VII. Cooperation & Competition

A. The Iterated Prisoner's Dilemma

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The Prisoners' Dilemma

- Devised by Melvin Dresher & Merrill Flood in 1950 at RAND Corporation
- Further developed by mathematician Albert W. Tucker in 1950 presentation to psychologists
- It "has given rise to a vast body of literature in subjects as diverse as philosophy, ethics, biology, sociology, political science, economics, and, of course, game theory." — S.J. Hagenmayer
- "This example, which can be set out in one page, could be the most influential one page in the social sciences in the latter half of the twentieth century." — R.A. McCain

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Prisoners' Dilemma: The Story

- Two criminals have been caught
- They cannot communicate with each other
- If both confess, they will each get 10 years
- If one confesses and accuses other:
  - confessor goes free
  - accused gets 20 years
- If neither confesses, they will both get 1 year on a lesser charge

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Prisoners' Dilemma Payoff Matrix

		Bob	
		cooperate	defect
Ann	cooperate	–1, –1	–20, 0
	defect	0, –20	–10, –10

- defect = confess, cooperate = don't
- payoffs < 0 because punishments (losses)

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Ann's "Rational" Analysis (Dominant Strategy)

		Bob	
		cooperate	defect
Ann	cooperate	–1, –1	–20, 0
	defect	0, –20	–10, –10

- if cooperates, may get 20 years
- if defects, may get 10 years
- ∴, best to defect

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Bob's "Rational" Analysis (Dominant Strategy)

		Bob	
		cooperate	defect
Ann	cooperate	–1, –1	–20, 0
	defect	0, –20	–10, –10

- if he cooperates, may get 20 years
- if he defects, may get 10 years
- ∴, best to defect

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Suboptimal Result of "Rational" Analysis

		Bob	
		cooperate	defect
Ann	cooperate	–1, –1	–20, 0
	defect	0, –20	–10, –10

- each acts individually rationally ⇒ get 10 years (dominant strategy equilibrium)
- "irrationally" decide to cooperate ⇒ only 1 year

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Summary

- Individually rational actions lead to a result that all agree is less desirable
- In such a situation you cannot act unilaterally in your own best interest
- Just one example of a (game-theoretic) *dilemma*
- Can there be a situation in which it would make sense to cooperate unilaterally?
  - **Yes**, if the players can expect to interact again in the future

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The Iterated Prisoners' Dilemma

and Robert Axelrod's Experiments

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Assumptions

- No mechanism for enforceable threats or commitments
- No way to foresee a player's move
- No way to eliminate other player or avoid interaction
- No way to change other player's payoffs
- Communication only through direct interaction

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Axelrod's Experiments

- Intuitively, expectation of future encounters may affect rationality of defection
- Various programs compete for 200 rounds
  - encounters each other and self
- Each program can remember:
  - its own past actions
  - its competitors' past actions
- 14 programs submitted for first experiment

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IPD Payoff Matrix

		B	
		cooperate	defect
A	cooperate	3, 3	0, 5
	defect	5, 0	1, 1

N.B. Unless DC + CD < 2 CC (i.e. $T + S < 2R$), can win by alternating defection/cooperation

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Indefinite Number of Future Encounters

- Cooperation depends on expectation of indefinite number of future encounters
- Suppose a known finite number of encounters:
  - No reason to C on last encounter
  - Since expect D on last, no reason to C on next to last
  - And so forth: there is no reason to C at all

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Analysis of Some Simple Strategies

- Three simple strategies:
  - **ALL-D**: always defect
  - **ALL-C**: always cooperate
  - **RAND**: randomly cooperate/defect
- Effectiveness depends on environment
  - **ALL-D** optimizes local (individual) fitness
  - **ALL-C** optimizes global (population) fitness
  - **RAND** compromises

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Expected Scores

⇓ playing ⇒	ALL-C	RAND	ALL-D	Average
ALL-C	3.0	1.5	0.0	1.5
RAND	4.0	2.25	0.5	2.25
ALL-D	5.0	3.0	1.0	3.0

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Result of Axelrod's Experiments

- Winner is Rapoport's **TFT** (Tit-for-Tat)
  - cooperate on first encounter
  - reply in kind on succeeding encounters
- Second experiment:
  - 62 programs
  - all know **TFT** was previous winner
  - **TFT** wins again

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Expected Scores

⇓ playing ⇒	ALL-C	RAND	ALL-D	TFT	Avg
ALL-C	3.0	1.5	0.0	3.0	1.875
RAND	4.0	2.25	0.5	2.25	2.25
ALL-D	5.0	3.0	1.0	$1+4/N$	2.5+
TFT	3.0	2.25	$1-1/N$	3.0	2.3125–

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Demonstration of Iterated Prisoners' Dilemma

Run NetLogo demonstration
PD N-Person Iterated.nlogo

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Characteristics of Successful Strategies

- *Don't be envious*
  - at best **TFT** ties other strategies
- *Be nice*
  - i.e. don't be first to defect
- *Reciprocate*
  - reward cooperation, punish defection
- *Don't be too clever*
  - sophisticated strategies may be unpredictable & look random; be clear

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Tit-for-Two-Tats

- More forgiving than **TFT**
- Wait for two successive defections before punishing
- Beats **TFT** in a noisy environment
- E.g., an unintentional defection will lead **TFT**s into endless cycle of retaliation
- May be exploited by feigning accidental defection

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Effects of Many Kinds of Noise Have Been Studied

- Misimplementation noise
- Misperception noise
  - noisy channels
- Stochastic effects on payoffs
- General conclusions:
  - sufficiently little noise ⇒ generosity is best
  - greater noise ⇒ generosity avoids unnecessary conflict but invites exploitation

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More Characteristics of Successful Strategies

- Should be a generalist (robust)
  - i.e. do sufficiently well in wide variety of environments
- Should do well with its own kind
  - since successful strategies will propagate
- Should be cognitively simple
- Should be evolutionary stable strategy
  - i.e. resistant to invasion by other strategies

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Kant's Categorical Imperative

"Act on maxims that can at the same time have for their object themselves as universal laws of nature."

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Ecological & Spatial Models

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Ecological Model

- What if more successful strategies spread in population at expense of less successful?
- Models success of programs as fraction of total population
- Fraction of strategy = probability random program obeys this strategy

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Variables

- $P_i(t)$ = probability = proportional population of strategy i at time t
- $S_i(t)$ = score achieved by strategy i
- $R_{ij}(t)$ = relative score achieved by strategy i playing against strategy j over many rounds
  - fixed (not time-varying) for now

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Computing Score of a Strategy

- Let n = number of strategies in ecosystem
- Compute score achieved by strategy i:

$$S_i(t) = \sum_{k=1}^{n} R_{ik}(t) P_k(t)$$

$$\mathbf{S}(t) = \mathbf{R}(t)\mathbf{P}(t)$$

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Updating Proportional Population

$$P_i(t+1) = \frac{P_i(t)S_i(t)}{\sum_{j=1}^{n} P_j(t)S_j(t)}$$

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Some Simulations

- Usual Axelrod payoff matrix
- 200 rounds per step

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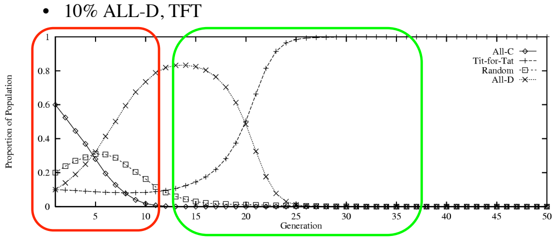
Demonstration Simulation

- 60% ALL-C
- 20% RAND
- 10% ALL-D, TFT



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NetLogo Demonstration of Ecological IPD

Run EIPD.nlogo

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Collectively Stable Strategy

- Let w = probability of future interactions
- Suppose cooperation based on reciprocity has been established
- Then no one can do better than **TFT** provided:

$$w \geq \max\left(\frac{T-R}{R-S}, \frac{T-R}{T-P}\right)$$

- The **TFT** users are in a Nash equilibrium

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"Win-Stay, Lose-Shift" Strategy

- Win-stay, lose-shift strategy:
  - begin cooperating
  - if other cooperates, continue current behavior
  - if other defects, switch to opposite behavior
- Called **PAV** (because suggests Pavlovian learning)

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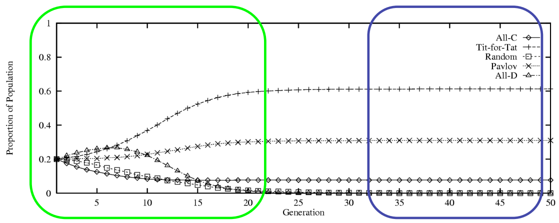
Simulation without Noise

- 20% each
- no noise



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Effects of Noise

- Consider effects of noise or other sources of error in response
- **TFT**:
  - cycle of alternating defections (CD, DC)
  - broken only by another error
- **PAV**:
  - eventually self-corrects (CD, DC, DD, CC)
  - can exploit **ALL-C** in noisy environment
- Noise added into computation of $R_{ij}(t)$

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Flake's Simulation with Noise

- R(t) is computed over r rounds
- $A_{ik}(j)$ = action of strategy i playing against strategy k in round j
  - Normal strategy i action with probability $1 - p_n$
  - Random C/D with probability p_n
- Note that this overestimates effects of noise

$$R_{ik}(t) = \sum_{j=1}^{r} \text{payoff}\left[A_{ik}(j) A_{ki}(j)\right]$$

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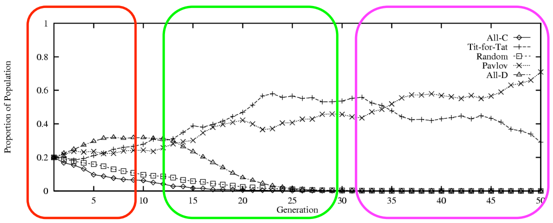
Simulation with Noise

- 20% each
- 0.5% noise



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Run Flake's EIPD with Noise

EIPD-cbn.nlogo

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Spatial Effects

- Previous simulation assumes that each agent is equally likely to interact with each other
- So strategy interactions are proportional to fractions in population
- More realistically, interactions with "neighbors" are more likely
  - "Neighbor" can be defined in many ways
- Neighbors are more likely to use the same strategy

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Spatial Simulation

- Toroidal grid
- Agent interacts only with eight neighbors
- Agent adopts strategy of most successful neighbor
- Ties favor current strategy

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NetLogo Simulation of Spatial IPD

Run SIPD-async-alter.nlogo

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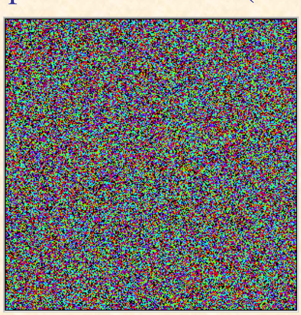
Typical Simulation (t = 1)



Colors:

ALL-C
TFT
RAND
PAV
ALL-D

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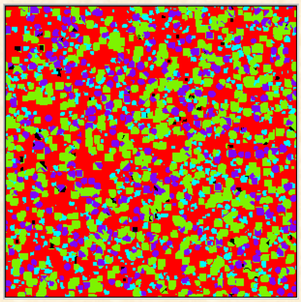
Typical Simulation (*t* = 5)



Colors:

ALL-C
TFT
RAND
PAV
ALL-D

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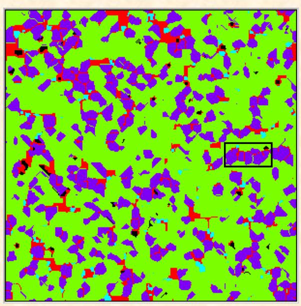
Typical Simulation (*t* = 10)



Colors:

ALL-C
TFT
RAND
PAV
ALL-D

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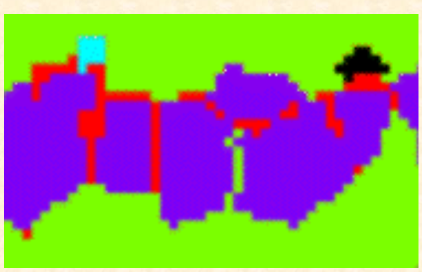
Typical Simulation (*t* = 10) Zooming In



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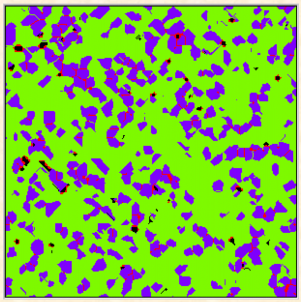
Typical Simulation (t = 20)



Colors:

ALL-C
TFT
RAND
PAV
ALL-D

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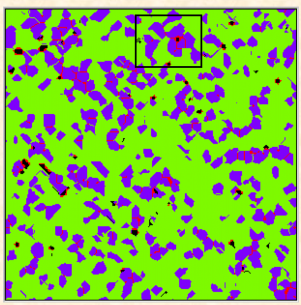
Typical Simulation (t = 50)



Colors:

ALL-C
TFT
RAND
PAV
ALL-D

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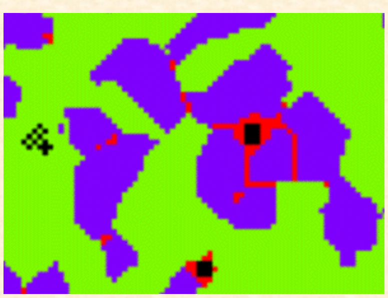
Typical Simulation (t = 50) Zoom In



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SIPD Without Noise

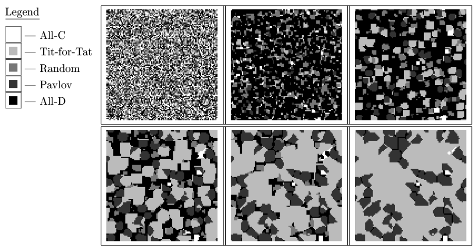

Figure 17.4 Competition in the spatial iterated Prisoner's Dilemma without noise

Conclusions: Spatial IPD

- Small clusters of cooperators can exist in hostile environment
- Parasitic agents can exist only in limited numbers
- Stability of cooperation depends on expectation of future interaction
- Adaptive cooperation/defection beats unilateral cooperation or defection

Additional Bibliography

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3. Axelrod, R. *The Evolution of Cooperation*. Basic Books, 1984.
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Part VIIB