CS580: Foundations

Exam #2

Thursday, March 25, 2004

Name (PRINTED): _____________________________________

Signature: _____________________________________________

INSTRUCTIONS:

Be concise and clear. This is a closed-book, closed-notes exam. Answer all questions.

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<th>Problem #</th>
<th>Max Score</th>
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1. (50 points)

For each of the following problems, answer True, False, or Open Question. Do not assume any unproven hypotheses. Do not give reasons for your answers.

_____ PATH ∈ NP
_____ PATH ∈ PSPACE
_____ SAT ≤_p PATH
_____ SAT ≤_p HAMPATH
_____ VERTEX-COVER ∈ P
_____ VERTEX-COVER ∈ NP
_____ VERTEX-COVER ∈ PSPACE
_____ VERTEX-COVER is NP-complete
_____ EQREX↑ ∈ PSPACE
_____ PSPACE = NPSPACE
_____ SAT ≤_p EQREX↑
_____ EQREX↑ is NP-complete
_____ CLIQUE ≤_p HAMPATH
_____ CLIQUE ≤_p HAMPATH
_____ CLIQUE ≤_p PATH
_____ SAT ≤_p HAMPATH
_____ TQBF ≤_p PATH
_____ TQBF ≤_p TQBF
_____ DTIME(n^2) ⊆ DTIME(n^2 log n)
_____ DSPACE(n) ⊆ NSPACE(n^2)
_____ NTIME(f(n)) ⊆ DTIME(c^f(n))
_____ DSPACE(n^2) ⊆ DSPACE(n^2.1)
_____ DTIME(n^2) ⊆ DTIME(n^2.1)

_____ Player E has a winning strategy in the following formula:

∃x_1 ∀x_2 ∃x_3 [(x_1 ∧ x_2) ∧ (x_2 ∨ x_3) ∨ (x_2 ∨ x_3)]

_____ Player A has a winning strategy in the following formula:

∃x_1 ∀x_2 ∃x_3 [(x_1 ∨ x_2) ∧ (x_2 ∨ x_3) ∧ (x_2 ∨ x_3)]
2. (15 points)

Define 4-COLOR as follows:

4-COLOR = \{<G> \mid \text{the vertices of } G \text{ can be colored with 4 colors such that no two vertices joined by an edge have the same color}\}.

4-COLOR is known to be NP-complete (which you can assume without proof).

Give the algorithm that, if P = NP, generates a valid coloring of G ∈ 4-COLOR (if one exists) in polynomial time. (The algorithm should output the specific coloring for each vertex.)
3. (20 points)

Consider the following solitaire puzzle. You are given an \( m \times m \) board which has an initial configuration of red and blue stones already placed on it. For each column you must remove either all of the red stones or all of the blue stones in that column. (If a column already has only red stones or only blue stones in it then you do not have to remove any further stones from that column.) The object is to leave at least one stone in each row. This may or may not be possible depending upon the initial configuration. Let \( \text{SOL-PUZZLE} = \{<G> \mid G \text{ is a puzzle with a solution.}\} \)

Show that \( \text{SOL-PUZZLE} \) is NP-complete. (Hint: Reduce from 3SAT.)
4. Let $\text{NEQ}_{nfa} = \{<M_1, M_2> | M_1 \text{ and } M_2 \text{ are NFA's with } L(M_1) \neq L(M_2)\}$.

(a) (5 points)
What is wrong with the following false proof that $\text{NEQ}_{nfa} \in \text{NP}$?

Consider the following nondeterministic polynomial time machine for $\text{NEQ}_{nfa}$. “On input $<M_1, M_2>$, nondeterministically guess a string $x$ and simulate $M_1$ and $M_2$ on $x$. Accept iff one of the machines accepts and one rejects.”

(b) (10 points)
Give a correct proof that $\text{NEQ}_{nfa} \in \text{PSPACE}$. 