1. Prove that the class of languages $\mathcal{NP}$ is closed under each of the following operations:
   a. Union
   b. Intersection
   c. Concatenation
   d. Kleene star.

2. Show that the Hamiltonian Path problem can be solved in polynomial time on directed acyclic graphs. Give the run time complexity of your solution.

3. Prove that $P \subseteq \text{co-NP}$.

4. Show that the $\leq_p$ relative is a transitive relation on languages. That is, show that if:
   \[ L_1 \leq_p L_2 \text{ and } L_2 \leq_p L_3 \text{ then } L_1 \leq_p L_3. \]

5. Let 5-CLIQUE = \{<G> | G is an undirected graph having a complete subgraph with 5 nodes\}. Show that 5-CLIQUE is in P.

6. Let HALF-CLIQUE = \{<G> | G is an undirected graph having a complete subgraph with $\lfloor n/2 \rfloor$ nodes, where $n$ is the number of nodes in G\}. Show that HALF-CLIQUE is NP-complete.

7. Let 3-COLOR = \{<G> | the nodes of G can be colored with 3 colors such that no two nodes joined by an edge have the same color\}. Show that 3-COLOR is NP-complete.
   
   Hint: In your reduction, let “True”, “False”, and “Red” be the 3 colors and use the following “gadget” structures in your construction:

   ![Diagram](image)