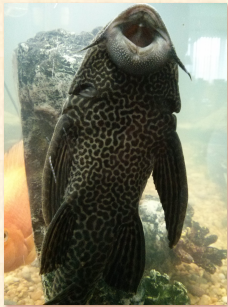# B.
Pattern Formation

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1

Differentiation
& Pattern Formation

- A central problem in development: How do cells differentiate to fulfill different purposes?
- How do complex systems generate spatial & temporal structure?
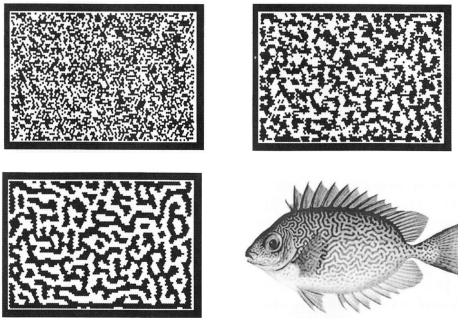- CAs are natural models of intercellular communication

1/30/17   photos ©2000, S. Cazamine

2

Plecostomus

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3

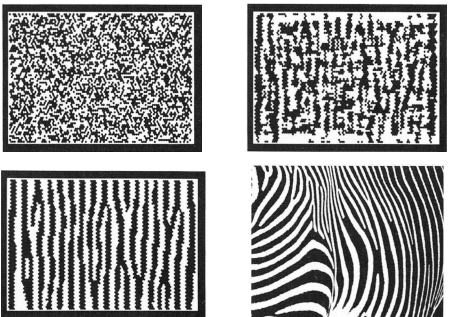## Vermiculated Rabbit Fish

1/30/17   figs. from Camazine & al.: *Self-Org. Biol. Sys.*   4

Zebra

1/30/17   figs. from Camazine & al.: *Self-Org. Biol. Sys.*   5

Activation & Inhibition in Pattern Formation

• Color patterns typically have a characteristic length scale
• Independent of cell size and animal size
• Achieved by:
    – short-range activation ⇒ local uniformity
    – long-range inhibition ⇒ separation

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Interaction Parameters

- R_1 and R_2 are the interaction ranges
- J_1 and J_2 are the interaction strengths

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CA Activation/Inhibition Model

- Let states $s_i \in \{-1, +1\}$
- and h be a bias parameter
- and r_{ij} be the distance between cells i and j
- Then the state update rule is:

$$s_i(t+1) = \mathrm{sign}\left[h + J_1 \sum_{r_{ij} < R_1} s_j(t) + J_2 \sum_{R_1 \le r_{ij} < R_2} s_j(t) \right]$$

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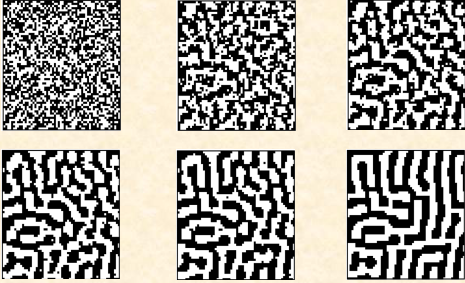
Demonstration of NetLogo Program for Activation/Inhibition Pattern Formation

RunAICA.nlogo

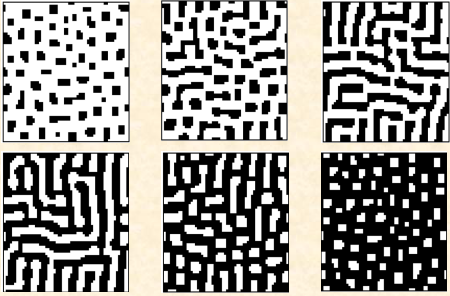1/30/17   9

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Example
$(R_1=1, R_2=6, J_1=1, J_2=-0.1, h=0)$
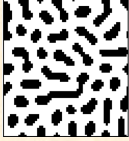
1/30/17   figs. from Bar-Yam   10

Effect of Bias
$(h = -6, -3, -1; 1, 3, 6)$

1/30/17   figs. from Bar-Yam   11

Effect of Interaction Ranges

$R_2 = 6$
$R_1 = 1$
$h = 0$

$R_2 = 8$
$R_1 = 1$
$h = 0$

$R_2 = 6$
$R_1 = 1.5$
$h = 0$

$R_2 = 6$
$R_1 = 1.5$
$h = -3$

1/30/17   figs. from Bar-Yam   12

Differential Interaction Ranges

- How can a system using strictly local interactions discriminate between states at long and short range?
- E.g. cells in developing organism
- Can use two different *morphogens* diffusing at two different rates
  - activator diffuses slowly (short range)
  - inhibitor diffuses rapidly (long range)

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Digression on Diffusion

- Simple 2-D diffusion equation:

$$\dot{A}(x,y) = D\nabla^2 A(x,y)$$

- Recall the 2-D Laplacian:

$$\nabla^2 A(x,y) = \frac{\partial^2 A(x,y)}{\partial x^2} + \frac{\partial^2 A(x,y)}{\partial y^2}$$

- The Laplacian (like 2nd derivative) is:
  - positive in a local minimum
  - negative in a local maximum

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Reaction-Diffusion System

$$\frac{\partial A}{\partial t} = D_A \nabla^2 A + f_A(A,I)$$

diffusion   reaction

$$\frac{\partial I}{\partial t} = D_I \nabla^2 I + f_I(A,I)$$

$$\frac{\partial}{\partial t}\begin{pmatrix} A \\ I \end{pmatrix} = \begin{pmatrix} D_A & 0 \\ 0 & D_I \end{pmatrix}\begin{pmatrix} \nabla^2 A \\ \nabla^2 I \end{pmatrix} + \begin{pmatrix} f_A(A,I) \\ f_I(A,I) \end{pmatrix}$$

$$\dot{\mathbf{c}} = \mathbf{D}\nabla^2\mathbf{c} + \mathbf{f}(\mathbf{c}), \text{ where } \mathbf{c} = \begin{pmatrix} A \\ I \end{pmatrix}$$

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General Reaction-Diffusion System

$$\frac{\partial c_i}{\partial t} = \sum_{\alpha} k_{\alpha} \nu_{i\alpha} \left(\prod_{k=1}^{n} c_k^{m_{k\alpha}} \right) - \nabla \cdot \mathbf{j}_i$$

where $\mathbf{j}_i = \vec{\mu}_i c_i - \mathbf{div}\ \mathbf{D}_i c_i$ (flux)

where k_{α} = rate constant for reaction α

and $\nu_{i\alpha}$ = stoichiometric coefficient

and $m_{k\alpha}$ = a non-negative integer

and $\vec{\mu}_i$ = drift vector

and $\mathbf{D}_i$ = diffusivity matrix

where $\mathbf{div}\ \mathbf{D}c = \sum_j \mathbf{e}_j \sum_k D_{jk} \partial c / \partial x_k$

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Framework for Complexity

- change = source terms + transport terms
- source terms = local coupling
  = interactions local to a small region
- transport terms = spatial coupling
  = interactions with contiguous regions
  = advection + diffusion
  – advection: non-dissipative, time-reversible
  – diffusion: dissipative, irreversible

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NetLogo Simulation of Reaction-Diffusion System

1. Diffuse activator in X and Y directions
2. Diffuse inhibitor in X and Y directions
3. Each patch performs:
   stimulation = bias + activator – inhibitor + noise
   if stimulation > 0 then
       set activator and inhibitor to 100
   else
       set activator and inhibitor to 0

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Demonstration of NetLogo Program for Activator/Inhibitor Pattern Formation

Run Pattern.nlogo

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Continuous-time Activator-Inhibitor System

- Activator A and inhibitor I may diffuse at different rates in x and y directions
- Cell becomes more active if activator + bias exceeds inhibitor
- Otherwise, less active
- A and I are limited to [0, 100] (depletion/saturation)

$$\frac{\partial A}{\partial t} = d_{Ax}\frac{\partial^2 A}{\partial x^2} + d_{Ay}\frac{\partial^2 A}{\partial y^2} + k_A(A + B - I)$$

$$\frac{\partial I}{\partial t} = d_{Ix}\frac{\partial^2 I}{\partial x^2} + d_{Iy}\frac{\partial^2 I}{\partial y^2} + k_I(A + B - I)$$

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Demonstration of NetLogo Program for Activator/Inhibitor Pattern Formation with Continuous State Change

Run Activator-Inhibitor.nlogo

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Turing Patterns

- Alan Turing studied the mathematics of reaction-diffusion systems

- Turing, A. (1952). The chemical basis of morphogenesis. *Philosophical Transactions of the Royal Society* **B 237**: 37–72.

- The resulting patterns are known as *Turing patterns*

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Observations

- With local activation and lateral inhibition
- And with a random initial state
- You can expect to get Turing patterns
- These are stationary states (dynamic equilibria)
- Macroscopically, Class I behavior
  - Microscopically, may be class III

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A Key Element of Self-Organization

- Activation vs. Inhibition
- Cooperation vs. Competition
- Amplification vs. Stabilization
- Growth vs. Limit
- Positive Feedback vs. Negative Feedback
  - Positive feedback creates
  - Negative feedback shapes

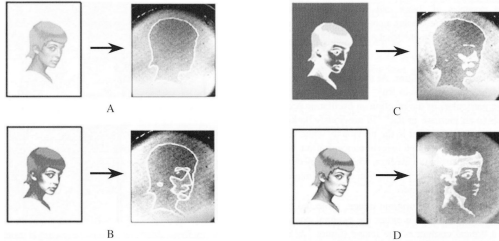1/30/17   24

Reaction-Diffusion Computing

- Has been used for image processing
  - diffusion $\Rightarrow$ noise filtering
  - reaction $\Rightarrow$ contrast enhancement
- Depending on parameters, RD computing can:
  - restore broken contours
  - detect edges
  - improve contrast

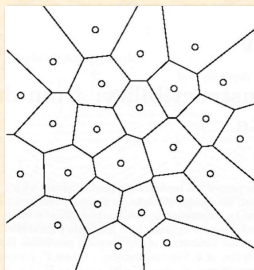1/30/17   25

Image Processing in BZ Medium

- (A) boundary detection, (B) contour enhancement, (C) shape enhancement, (D) feature enhancement

1/30/17   Image < Adamatzky, *Comp. in Nonlinear Media & Autom. Coll.*   26

Voronoi Diagrams

- Given a set of generating points:
- Construct a polygon around each generating point of set, so all points in a polygon are closer to its generating point than to any other generating points.

1/30/17   Image < Adamatzky & al., *Reaction-Diffusion Computers*   27
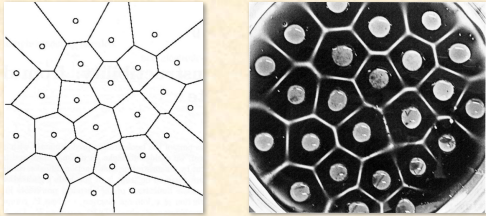
Some Uses of Voronoi Diagrams

- Collision-free path planning
- Determination of service areas for power substations
- Nearest-neighbor pattern classification
- Determination of largest empty figure

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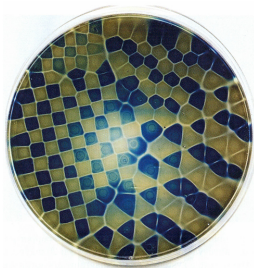## Computation of Voronoi Diagram by Reaction-Diffusion Processor

1/30/17   Image < Adamatzky & al., *Reaction-Diffusion Computers*   29
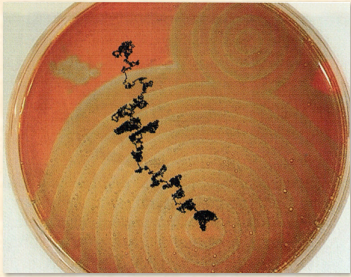
Mixed Cell Voronoi Diagram

1/30/17   Image < Adamatzky & al., *Reaction-Diffusion Computers*   30

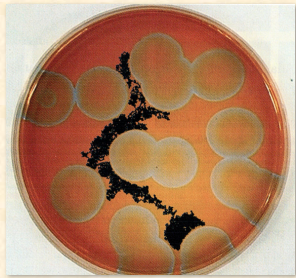### Path Planning via BZ medium: No Obstacles

1/30/17   Image < Adamatzky & al., *Reaction-Diffusion Computers*   31

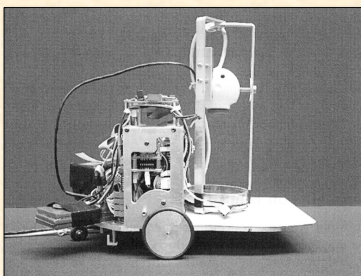### Path Planning via BZ medium: Circular Obstacles

1/30/17   Image < Adamatzky & al., *Reaction-Diffusion Computers*   32
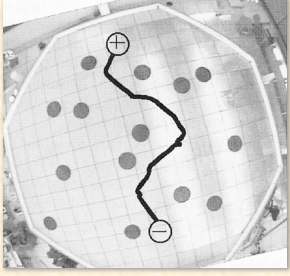
Mobile Robot with Onboard Chemical Reactor

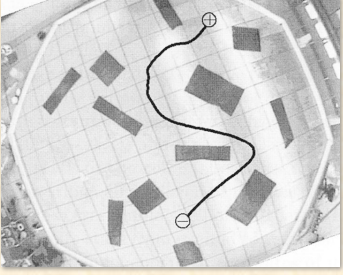1/30/17   Image < Adamatzky & al., *Reaction-Diffusion Computers*   33
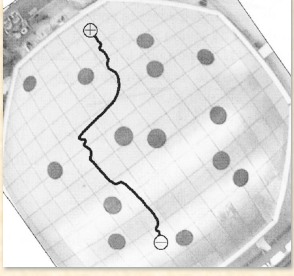
Actual Path: Pd Processor

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Image < Adamatzky & al., *Reaction-Diffusion Computers*
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Actual Path: Pd Processor

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Image < Adamatzky & al., *Reaction-Diffusion Computers*
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Actual Path: BZ Processor

1/30/17
Image < Adamatzky & al., *Reaction-Diffusion Computers*
36

Bibliography for Reaction-Diffusion Computing

1. Adamatzky, Adam. *Computing in Nonlinear Media and Automata Collectives*. Bristol: Inst. of Physics Publ., 2001.

2. Adamatzky, Adam, De Lacy Costello, Ben, & Asai, Tetsuya. *Reaction Diffusion Computers*. Amsterdam: Elsevier, 2005.

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