

# Lecture 21

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(fig. from Haykin *Neur. Netw.*)

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Energy Surface

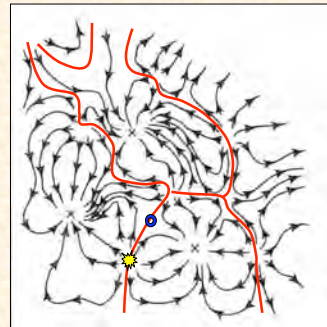


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(fig. from Haykin *Neur. Netw.*)

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Energy Surface + Flow Lines



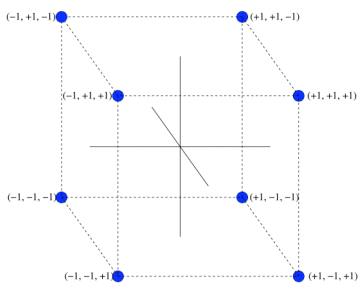
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(fig. from Haykin *Neur. Netw.*)

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Flow Lines

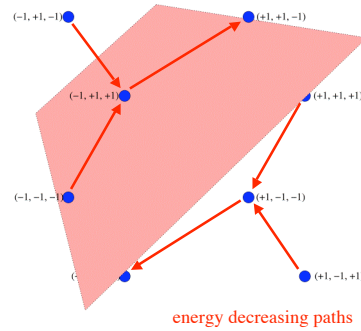
Basins of Attraction



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Bipolar State Space



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Basins in Bipolar State Space

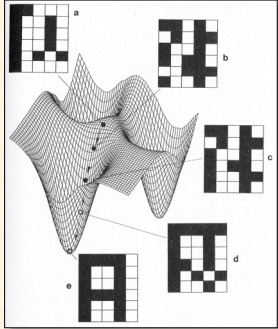
energy decreasing paths

Demonstration of Hopfield Net Dynamics II

[Run initialized Hopfield.nlogo](#)

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Storing Memories as Attractors



(fig. from Solé & Goodwin)

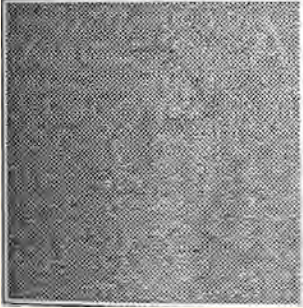
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Demonstration of Hopfield Net

[Run Malasri Hopfield Demo](#)

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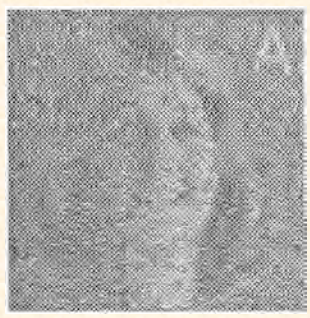
Example of Pattern Restoration



(fig. from Arbib 1995)

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Example of Pattern Restoration



(fig. from Arbib 1995)

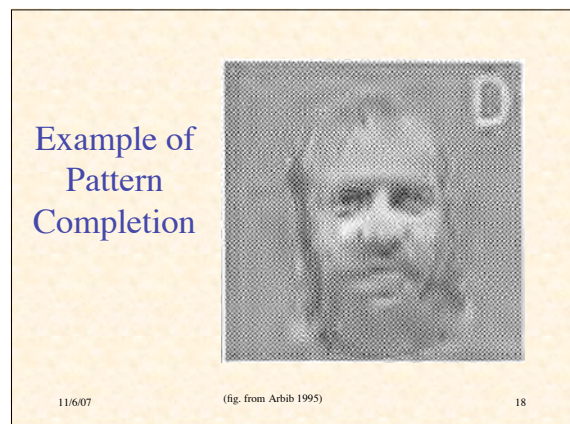
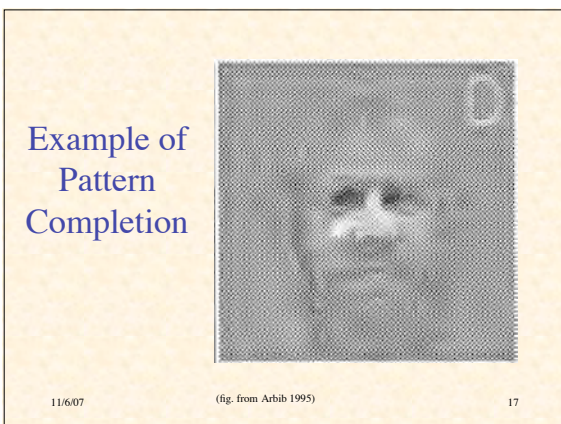
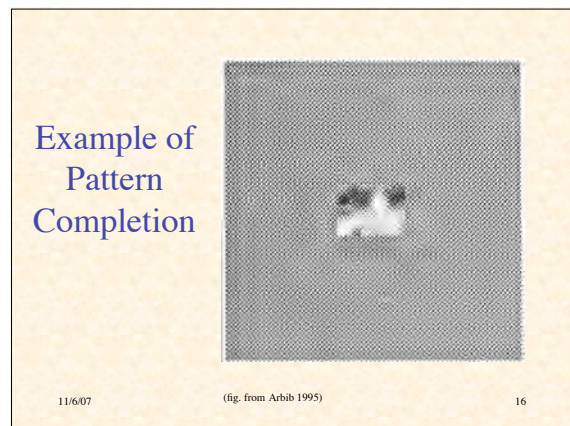
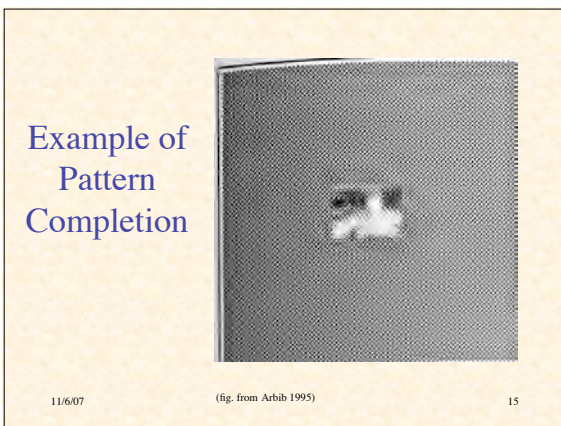
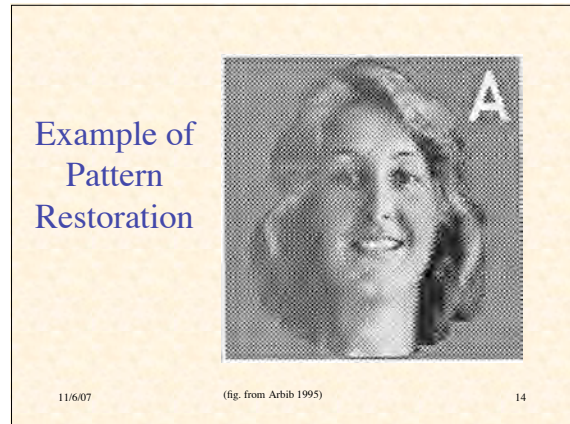
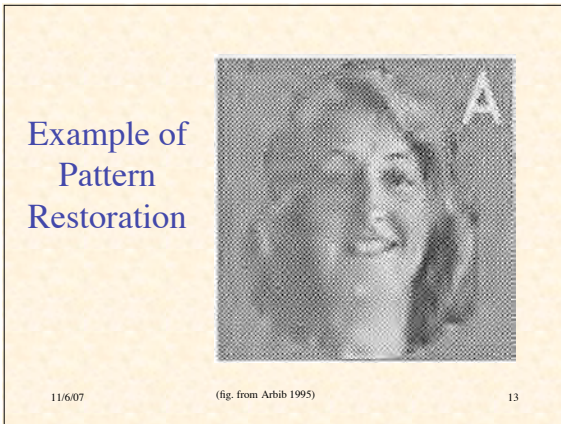
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Example of Pattern Restoration




(fig. from Arbib 1995)

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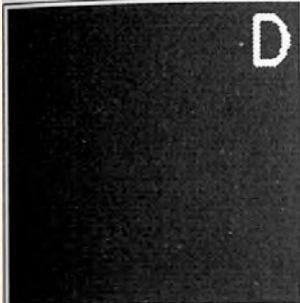


Example of  
Pattern  
Completion



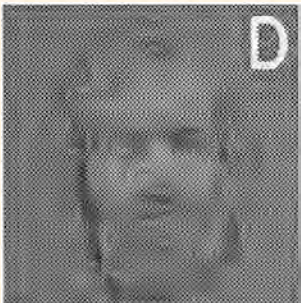
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Example of  
Association



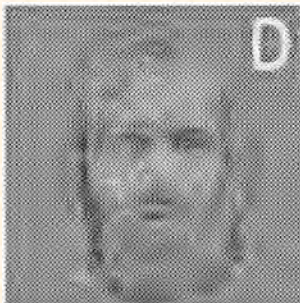
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Example of  
Association



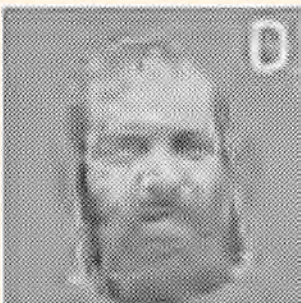
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Example of  
Association



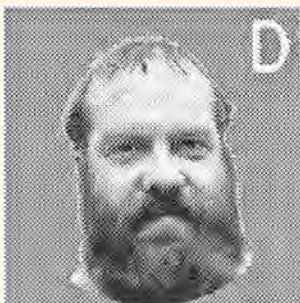
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Example of  
Association



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Example of  
Association



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### Applications of Hopfield Memory

- Pattern restoration
- Pattern completion
- Pattern generalization
- Pattern association

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### Hopfield Net for Optimization and for Associative Memory

- For optimization:
  - we know the weights (couplings)
  - we want to know the minima (solutions)
- For associative memory:
  - we know the minima (retrieval states)
  - we want to know the weights

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### Hebb's Rule

“When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth or metabolic change takes place in one or both cells such that A's efficiency, as one of the cells firing B, is increased.”

—Donald Hebb (*The Organization of Behavior*, 1949, p. 62)

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### Example of Hebbian Learning: Pattern Imprinted

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### Example of Hebbian Learning: Partial Pattern Reconstruction

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### Mathematical Model of Hebbian Learning for One Pattern

Let  $W_{ij} = \begin{cases} x_i x_j, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases}$

Since  $x_i x_i = x_i^2 = 1$ ,  $\mathbf{W} = \mathbf{xx}^T - \mathbf{I}$

For simplicity, we will include self-coupling:

$$\mathbf{W} = \mathbf{xx}^T$$

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### A Single Imprinted Pattern is a Stable State

- Suppose  $\mathbf{W} = \mathbf{x}\mathbf{x}^T$
- Then  $\mathbf{h} = \mathbf{W}\mathbf{x} = \mathbf{x}\mathbf{x}^T\mathbf{x} = n\mathbf{x}$   
since  $\mathbf{x}^T\mathbf{x} = \sum_{i=1}^n x_i^2 = \sum_{i=1}^n (\pm 1)^2 = n$
- Hence, if initial state is  $\mathbf{s} = \mathbf{x}$ , then new state is  $\mathbf{s}' = \text{sgn}(n\mathbf{x}) = \mathbf{x}$
- May be other stable states (e.g.,  $-\mathbf{x}$ )

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### Questions

- How big is the basin of attraction of the imprinted pattern?
- How many patterns can be imprinted?
- Are there unneeded *spurious* stable states?
- These issues will be addressed in the context of multiple imprinted patterns

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