D.
Pattern Formation

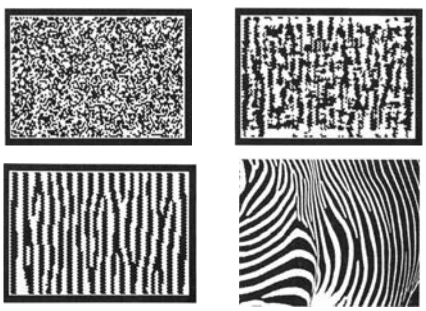
9/17/08 1

Differentiation & Pattern Formation

- A central problem in development: How do cells differentiate to fulfill different purposes?
- How do complex systems generate spatial & temporal structure?
- CAs are natural models of intercellular communication

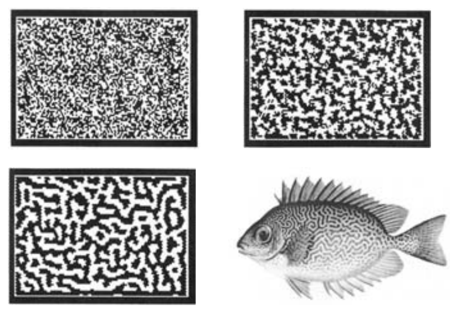
9/17/08 photos ©2000, S. Cazamine 2

Zebra



9/17/08 figs. from Camazine & al.: *Self-Org. Biol. Sys.* 3

Vermiculated Rabbit Fish



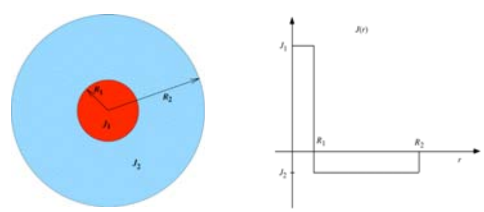
9/17/08 figs. from Camazine & al.: *Self-Org. Biol. Sys.* 4

Activation & Inhibition in Pattern Formation

- Color patterns typically have a characteristic length scale
- Independent of cell size and animal size
- Achieved by:
  - short-range activation ⇒ local uniformity
  - long-range inhibition ⇒ separation

9/17/08 5

Interaction Parameters



- R_1 and R_2 are the interaction ranges
- J_1 and J_2 are the interaction strengths

9/17/08 6

CA Activation/Inhibition Model

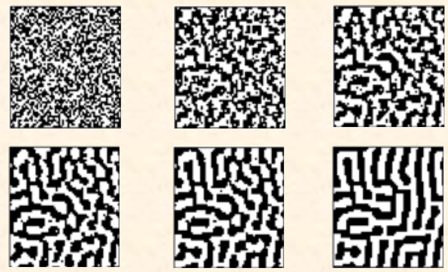
- Let states $s_i \in \{-1, +1\}$
- and h be a bias parameter
- and r_{ij} be the distance between cells i and j
- Then the state update rule is:

$$s_i(t+1) = \operatorname{sign}\left[h + J_1 \sum_{r_{ij} < R_1} s_j(t) + J_2 \sum_{R_1 \le r_{ij} < R_2} s_j(t)\right]$$

9/17/08 7

Example

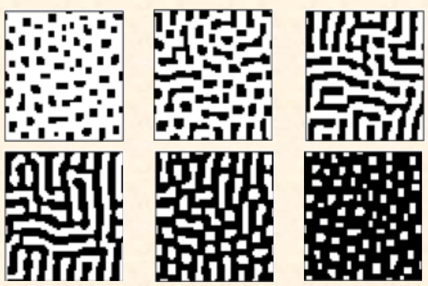
(R_1=1, R_2=6, J_1=1, J_2=–0.1, h=0)



9/17/08 figs. from Bar-Yam 8

Effect of Bias

(h = –6, –3, –1; 1, 3, 6)



9/17/08 figs. from Bar-Yam 9

Effect of Interaction Ranges

9/17/08 figs. from Bar-Yam 10

Demonstration of NetLogo Program for Activation/Inhibition Pattern Formation: Fur

Run AICA.nlogo

9/17/08 11

Differential Interaction Ranges

- How can a system using strictly local interactions discriminate between states at long and short range?
- E.g. cells in developing organism
- Can use two different *morphogens* diffusing at two different rates
  - activator diffuses slowly (short range)
  - inhibitor diffuses rapidly (long range)

9/17/08 12

Digression on Diffusion

- Simple 2-D diffusion equation:

$$\dot{A}(x,y) = c\nabla^2 A(x,y)$$

- Recall the 2-D Laplacian:

$$\nabla^2 A(x,y) = \frac{\partial^2 A(x,y)}{\partial x^2} + \frac{\partial^2 A(x,y)}{\partial y^2}$$

- The Laplacian (like 2nd derivative) is:
  - positive in a local minimum
  - negative in a local maximum

9/17/08 13

Reaction-Diffusion System

diffusion

$$\frac{\partial A}{\partial t} = d_A \nabla^2 A + f_A(A,I)$$
$$\frac{\partial I}{\partial t} = d_I \nabla^2 I + f_I(A,I)$$

reaction

$$\frac{\partial}{\partial t}\begin{pmatrix} A \\ I \end{pmatrix} = \begin{pmatrix} d_A & 0 \\ 0 & d_I \end{pmatrix}\begin{pmatrix} \nabla^2 A \\ \nabla^2 I \end{pmatrix} + \begin{pmatrix} f_A(A,I) \\ f_I(A,I) \end{pmatrix}$$

$$\dot{\mathbf{c}} = \mathbf{D}\nabla^2\mathbf{c} + \mathbf{f}(\mathbf{c}), \text{ where } \mathbf{c} = \begin{pmatrix} A \\ I \end{pmatrix}$$

9/17/08 14

Example: Activation-Inhibition System

- Let σ be the logistic sigmoid function
- Activator *A* and inhibitor *I* may diffuse at different rates in *x* and *y* directions
- Cell is "on" if activator + bias exceeds inhibitor

$$\frac{\partial A}{\partial t} = d_{Ax}\frac{\partial^2 A}{\partial x^2} + d_{Ay}\frac{\partial^2 A}{\partial y^2} + k_A \sigma\left[m_A(A+B-I)\right]$$
$$\frac{\partial I}{\partial t} = d_{Ix}\frac{\partial^2 I}{\partial x^2} + d_{Iy}\frac{\partial^2 I}{\partial y^2} + k_I \sigma\left[m_I(A+B-I)\right]$$

9/17/08 15

NetLogo Simulation of Reaction-Diffusion System

1. Diffuse activator in X and Y directions
2. Diffuse inhibitor in X and Y directions
3. Each patch performs:

```
stimulation = bias + activator – inhibitor + noise
if stimulation > 0 then
    set activator and inhibitor to 100
else
    set activator and inhibitor to 0
```

9/17/08 16

Demonstration of NetLogo Program for Activation/Inhibition Pattern Formation

Run Pattern.nlogo

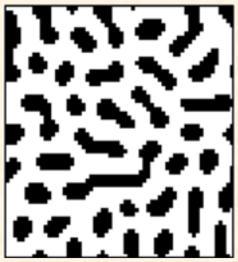
9/17/08 17

Abstract Activation/Inhibition Spaces

- Consider two axes of cultural preference
  - E.g. hair length & interpersonal distance
  - Fictitious example!
- Suppose there are no objective reasons for preferences
- Suppose people approve/encourage those with similar preferences
- Suppose people disapprove/discourage those with different preferences
- What is the result?

9/17/08 18

Emergent Regions of Acceptable Variation



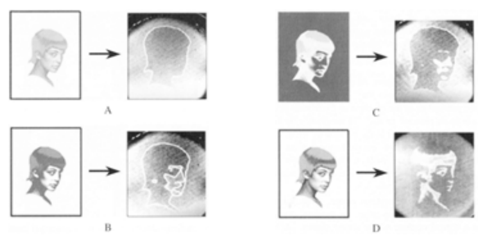
A Key Element of Self-Organization

- Activation vs. Inhibition
- Cooperation vs. Competition
- Amplification vs. Stabilization
- Growth vs. Limit
- Positive Feedback vs. Negative Feedback
  - Positive feedback creates
  - Negative feedback shapes

Reaction-Diffusion Computing

- Has been used for image processing
  - diffusion ⇒ noise filtering
  - reaction ⇒ contrast enhancement
- Depending on parameters, RD computing can:
  - restore broken contours
  - detect edges
  - improve contrast

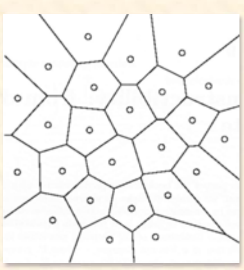
Image Processing in BZ Medium



- (A) boundary detection, (B) contour enhancement, (C) shape enhancement, (D) feature enhancement

 Image < Adamatzky, *Comp. in Nonlinear Media & Autom. Coll.*

Voronoi Diagrams



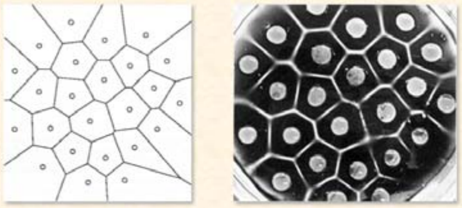
- Given a set of generating points:
- Construct polygon around each gen. point of set, so all points in poly. are closer to its generating point than to any other generating points.

 Image < Adamatzky & al., *Reaction-Diffusion Computers*

Some Uses of Voronoi Diagrams

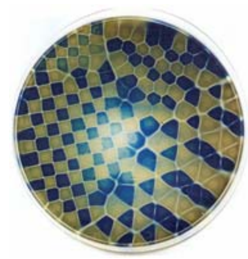
- Collision-free path planning
- Determination of service areas for power substations
- Nearest-neighbor pattern classification
- Determination of largest empty figure

Computation of Voronoi Diagram by Reaction-Diffusion Processor



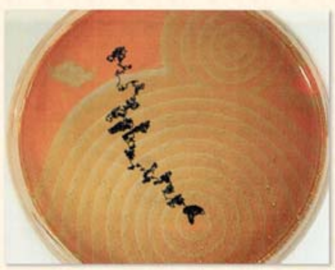
9/17/08 Image < Adamatzky & al., *Reaction-Diffusion Computers* 25

Mixed Cell Voronoi Diagram



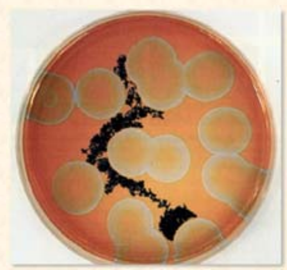
9/17/08 Image < Adamatzky & al., *Reaction-Diffusion Computers* 26

Path Planning via BZ medium: No Obstacles



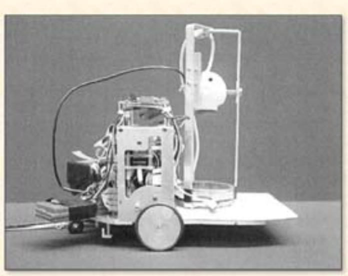
9/17/08 Image < Adamatzky & al., *Reaction-Diffusion Computers* 27

Path Planning via BZ medium: Circular Obstacles



9/17/08 Image < Adamatzky & al., *Reaction-Diffusion Computers* 28

Mobile Robot with Onboard Chemical Reactor

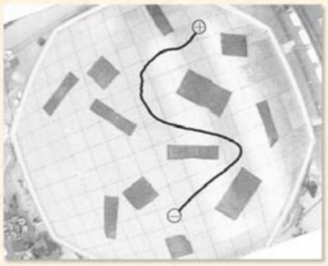


9/17/08 Image < Adamatzky & al., *Reaction-Diffusion Computers* 29

Actual Path: Pd Processor

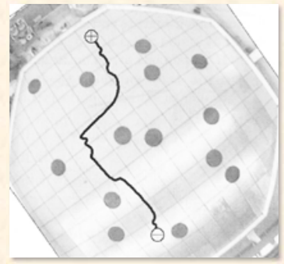
9/17/08 Image < Adamatzky & al., *Reaction-Diffusion Computers* 30

Actual Path: Pd Processor



9/17/08 Image < Adamatzky & al., *Reaction-Diffusion Computers* 31

Actual Path: BZ Processor



9/17/08 Image < Adamatzky & al., *Reaction-Diffusion Computers* 32

Bibliography for Reaction-Diffusion Computing

1. Adamatzky, Adam. *Computing in Nonlinear Media and Automata Collectives*. Bristol: Inst. of Physics Publ., 2001.
2. Adamatzky, Adam, De Lacy Costello, Ben, & Asai, Tetsuya. *Reaction Diffusion Computers*. Amsterdam: Elsevier, 2005.

9/17/08 33

Segmentation

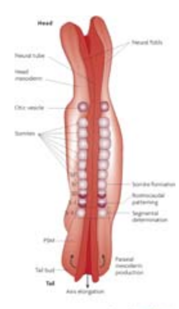
(in embryological development)

9/17/08 34

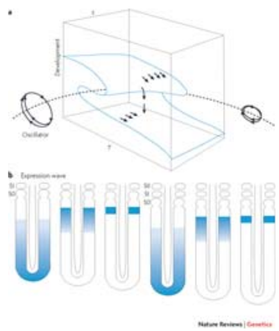
Vertebrae

- Humans: 33, chickens: 55, mice: 65, corn snake: 315
- Characteristic of species
- How does an embryo "count" them?
- "Clock and wavefront model" of Cooke & Zeeman (1976).

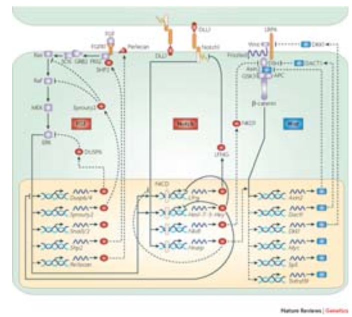
9/17/08 35



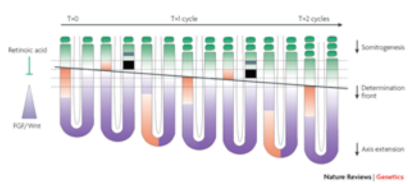
9/17/08 36



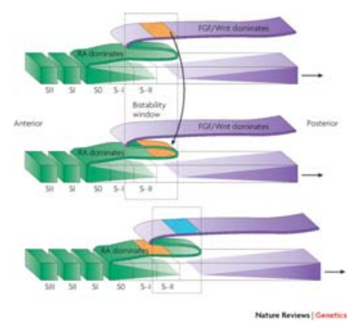
9/17/08 37



9/17/08 38



9/17/08 39



9/17/08 40

NetLogo Simulation of Segmentation

[Run Segmentation.nlogo]

9/17/08 41

Segmentation References

1. Cooke, J., & Zeeman, E.C. (1976). A clock and wavefront model for control of the number of repeated structures during animal morphogenesis. *J. Theor. Biol.* **58**: 455–76.
2. Dequéant, M.-L., & Pourquié, O. (2008). Segmental patterning of the vertebrate embryonic axis. *Nature Reviews Genetics* **9**: 370–82.
3. Gomez, C., Özbudak, E.M., Wunderlich, J., Baumann, D., Lewis, J., & Pourquié, O. (2008). Control of segment number in vertebrate embryos. *Nature* **454**: 335–9.

9/17/08 42

Additional Bibliography

1. Kessin, R. H. *Dictyostelium: Evolution, Cell Biology, and the Development of Multicellularity*. Cambridge, 2001.
2. Gerhardt, M., Schuster, H., & Tyson, J. J. "A Cellular Automaton Model of Excitable Media Including Curvature and Dispersion," *Science* **247** (1990): 1563-6.
3. Tyson, J. J., & Keener, J. P. "Singular Perturbation Theory of Traveling Waves in Excitable Media (A Review)," *Physica D* **32** (1988): 327-61.
4. Camazine, S., Deneubourg, J.-L., Franks, N. R., Sneyd, J., Theraulaz, G.,& Bonabeau, E. *Self-Organization in Biological Systems*. Princeton, 2001.
5. Pálsson, E., & Cox, E. C. "Origin and Evolution of Circular Waves and Spiral in *Dictyostelium discoideum* Territories," *Proc. Natl. Acad. Sci. USA*: **93** (1996): 1151-5.
6. Solé, R., & Goodwin, B. *Signs of Life: How Complexity Pervades Biology*. Basic Books, 2000.

continue to "Part 3A"