II. Cellular Automata

Cellular Automata (CAs)

- Invented by von Neumann in 1940s to study reproduction
- He succeeded in constructing a self-reproducing CA
- Have been used as:
  - massively parallel computer architecture
  - model of physical phenomena (Fredkin, Wolfram)
- Currently being investigated as model of quantum computation (QCAs)

Structure

- Discrete space (lattice) of regular cells
  - 1D, 2D, 3D, ...
  - rectangular, hexagonal, ...
- At each unit of time a cell changes state in response to:
  - its own previous state
  - states of neighbors (within some “radius”)
- All cells obey same state update rule
  - an FSA
- Synchronous updating

Example: Conway’s Game of Life

- Invented by Conway in late 1960s
- A simple CA capable of universal computation
- Structure:
  - 2D space
  - rectangular lattice of cells
  - binary states (alive/dead)
  - neighborhood of 8 surrounding cells (& self)
  - simple population-oriented rule
State Transition Rule

- Live cell has 2 or 3 live neighbors ⇒ stays as is (stasis)
- Live cell has < 2 live neighbors ⇒ dies (loneliness)
- Live cell has > 3 live neighbors ⇒ dies (overcrowding)
- Empty cell has 3 live neighbors ⇒ comes to life (reproduction)

Demonstration of Life

Go to CBN
Online Experimentation Center
(mitpress.mit.edu/books/FLAOH/cbnhtml/java.html)

Some Observations About Life

1. Long, chaotic-looking initial transient
   - unless initial density too low or high
2. Intermediate phase
   - isolated islands of complex behavior
   - matrix of static structures & “blinkers”
   - gliders creating long-range interactions
3. Cyclic attractor
   - typically short period

From Life to CAs in General

- What gives Life this very rich behavior?
- Is there some simple, general way of characterizing CAs with rich behavior?
- It belongs to Wolfram’s Class IV
**Langton’s Investigation**

*Under what conditions can we expect a complex dynamics of information to emerge spontaneously and come to dominate the behavior of a CA?*

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**Wolfram’s Classification**

- **Class I:** evolve to fixed, homogeneous state
  - limit point
- **Class II:** evolve to simple separated periodic structures
  - limit cycle
- **Class III:** yield chaotic aperiodic patterns
  - strange attractor (chaotic behavior)
- **Class IV:** complex patterns of localized structure
  - long transients, no analog in dynamical systems

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**Approach**

- Investigate 1D CAs with:
  - random transition rules
  - starting in random initial states
- Systematically vary a simple parameter characterizing the rule
- Evaluate qualitative behavior (Wolfram class)
Assumptions

- Periodic boundary conditions
  - no special place
- Strong quiescence:
  - if all the states in the neighborhood are the same, then the new state will be the same
  - persistence of uniformity
- Spatial isotropy:
  - all rotations of neighborhood state result in same new state
  - no special direction
- Totalistic [not used by Langton]:
  - depend only on sum of states in neighborhood
  - implies spatial isotropy

Langton’s Lambda

- Designate one state to be quiescent state
- Let $K =$ number of states
- Let $N = 2r + 1 =$ area of neighborhood
- Let $T = K^N =$ number of entries in table
- Let $n_q =$ number mapping to quiescent state
- Then
  \[ \lambda = \frac{T - n_q}{T} \]

Range of Lambda Parameter

- If all configurations map to quiescent state:
  \[ \lambda = 0 \]
- If no configurations map to quiescent state:
  \[ \lambda = 1 \]
  - If every state is represented equally:
    \[ \lambda = 1 - \frac{1}{K} \]
  - A sort of measure of “excitability”