Part 3: Autonomous Agents

Lecture 21

Energy Surface

Energy Surface + Flow Lines

Basins of Attraction

Bipolar State Space

Basins in Bipolar State Space

energy decreasing paths
Demonstration of Hopfield Net Dynamics II

Run initialized Hopfield.nlogo

Storing Memories as Attractors

Example of Pattern Restoration

Example of Pattern Restoration
Example of Pattern Completion

Example of Association

Example of Association

Example of Association

Example of Association

Example of Association
Part 3: Autonomous Agents

Applications of Hopfield Memory

- Pattern restoration
- Pattern completion
- Pattern generalization
- Pattern association

Hopfield Net for Optimization and for Associative Memory

- For optimization:
  - we know the weights (couplings)
  - we want to know the minima (solutions)
- For associative memory:
  - we know the minima (retrieval states)
  - we want to know the weights

Hebb’s Rule

“When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth or metabolic change takes place in one or both cells such that A’s efficiency, as one of the cells firing B, is increased.”

—Donald Hebb (The Organization of Behavior, 1949, p. 62)

Example of Hebbian Learning: Pattern Imprinted

Example of Hebbian Learning: Partial Pattern Reconstruction

Mathematical Model of Hebbian Learning for One Pattern

Let $W_{ij} = \begin{cases} x_i x_j, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases}$

Since $x_i x_j = x_i^2 = 1$, $W = xx^T - I$

For simplicity, we will include self-coupling:

$W = xx^T$
A Single Imprinted Pattern is a Stable State

- Suppose $W = xx^T$
- Then $h = Wx = xx^T x = n x$
  
  since
  
  $x^T x = \sum_{i=1}^n x_i^2 = \sum_{i=1}^n (\pm 1)^2 = n$

- Hence, if initial state is $s = x$, then new state is $s' = \text{sgn}(n x) = x$
- May be other stable states (e.g., $-x$)

Questions

- How big is the basin of attraction of the imprinted pattern?
- How many patterns can be imprinted?
- Are there unneeded spurious stable states?
- These issues will be addressed in the context of multiple imprinted patterns