D. Pattern Formation
Differentiation & Pattern Formation

- A central problem in development: How do cells differentiate to fulfill different purposes?
- How do complex systems generate spatial & temporal structure?
- CAs are natural models of intercellular communication
Zebra

Vermiculated Rabbit Fish

Activation & Inhibition in Pattern Formation

• Color patterns typically have a characteristic length scale

• Independent of cell size and animal size

• Achieved by:
  – short-range activation ⇒ local uniformity
  – long-range inhibition ⇒ separation
Interaction Parameters

- $R_1$ and $R_2$ are the interaction ranges
- $J_1$ and $J_2$ are the interaction strengths
CA Activation/Inhibition Model

• Let states $s_i \in \{-1, +1\}$
• and $h$ be a bias parameter
• and $r_{ij}$ be the distance between cells $i$ and $j$
• Then the state update rule is:

$$s_i(t + 1) = \text{sign} \left[ h + J_1 \sum_{r_{ij} < R_1} s_j(t) + J_2 \sum_{R_1 \leq r_{ij} < R_2} s_j(t) \right]$$
Example

\[(R_1=1, R_2=6, J_1=1, J_2=-0.1, h=0)\]
Effect of Bias

\( (h = -6, -3, -1; 1, 3, 6) \)
Effect of Interaction Ranges

\[ R_2 = 6 \]
\[ R_1 = 1 \]
\[ h = 0 \]

\[ R_2 = 6 \]
\[ R_1 = 1.5 \]
\[ h = 0 \]

\[ R_2 = 8 \]
\[ R_1 = 1 \]
\[ h = 0 \]

\[ R_2 = 6 \]
\[ R_1 = 1.5 \]
\[ h = 0 \]

\[ R_2 = 6 \]
\[ R_1 = 1.5 \]
\[ h = -3 \]
Demonstration of NetLogo Program for Activation/Inhibition Pattern Formation: Fur

Run AICA.nlogo
Differential Interaction Ranges

• How can a system using strictly local interactions discriminate between states at long and short range?
• E.g. cells in developing organism
• Can use two different *morphogens* diffusing at two different rates
  – activator diffuses slowly (short range)
  – inhibitor diffuses rapidly (long range)
Digression on Diffusion

• Simple 2-D diffusion equation:

$$\dot{A}(x, y) = c \nabla^2 A(x, y)$$

• Recall the 2-D Laplacian:

$$\nabla^2 A(x, y) = \frac{\partial^2 A(x, y)}{\partial x^2} + \frac{\partial^2 A(x, y)}{\partial y^2}$$

• The Laplacian (like 2nd derivative) is:
  – positive in a local minimum
  – negative in a local maximum
Reaction-Diffusion System

\[
\begin{align*}
\frac{\partial A}{\partial t} &= d_A \nabla^2 A + f_A(A, I) \\
\frac{\partial I}{\partial t} &= d_I \nabla^2 I + f_I(A, I)
\end{align*}
\]

\[
\frac{\partial}{\partial t} \begin{pmatrix} A \\ I \end{pmatrix} =
\begin{pmatrix} d_A & 0 \\ 0 & d_I \end{pmatrix} \begin{pmatrix} \nabla^2 A \\ \nabla^2 I \end{pmatrix} +
\begin{pmatrix} f_A(A, I) \\ f_I(A, I) \end{pmatrix}
\]

\[
\dot{c} = D \nabla^2 c + f(c), \quad \text{where } c = \begin{pmatrix} A \\ I \end{pmatrix}
\]
Example: Activation-Inhibition System

- Let $\sigma$ be the logistic sigmoid function
- Activator $A$ and inhibitor $I$ may diffuse at different rates in $x$ and $y$ directions
- Cell is “on” if activator + bias exceeds inhibitor

\[
\frac{\partial A}{\partial t} = d_{Ax} \frac{\partial^2 A}{\partial x^2} + d_{Ay} \frac{\partial^2 A}{\partial y^2} + k_A \sigma[m_A(A + B - I)]
\]

\[
\frac{\partial I}{\partial t} = d_{Ix} \frac{\partial^2 I}{\partial x^2} + d_{Iy} \frac{\partial^2 I}{\partial y^2} + k_I \sigma[m_I(A + B - I)]
\]
NetLogo Simulation of Reaction-Diffusion System

1. Diffuse activator in X and Y directions
2. Diffuse inhibitor in X and Y directions
3. Each patch performs:
   stimulation = bias + activator – inhibitor + noise
   if stimulation > 0 then
     set activator and inhibitor to 100
   else
     set activator and inhibitor to 0
Demonstration of NetLogo Program for Activation/Inhibition Pattern Formation

Run Pattern.nlogo
Abstract Activation/Inhibition Spaces

• Consider two axes of cultural preference
  – E.g. hair length & interpersonal distance
  – Fictitious example!
• Suppose there are no objective reasons for preferences
• Suppose people approve/encourage those with similar preferences
• Suppose people disapprove/discourage those with different preferences
• What is the result?
Emergent Regions of Acceptable Variation
A Key Element of Self-Organization

- Activation vs. Inhibition
- Cooperation vs. Competition
- Amplification vs. Stabilization
- Growth vs. Limit
- Positive Feedback vs. Negative Feedback
  - Positive feedback creates
  - Negative feedback shapes
Reaction-Diffusion Computing

• Has been used for image processing
  – diffusion ⇒ noise filtering
  – reaction ⇒ contrast enhancement

• Depending on parameters, RD computing can:
  – restore broken contours
  – detect edges
  – improve contrast
Image Processing in BZ Medium

- (A) boundary detection, (B) contour enhancement,
  (C) shape enhancement, (D) feature enhancement
Voronoi Diagrams

- Given a set of generating points:
- Construct polygon around each gen. point of set, so all points in poly. are closer to its generating point than to any other generating points.

Image < Adamatzky & al., Reaction-Diffusion Computers
Some Uses of Voronoi Diagrams

• Collision-free path planning
• Determination of service areas for power substations
• Nearest-neighbor pattern classification
• Determination of largest empty figure
Computation of Voronoi Diagram by Reaction-Diffusion Processor
Mixed Cell Voronoi Diagram
Path Planning via BZ medium: No Obstacles

Image < Adamatzky & al., *Reaction-Diffusion Computers*
Path Planning via BZ medium: Circular Obstacles

Image < Adamatzky & al., Reaction-Diffusion Computers
Mobile Robot with Onboard Chemical Reactor
Actual Path: Pd Processor

Image < Adamatzky & al., Reaction-Diffusion Computers
Actual Path: Pd Processor
Actual Path: BZ Processor
Bibliography for Reaction-Diffusion Computing


Segmentation

(in embryological development)
Vertebræ

- Characteristic of species
- How does an embryo “count” them?
- “Clock and wavefront model” of Cooke & Zeeman (1976).
NetLogo Simulation of Segmentation

Run Segmentation.nlogo
Segmentation References


Additional Bibliography


*continue to “Part 3A”*