1. If $M$ is a DFA accepting language $L$, then exchanging the final and non-final states in $M$ gives a new DFA accepting the complement of $L$. Show, by giving an example, that this is not true in general for NFAs.

2. Give state diagrams of DFAs recognizing the following languages. In all cases, the alphabet is $\{0, 1\}$.
   
   a. $\{ w \mid w \text{ contains at least 3 } 1\text{s} \}$.
   
   b. $\{ w \mid w \text{ does not contain the substring } 110 \}$.
   
   c. $\{ w \mid \text{the length of } w \text{ is at most } 5 \}$.
   
   d. $\{ w \mid w \text{ contains an even number of } 0\text{s or exactly two } 1\text{s} \}$.
   
   e. $\{ w \mid w \text{ contains at most one pair of consecutive } 0\text{’s and at most one pair of consecutive } 1\text{’s} \}$.
   
   f. $\{ w \mid w, \text{ when interpreted as an integer, is divisible by } 5 \}$ (The most significant digit is the first to be read.)

3. Give regular expressions generating the languages of problems 2a-e above. Provide justification that each regular expression is correct.

4. Below is the transition table of an NFA with start state $p$ and accepting states $q$ and $s$. Use subset construction to find an equivalent DFA.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>${q, s}$</td>
<td>${q}$</td>
</tr>
<tr>
<td>$q$</td>
<td>${r}$</td>
<td>${q, r}$</td>
</tr>
<tr>
<td>$r$</td>
<td>${s}$</td>
<td>${p}$</td>
</tr>
<tr>
<td>$s$</td>
<td>${}$</td>
<td>${p}$</td>
</tr>
</tbody>
</table>

5. Design an NFA to recognize the language below, where the alphabet is $\{0, 1\}$. Your NFA should have no more than 13 states and 15 arcs. You may represent your NFA by a transition diagram.

   $\{ w \mid w \text{ ends in } 010 \text{ and has } 011 \text{ somewhere preceding, or } w \text{ ends in } 101 \text{ and has } 100 \text{ somewhere preceding} \}$

6. Prove that every NFA can be converted to an equivalent one that has a single accept state.

7. Give a counterexample to show that the following construction fails to prove the closure of the class of regular languages under Kleene closure (or “star operation”). (In other words, you must present a finite automaton, $N_1$, for which the constructed automaton $N$ does not recognize the star of $N_1$’s language, $A_1^*$.)

   Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize $A_1$. Construct $N = (Q, \Sigma, \delta, q_I, F)$ as follows. $N$ is supposed to recognize $A_1^*$.

   - The states of $N$ are the states of $N_1$.
   - The start state of $N$ is the same as the start state of $N_1$.
   - $F = \{q_I\} \cup F_1$.
   - Define $\delta$ so that for any $q \in Q$ and any $a \in \Sigma^*$,
     
     $\delta(q, a) = \begin{cases} 
     \delta_1(q, a) & q \notin F_1 \text{ or } a \neq \varepsilon \\
     \delta_1(q, a) \cup \{q_I\} & q \in F_1 \text{ and } a = \varepsilon 
     \end{cases}$