P, NP, co-NP: Known, Unknown, and Implications thereof

The following categories list fundamental facts about P, NP, and co-NP that are known to be true, are unknown (i.e., open problems), and implications about what we do and don’t know. You should be comfortable with your knowledge of P, NP, and co-NP as it relates to the information in this document.

**Known**

- $P \subseteq NP$
- $P \subseteq co-NP$
- $P \subseteq NP \cap co-NP$
- P is closed under complementation, Kleene star, union, concatenation, and intersection.
- NP is closed under Kleene star, union, concatenation, and intersection.
- If $L_1$ is NP-complete, and $L_1 \leq^p L_2$, then $L_2$ is NP-complete.

**Unknown/Open Questions**

- $P = NP$? (or, equivalently, $P \neq NP$?)
- Is NP closed under complementation? (i.e., is $NP = co-NP$?)
- $P = NP \cap co-NP$?

**Implications from what is known**

- If $NP \neq co-NP$, then $P \neq NP$.
- If NP-complete problem is in P, then $P = NP$.
- If $P \neq NP$, then there exists a problem in NP that is not NP-complete.
- If $A$ is NP-complete, and $\overline{A}$ is in NP, then $NP = co-NP$. 