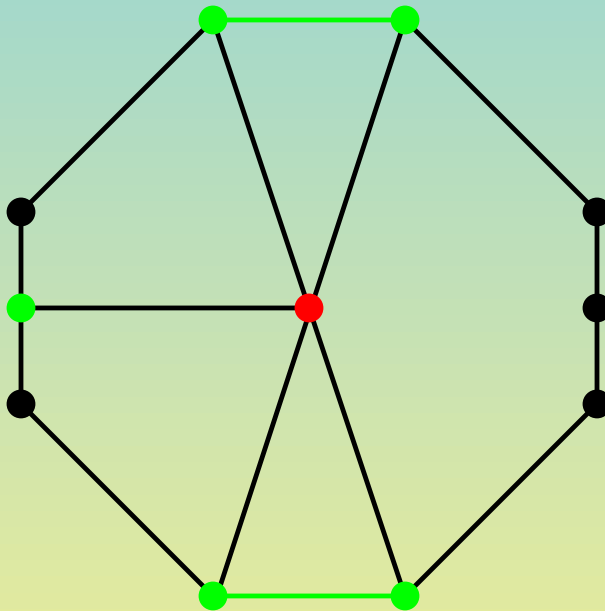
A **wheel** consists of a hole of length ≥ 6 (“**rim**”) and a vertex (“**hub**”) forming ≥ 2 triangles.



THEOREM If a Berge graph has a wheel, then it or its complement has a prism, a skew partition or a 2-join.

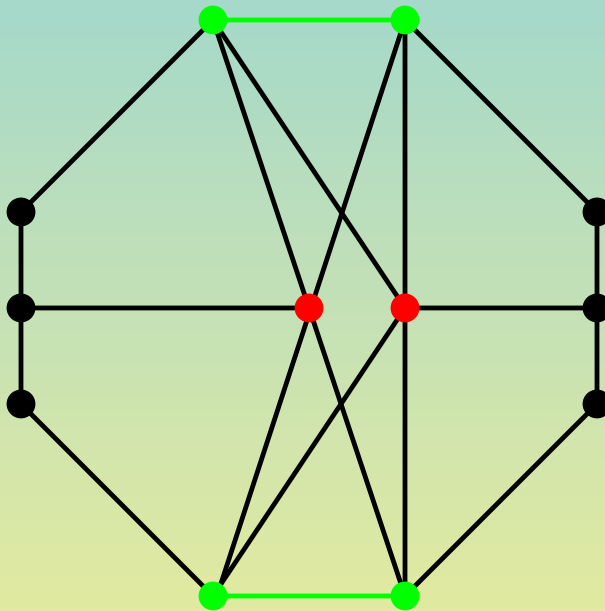
STRATEGY

Take a maximal coconnected set of hubs such that their common neighbors on the rim form ≥ 2 edges. Take all common neighbors of hubs. This tends to be a skew cutset.



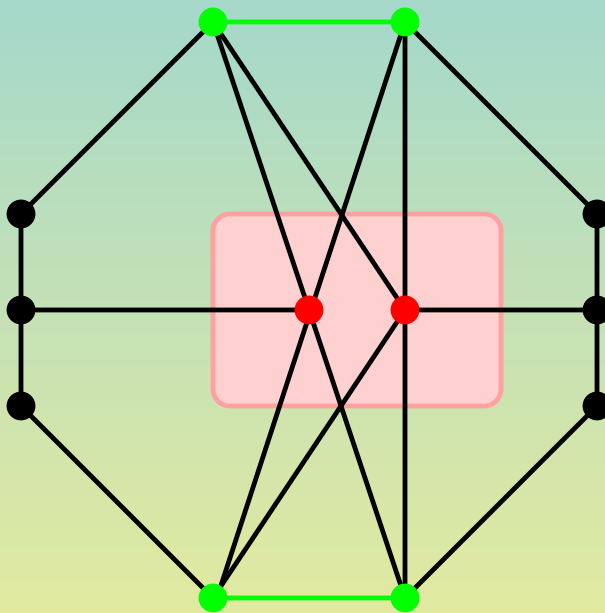
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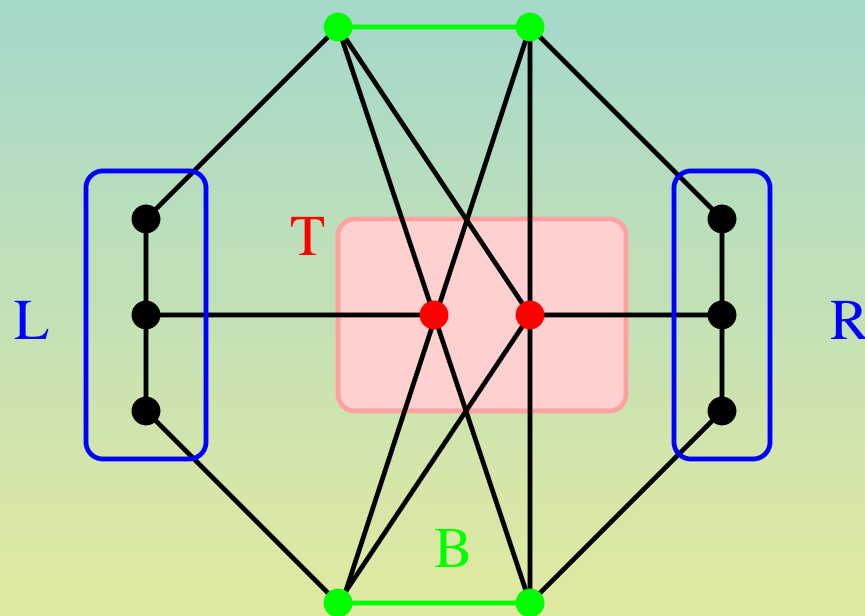
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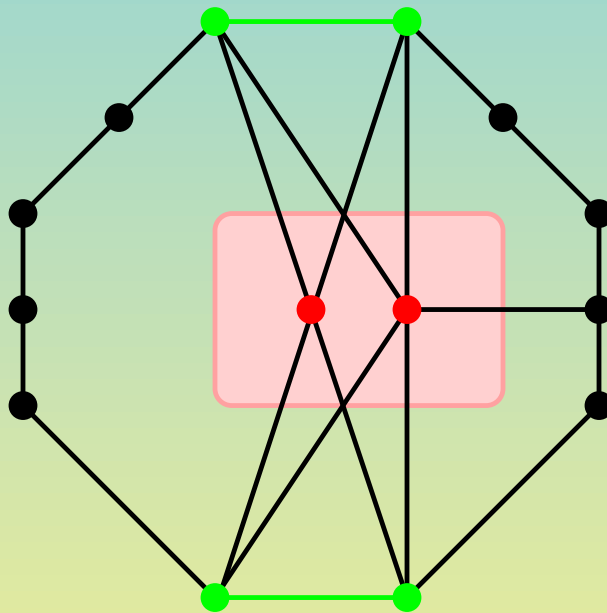
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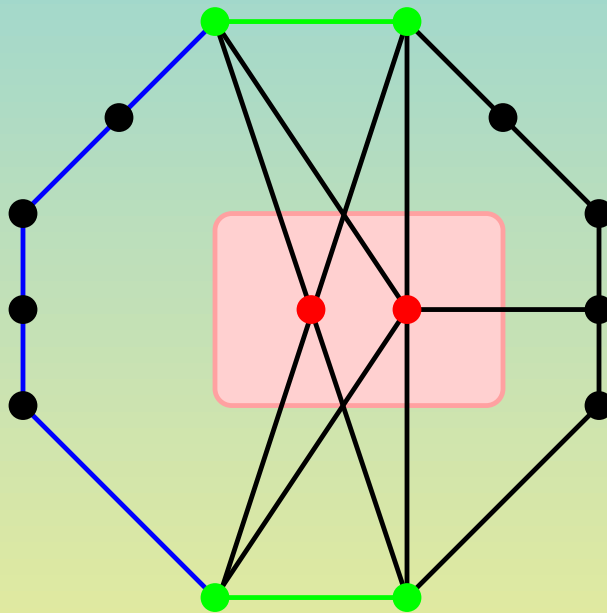
APPLICATION OF ODD PATH LEMMA

Every subpath of the rim between common neighbors of hubs is even or an edge.



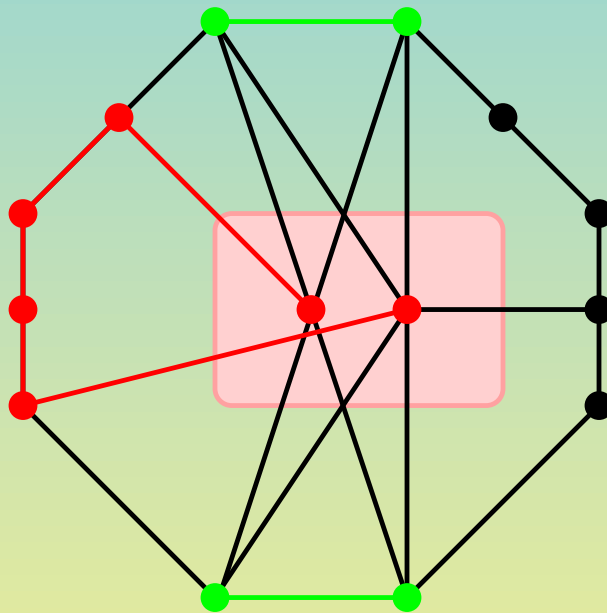
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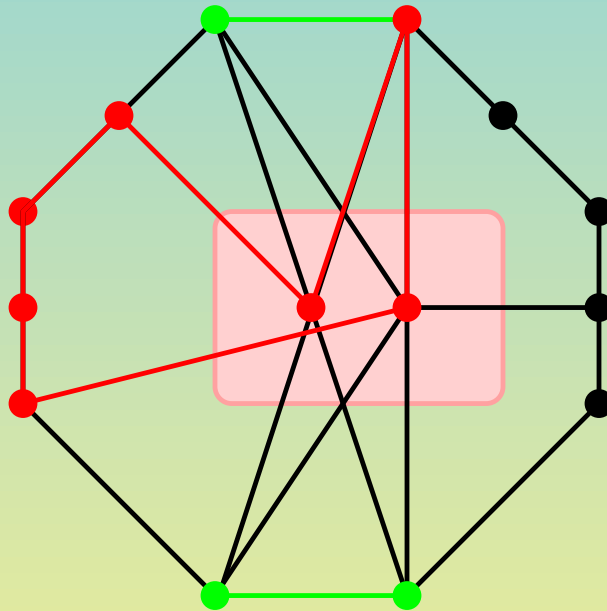
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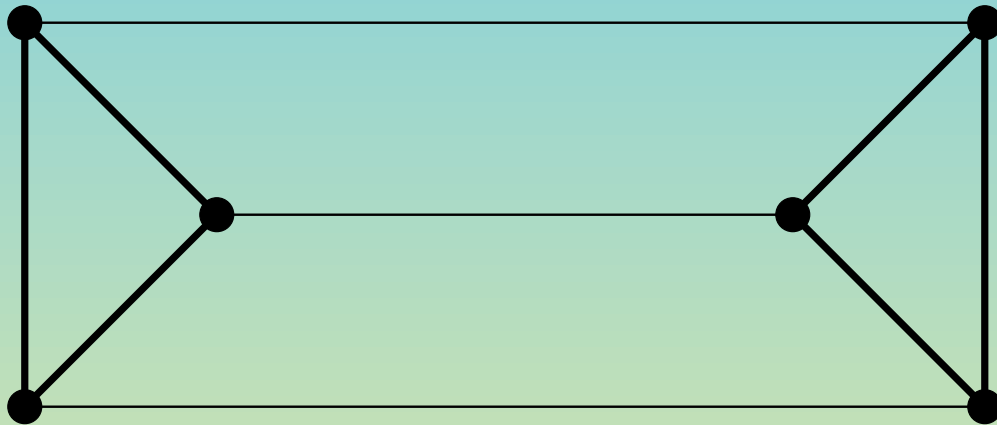
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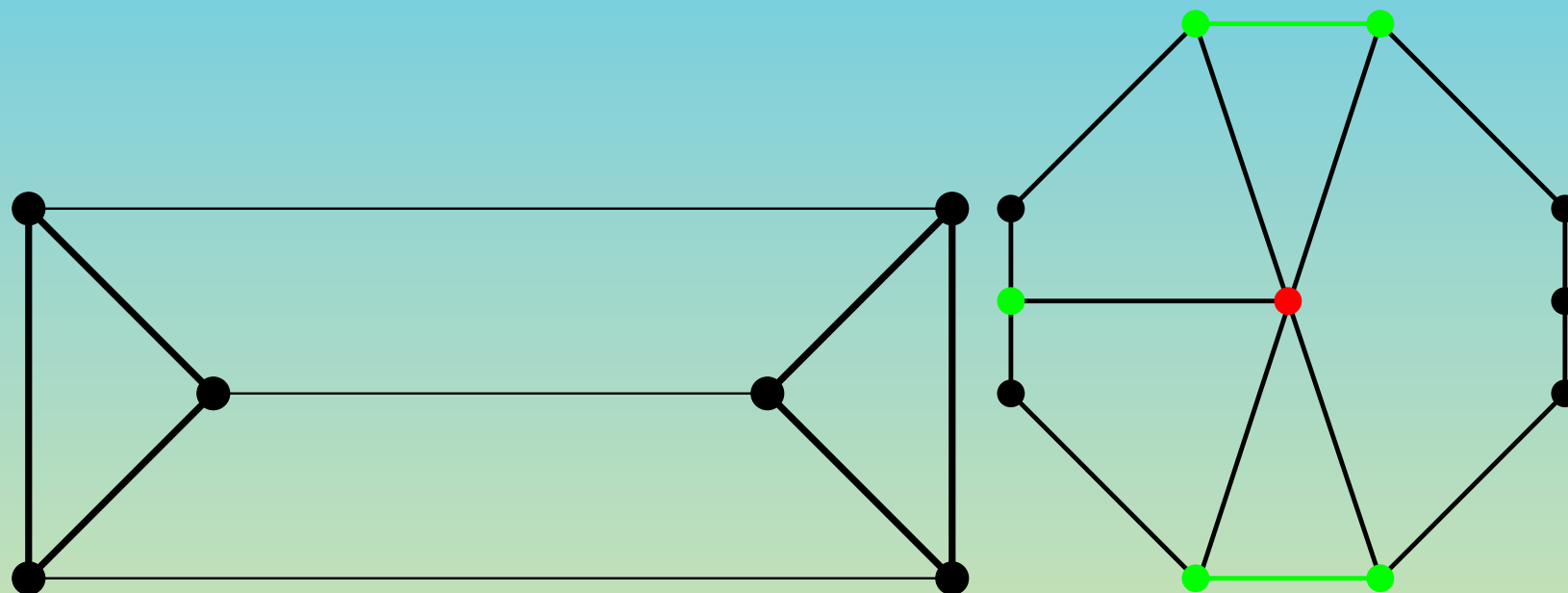
For the SPGC we may assume G has no **even pair**: a pair of vertices such that every induced path between them is even.

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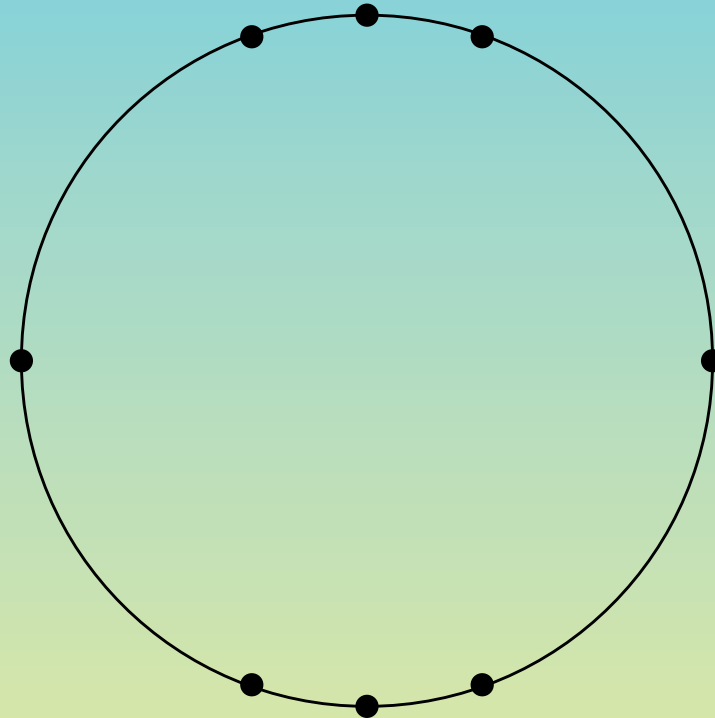
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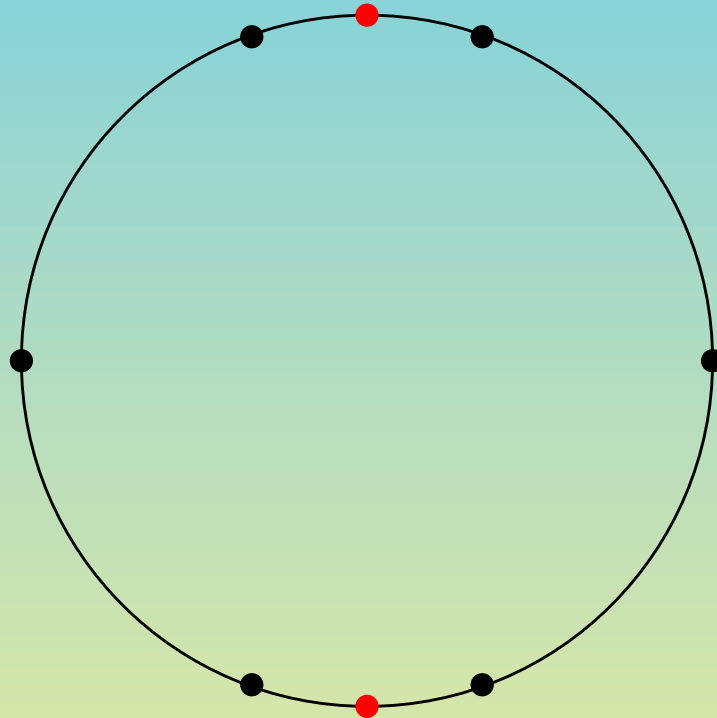
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PROOF WMA G has a hole of length at least six.



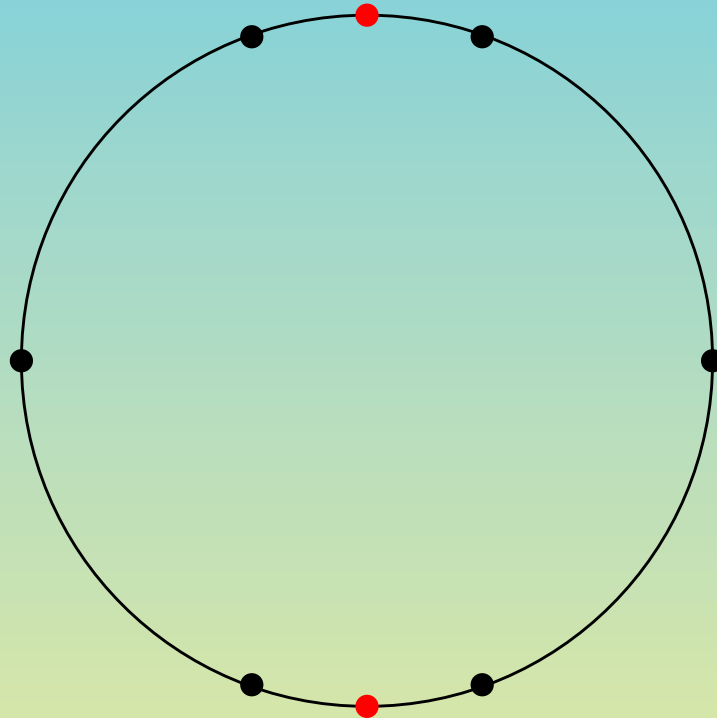
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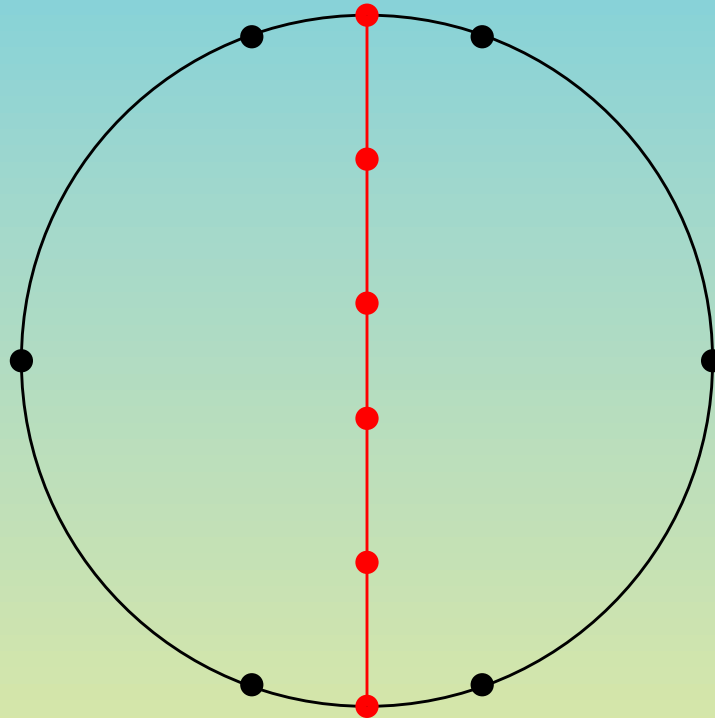
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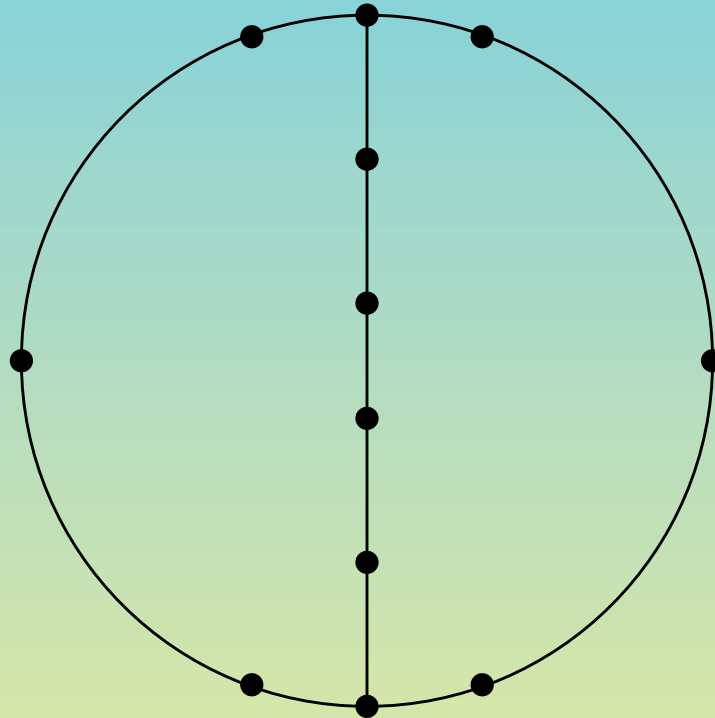
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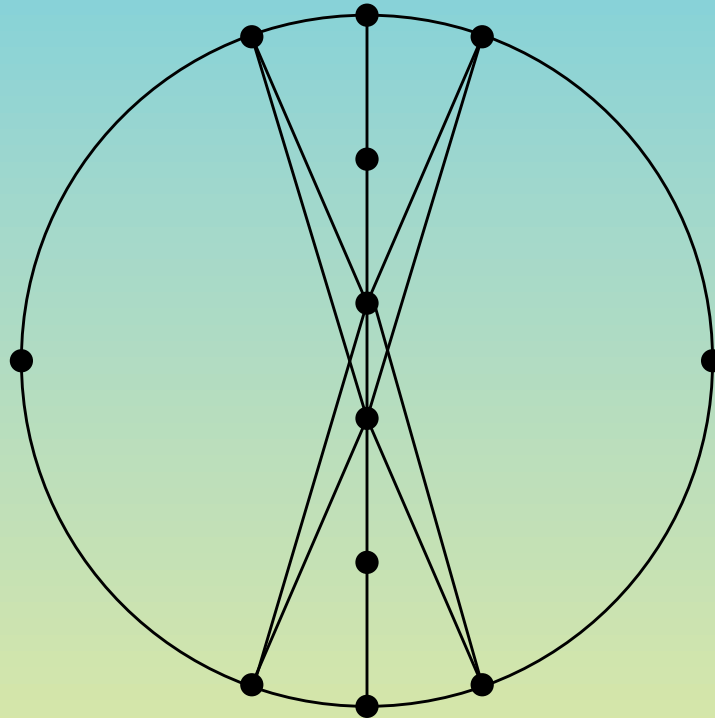
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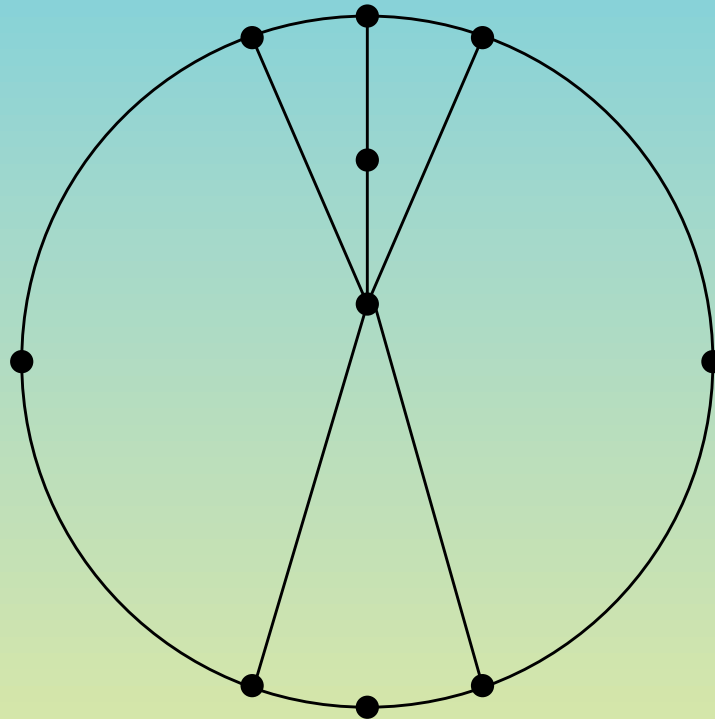
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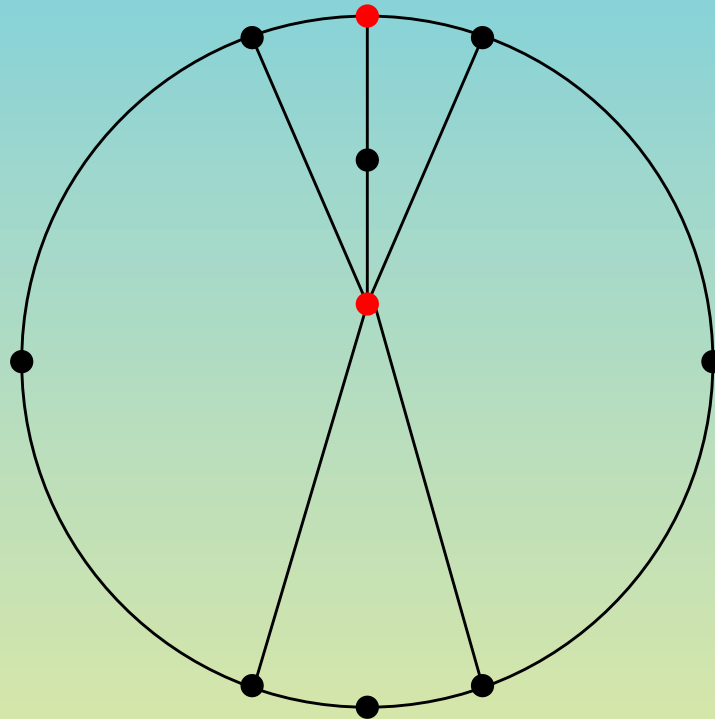
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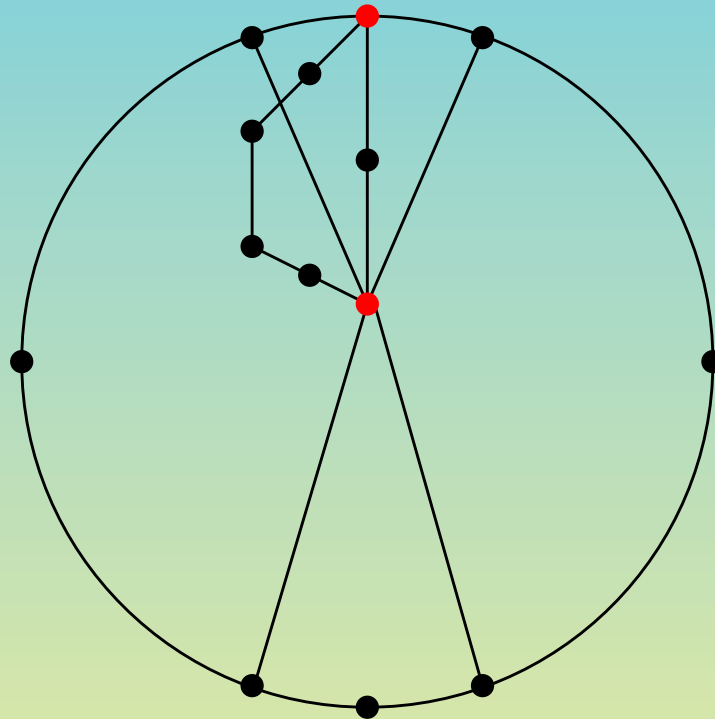
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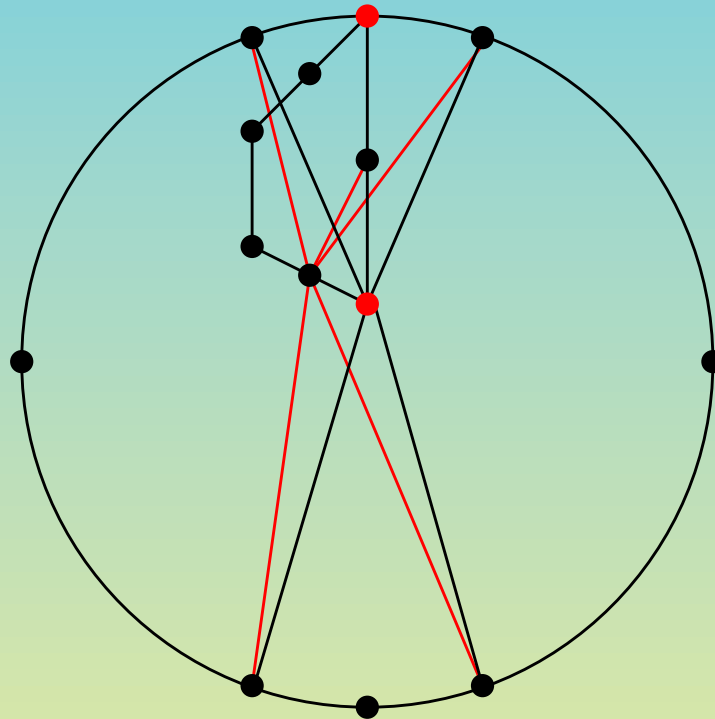
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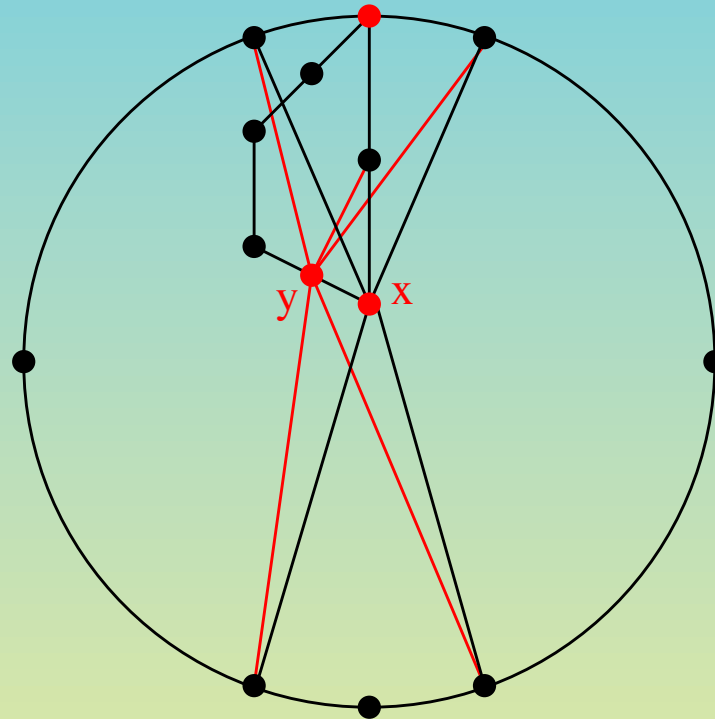
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ALGORITHMS

CONJECTURE Perfection can be tested in polynomial time.

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THEOREM (Conforti, Cornuéjols, Kapoor, Vušković)
There is a polynomial-time algorithm to test if an input graph has an **even** hole.