A presentation on
Randomized search strategies
with imperfect sensors

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Overview of Present (1993)

- Very inexpensive autonomous mobile robots with fairly limited sensor capabilities are coming soon (i.e., hardware).
- The challenges will be technical and management (i.e., the software).
  - E.G., making systems more user-friendly by defining “meaningful mission-oriented system-level parameters.”
Background

- **Coverage** – “application of the effects of some sensor or effector to some extended physical space.”
  - *Blanket* – static deployment to detect targets.
  - *Barrier* – minimize undetected penetration
  - *Sweep* – move through the area
  - Military apps. include: intelligent land mine deployment, shallow water mine sweeping, reconnaissance, sentry duty, comms. relay, maintenance inspection, carrier deck FOD disposal, ship hull cleaning.
This paper

- **Search** — the canonical type of coverage
  - a number of searching elements move about search area to find stationary or mobile targets.


- This paper:
  - Coordinated vs. random search models.
  - Randomized strategies for uniform search coverage
Target Detection Sensor
Abstractions

Consider the following search:

• 1+ searchers, 1+ stationary targets
• Targets a priori equally likely to be at any part of search area.
• Detection at any instant depends on:
  • relative geometry of searcher & target
  • physical characteristics of sensor, environment, target
Lateral Range Curve

Integrated encounter probability vs. searcher’s distance of closest approach to the target.

Gage, Fig. 1a & 1b
Definite Range Law or “Cookie Cutter”

Sensor characterized by single parameter: max detection range

Creates a “binary” detection

Gage, Figure 1c
Imperfect Sensor

Second parameter: mean probability of detection less-than-one over specified range.

Cookie Cutter (previous) would give unreasonably optimistic predictions when multiple coordinated passes are made through an area, so imperfect is more reasonable.

Gage, Figure 1d
The Search

Consider:

- N robots, each with an "imperfect" sensor
- r nominal range
- p detection probability (any target within r distance has p probability of being detected)
- Targets > r are not detected
- No false alarms

- No action on detected targets
- d total robot distance (each) during mission
- A search area
- S
  - average # of times each point in the search area is sensed
  - "Sweep fraction"
  - \( S = 2r d N/A \)
The Search: Coordinated and Random Search Processes

- Let $D =$ probability that any given target is detected.
- $D$ is equivalent to the expected fraction of targets detected.
- Calculate $D$ for
  1) perfectly coordinated search pattern
  2) completely random search pattern (which is less expensive to implement)
The Search: Urn Example

We have:

- An Urn - contains very large # of apparently identical marbles, a small percentage of which bear an invisible mark.
- A machine – can detect marks with probability of detection $p$ (no false positives)
Urn example continued

Our Strategies:

- **Coordinated search:**
  - Pull marbles at random, test them.
  - Place unmarked marbles in second urn.
  - When all marbles tested, repeat process.

- **Random search:**
  - Pull marbles at random, test them.
  - Return unmarked marbles to urn and mix.

Comparison:

- **Coordinated search is superior**
  - If $n$ passes are made, all marbles have been tested at least $n - 1$ times.
  - No such guarantee for random.

- **Random is cheaper**
  - No second urn required

Note: Coordinated search is best, but real world constraints (i.e. navigation inaccuracies) often yield results on par with random search.
Urn example continued (2)

Coordinated:
- \( D_c = 1 - (1 - p)^S \)
  - where \( S \) measures avg. # of times each marble has been tested
  - Only true for integers \( S \)
  - Upper bound; good for large \( S \)

Random
- \( D_r = 1 - e^{-pS} \)
  - Good for non-integer \( S \)

Gage, Figure 2
(for \( p = 0.8 \))
Different apps. Require different metrics

Calculate “search gain” $G$

- equate $D_c$ with $D_r$
- “Factor reduction in required search effort”

$G_{\text{many target}} = \frac{S_r}{S_c} = \frac{-\ln(1 - p)}{p}$

$G_{\text{single target}} = \frac{<S_r>}{<S_c>} = \frac{1/p}{1/p - 1/2} = \frac{2}{2-p}$
The Search: Measuring strategy effectiveness continued

- Gage, Figure 3: As $p$ increases, gain increases dramatically for multiple targets
Randomized Search Strategies

Remember the earlier question: How to generate a randomized search path which provides uniform coverage?

The path generator algorithm must be such that:

- A target cannot predict a searcher’s future path by knowing its prior path
- The path cannot be more efficient than random or the target could favor already searched locations
- The path cannot be a fixed reaction to the environment
- The periphery is not neglected
McNish (1987) used diffuse reflection – how light reflects off a matte surface – to provide uniform search coverage.

- Uses algorithm: \( \text{Prob}(\theta) = \frac{1}{2} \sin(\theta) \, d\theta \) where \( \theta \) measures the angle of the chord from the tangent to the boundary reflection point.
- Lalley and Robbins proved that as length of path goes to \( \infty \), search effort is the same at every point in \( D \).
Random Strategies: Generalization to Convex

Motivation – It is impossible to “tile” an area with circles; rectangles are preferable.

When the searcher knows the boundaries of the area, his position, and targets are stationary, apply the following generalization:

- Choose point P in convex search area D
- $\theta = \text{angle from tangent to boundary of } D \text{ at } P \text{ to any chord}$
- $K(\theta) = \text{length of the chord from } P \text{ across } D \text{ at angle } \theta$
- Draw PQ and PR at $\theta_1$ and $\theta_2$
Generalization to Convex (2)

Area \( (A) = \int_{\theta_1}^{\theta_2} \frac{1}{2} K(\theta)^2 \, d\theta \)

Effort(\( \theta \)) = K(\( \theta \)) width

\(<\text{Effort}(A)> = \int_{\theta_1}^{\theta_2} \text{Prob}(\theta) \text{ Effort}(\theta) \, d\theta \)

If \(<\text{Effort}(A)\) is proportional to \( A \) for any \( \theta_1 \) and \( \theta_2 \)
then
\[ \text{Prob}(\theta) = \text{constant} \, K(\theta) \]

Gage, Figure 4a
This strategy gives uniform effort over entire search area on per-chord basis.

For convex, need to know: position, distance to boundary in all directions.

For circle: just position and size of circle.

For non-convex: Impossible unless the area is composed of joined symmetric convex spaces.
Conclusion

Randomized is better for inexpensive robots for 2 reasons:

- $G$ decreases as $P$ decreases
- Cost/Benefit

Roboticists must analyze and select appropriate strategies for appropriate applications.
Summary

Issue: Find good random search strategies

Method:
- Use an “Imperfect Sensor” model
- One random strategy is *Diffuse Reflection*
  - Gives uniform search over entire area
- Generalize Diffuse Reflection for all convex areas

Results: Diffuse Reflection proven to be effective but no experiments detailed.
Evaluation of Method:

- Gage rightly emphasizes the need to analyze many strategies.
- Expands earlier work well.
- Only provides a solution for one small area of the issue, and thus is very limited in scope.

Contributions of this paper with regard to other work:

- Expansion of Gage’s earlier paper that dealt more with relationship between sensor cost and performance levels in designing multi-robot teams.
- Also expands McNish’s (1987) method of random path generation using diffuse reflection in a circular area to all convex areas, which is very useful.
Questions???