Implement the Particle Swarm Optimization (PSO) algorithm. Your program should take as arguments:

1. Number of epochs (iterations)
2. Number of particles
3. Inertia
4. Cognition parameter
5. Social parameter

Good resources for implementing this project are the following:

http://web.eecs.utk.edu/~kvanhorn/cs594_bio/project6/pso.html
http://en.wikipedia.org/wiki/Particle_swarm_optimization
http://www.swarmintelligence.org/tutorials.php

You will test your implementation on the two problems that follow. First, let

\[ m\text{dist} = \sqrt{\text{max}_x^2 + \text{max}_y^2} / 2 \]

\[ p\text{dist} = \sqrt{(p_x - 20)^2 + (p_y - 7)^2} \]

\[ n\text{dist} = \sqrt{(p_x + 20)^2 + (p_y + 7)^2} \]

where \( p_x \) and \( p_y \) are the \( x \) and \( y \) coordinates of a particle’s position, and \( \text{max}_x \) and \( \text{max}_y \) are the world width and world height.

**Problem 1:**

\[ Q(p_x, p_y) = 100 \cdot \left(1 - \frac{p\text{dist}}{m\text{dist}}\right) \]

**Problem 2:**

\[ Q(p_x, p_y) = 9 \cdot \text{max}(0, 10 - p\text{dist}^2) + 10 \cdot \left(1 - \frac{p\text{dist}}{m\text{dist}}\right) + 70 \cdot \left(1 - \frac{n\text{dist}}{m\text{dist}}\right) \]
Problem 1 has one maximum at (20, 7) and Problem 2 has two maxima: one at (20, 7) (a global maximum) and one at (−20, −7) (a local maximum).

For each problem, experiment with several different values and combinations of the following parameters:

1. Number of particles
2. Inertia
3. Cognitive parameter
4. Social parameter

Try to draw conclusions on how the parameters affect the performance of the algorithm.

The algorithm’s performance can be measured in several ways (besides number of epochs until convergence). First, it can be measured in terms of the difference of the average $x$ coordinate of the particles and the $x$ coordinate of the global maximum, similar for the $y$ coordinates. Second, performance can be measured in terms of the percentage of particles that have converged to within a certain epsilon (e.g. 0.001) of the global maximum. Finally, performance can be evaluated qualitatively in the form of a scatter plot of the particles (and the maxima) at the end of the iterations of the algorithm. Graphs can also be obtained from the first two measures, where the $x$ axis contains the number of epochs and the $y$ axis contains the performance measure. These graphs should be included in your report, along with a discussion of how the parameters influence the algorithm’s behavior.

For graduate credit:

In addition to the constant inertia implemented above, experiment with decreasing inertia. This means that the inertia will decrease over time (i.e. a certain amount per epoch), similar to temperature cooling in simulated annealing. The decrease can be linear or some percentage of the current inertia value, your choice. Experiment with and discuss how this affects the algorithm’s behavior for both problems.

In the PSO algorithm given in the Wikipedia article, in addition to global best and personal best, the update formula for velocity also includes neighborhood (local) best (and thus a third parameter in addition to the ones for cognition and social influence). A neighborhood of a particle is given by its two nearest topological neighbors (plus itself). Experiment with this variant of the velocity update formula and discuss the effect on the algorithm’s performance on the two problems. For extra credit, analyze $k$-nearest neighbors for different values of $k$ and/or neighborhoods based on Euclidean distance.

For extra credit (all students):

You will probably find that the algorithm does not perform optimally for the second problem. Many of the points may not converge to the global maximum. See if you can find ways to tweak the update formulas or find a set of parameters that will produce results that are close to optimal.