Problem Set 2

Since each problem will be graded separately, please turn in each on a separate page with your name.

1. Using the pumping lemma, show that \( \{a^mb^n \mid m > n\} \) is not regular.

2. Give an example non-regular language where the pumping lemma is true.

3. For any language \( L \), let PREFIX-REMOVE(\( L \)) = \{w \mid w \in L \) and no proper prefix of \( w \) is in \( L \} \). Show that the class of regular languages is closed under the PREFIX-REMOVE operation.

4. For any language \( L \), let \( L^R = \{w \mid \) the reverse of \( w \) is in \( L \} \). Show that if \( L \) is regular then so is \( L^R \).

5. Let \( \Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\} \). That is, \( \Sigma \) contains all size 2 columns of 0’s and 1’s. Let:

\[
A = \{ w \in \Sigma^* \mid \text{the second row of } w \text{ is 5 times the first row (both rows interpreted as binary numbers)} \}.
\]

So: \( \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \in A \), but: \( \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \not\in A \).

Show that \( A \) is regular. (Hint: Work with \( A^R \).)

6. Suppose one starts with an \( n \)-state DFA. Using the construction discussed in class, how big can the equivalent regular expression be? You do not need to give an exact answer here; a rough upper bound is sufficient.