Lecture 15

Ant Colony Optimization (ACO)

Developed in 1991 by Dorigo (PhD dissertation) in collaboration with Colorni & Maniezzo

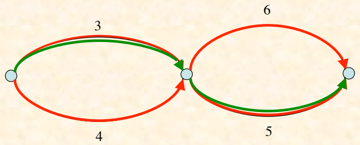
Basis of all Ant-Based Algorithms

- Positive feedback
- Negative feedback
- Cooperation

Positive Feedback

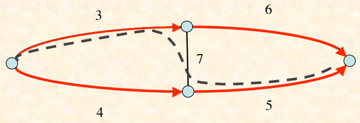
- To reinforce portions of good solutions that contribute to their goodness
- To reinforce good solutions directly
- Accomplished by *pheromone accumulation*

Reinforcement of Solution Components



Parts of good solutions *may* produce better solutions

Negative Reinforcement of Non-solution Components



Parts not in good solutions *tend* to be forgotten

Negative Feedback

- To avoid premature convergence (*stagnation*)
- Accomplished by *pheromone evaporation*

Cooperation

- For simultaneous exploration of different solutions
- Accomplished by:
  - *multiple ants* exploring solution space
  - *pheromone trail* reflecting multiple perspectives on solution space

Traveling Salesman Problem

- Given the travel distances between *N* cities
  - may be symmetric or not
- Find the shortest route visiting each city exactly once and returning to the starting point
- NP-hard
- Typical combinatorial optimization problem

Ant System for Traveling Salesman Problem (AS-TSP)

- During each iteration, each ant completes a tour
- During each tour, each ant maintains *tabu list* of cities already visited
- Each ant has access to
  - distance of current city to other cities
  - intensity of local pheromone trail
- Probability of next city depends on both

Transition Rule

- Let $\eta_{ij} = 1/d_{ij}$ = "nearness" of city *j* to current city *i*
- Let τ_{ij} = strength of trail from *i* to *j*
- Let J_i^k = list of cities ant *k* still has to visit after city *i* in current tour
- Then transition probability for ant *k* going from *i* to $j \in J_i^k$ in tour *t* is:

$$p_{ij}^k = \frac{\left[\tau_{ij}(t)\right]^\alpha \left[\eta_{ij}\right]^\beta}{\sum_{l \in J_i^k} \left[\tau_{il}(t)\right]^\alpha \left[\eta_{il}\right]^\beta}$$

Pheromone Deposition

- Let $T^k(t)$ be tour *t* of ant *k*
- Let $L^k(t)$ be the length of this tour
- After completion of a tour, each ant *k* contributes:

$$\Delta\tau_{ij}^k = \begin{cases} Q / L^k(t) & \text{if } (i,j) \in T^k(t) \\ 0 & \text{if } (i,j) \notin T^k(t) \end{cases}$$

Pheromone Decay

- Define total pheromone deposition for tour *t*:

$$\Delta\tau_{ij}(t) = \sum_{k=1}^{m} \Delta\tau_{ij}^{k}(t)$$

- Let ρ be decay coefficient
- Define trail intensity for next round of tours:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \Delta\tau_{ij}(t)$$

Number of Ants is Critical

- Too many:
  - suboptimal trails quickly reinforced
  - ∴ early convergence to suboptimal solution
- Too few:
  - don’t get cooperation before pheromone decays
- Good tradeoff:
  number of ants = number of cities
  ($m = n$)

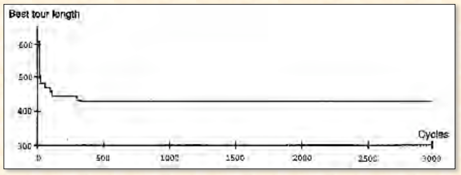
Improvement: “Elitist” Ants

- Add a few ($e \approx 5$) “elitist” ants to population
- Let T^+ be best tour so far
- Let L^+ be its length
- Each “elitist” ant reinforces edges in T^+ by Q/L^+
- Add *e* more “elitist” ants
- This applies accelerating positive feedback to best tour

Time Complexity

- Let *t* be number of tours
- Time is $\mathcal{O}(tn^2m)$
- If $m = n$ then $\mathcal{O}(tn^3)$
  - that is, cubic in number of cities

Convergence



- 30 cities (“Oliver30”)
- Best tour length
- Converged to optimum in 300 cycles

fig. < Dorigo et al. (1996)

Evaluation

- Both “very interesting and disappointing”
- For 30-cities:
  - beat genetic algorithm
  - matched or beat tabu search & simulated annealing
- For 50 & 75 cities and 3000 iterations
  - did not achieve optimum
  - but quickly found good solutions
- I.e., does not scale up well
- Like all general-purpose algorithms, it is out-performed by special purpose algorithms

Improving Network Routing

1. Nodes periodically send *forward ants* to some recently recorded destinations
2. Collect information on way
3. Die if reach already visited node
4. When reaches destination, estimates time and turns into *backward ant*
5. Returns by same route, updating routing tables

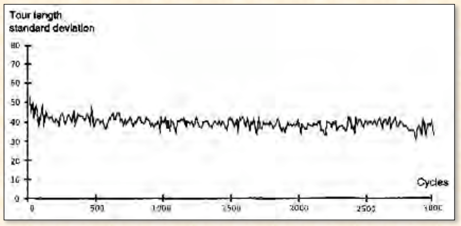
Some Applications of ACO

- Routing in telephone networks
- Vehicle routing
- Job-shop scheduling
- Constructing evolutionary trees from nucleotide sequences
- Various classic NP-hard problems
  - shortest common supersequence, graph coloring, quadratic assignment, …

Improvements as Optimizer

- Can be improved in many ways
- E.g., combine local search with ant-based methods
- As method of stochastic combinatorial optimization, performance is promising, comparable with best heuristic methods
- Much ongoing research in ACO
- But optimization is not a principal topic of this course

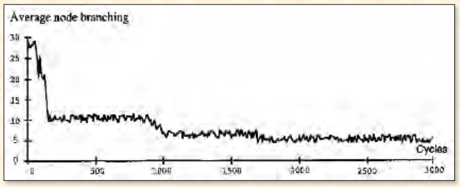
Nonconvergence



- Standard deviation of tour lengths
- Optimum = 420

fig. < Dorigo et al. (1996)

Average Node Branching Number



- Branching number = number of edges leaving a node with pheromone > threshold
- Branching number = 2 for fully converged solution

fig. < Dorigo et al. (1996)

The Nonconvergence Issue

- AS often does not converge to single solution
- Population maintains high diversity
- A bug or a feature?
- Potential advantages of nonconvergence:
  - avoids getting trapped in local optima
  - promising for dynamic applications
- Flexibility & robustness are more important than optimality in natural computation

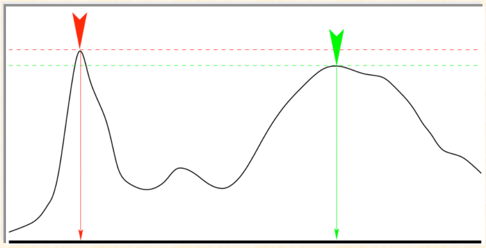
Natural Computation

Natural computation is computation that occurs in nature or is inspired by computation occurring in nature

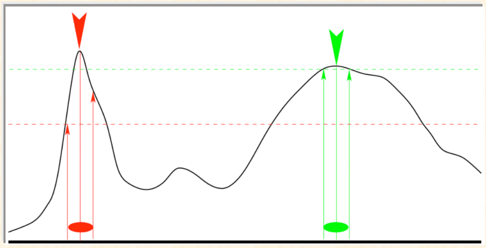
Optimization in Natural Computation

- Good, but suboptimal solutions may be preferable to optima if:
  - suboptima can be obtained more quickly
  - suboptima can be adapted more quickly
  - suboptima are more robust
  - an ill-defined suboptimum may be better than a sharp optimum
- "The best is often the enemy of the good"

Robust Optima



Effect of Error/Noise



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