Problem Set 4

Since each problem will be graded separately, please turn in each on a separate page with your name.

1. Let a $k$-PDA be a pushdown automaton that has $k$ stacks. Thus a 0-pda is an NFA and a 1-pda is a conventional PDA. We already know that 1-PDAs are more powerful (recognize a larger class of languages) than 0-PDAs.
   a) Show that 2-PDAs are more powerful than 1-PDAs.
   b) Show that 3-PDAs are not more powerful than 2-PDAs.
      (Hint: Simulate a Turing machine tape with 2 stacks.)

2. Let $A = \{<R>: R$ is a regular expression describing a language containing at least one string $w$ which has 111 as a substring (i.e., $w = x111y$ for some $x$ and $y$)\}. Show that $A$ is decidable.

3. A linearly bounded automaton (LBA) is a deterministic 1-tape Turing machine whose head is not permitted to move off the input portion of the tape (except to read one blank symbol past the right hand end of the input so that it can detect where the input ends.) Let $K_{LBA} = \{<M, w>: M$ is an LBA accepting $w$\}.
   a) For $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$, let $k = |Q|$ and $g = |F|$. Show that if $M$ runs for more than $k(n + 1)g^{n+1}$ steps on an input of length $n$ then it will run forever.
   b) Use a) to show $K_{LBA}$ is decidable.

4. Let $C$ be a language. Prove that $C$ is recursively enumerable iff there is a decidable language $D$ such that $C = \{x : \exists y (<x, y> \in D)\}$.

5. Consider the problem of testing whether a two tape Turing machine ever writes a non-blank symbol on its second tape. Formulate this problem as a language, and show that it is not decidable.

6. a) Show that the Post’s Correspondence Problem (PCP) is decidable over a unary alphabet (i.e., over the alphabet $\Sigma = \{1\}$).
   b) Show that PCP is undecidable over a binary alphabet (i.e., over the alphabet $\Sigma = \{0, 1\}$).

7. Define a two-headed finite automaton (2FA) to be a deterministic finite automaton which has two read-only, bidirectional heads that start at the left end of the input tape and which can be independently controlled to move in either direction. (Assume that the input is given with left and right delimiters around it.) A 2FA accepts its input by entering a special halt state. For example, it is easy to see that a 2FA can accept the language $\{a^n b^n c^n : n \geq 0\}$.
   a) Let $K_{2FA} = \{<M, x>: M$ is a 2FA and $M$ accepts $x$\}. Show $K_{2FA}$ is decidable.
   b) Let $E_{2FA} = \{<M>: M$ is a 2FA and $L(M) = \emptyset$\}. Show $E_{2FA}$ is not decidable.