# Part B
# Ants (Natural and Artificial)

# Real Ants

(especially the black garden ant,
*Lasius niger*)

# Adaptive Significance

- Selects most profitable from array of food sources
- Selects shortest route to it
  - longer paths abandoned within 1–2 hours
- Adjusts amount of exploration to quality of identified sources
- Collective decision making can be as accurate and effective as some vertebrate individuals

# Observations on Trail Formation

- Two equal-length paths presented at same time: ants choose one at random
- Sometimes the longer path is initially chosen
- Ants may remain "trapped" on longer path, once established
- Or on path to lower quality source, if it's discovered first
- But there may be advantages to sticking to paths
  - easier to follow
  - easier to protect trail & source
  - safer

# Process of Trail Formation

1. Trail laying

2. Trail following

# Trail Laying

- On discovering food, forager lays chemical trail while returning to nest
    - only ants who have found food deposit pheromone
- Others stimulated to leave nest by:
    - the trail
    - the recruitor exciting nestmates (sometimes)
- In addition to defining trail, pheromone:
    - serves as general orientation signal for ants outside nest
    - serves as arousal signal for ants inside

# Additional Complexities

- Some ants begin marking on return from discovering food
- Others on their first return trip to food
- Others not at all, or variable behavior
- Probability of trail laying decreases with number of trips

# Frequency of Trail Marking

- Ants modulate frequency of trail marking
- May reflect quality of source
  - hence more exploration if source is poor
- May reflect orientation to nest
  - ants keep track of general direction to nest
  - and of general direction to food source
  - trail laying is less intense if the angle to homeward direction is large

# Trail Following

- Ants preferentially follow stronger of two trails
  - show no preference for path they used previously
- Ant may double back, because of:
  - decrease of pheromone concentration
  - unattractive orientation

# Probability of Choosing One of Two Branches

- Let $C_L$ and $C_R$ be units of pheromone deposited on left & right branches
- Let $P_L$ and $P_R$ be probabilities of choosing them
- Then:

$$P_L = \frac{(C_L + 6)^2}{(C_L + 6)^2 + (C_R + 6)^2}$$

- Nonlinearity amplifies probability

# Additional Adaptations

- If a source is crowded, ants may return to nest or explore for other sources

- New food sources are preferred if they are near to existing sources

- Foraging trails may rotate systematically around a nest

# Pheromone Evaporation

- Trails can persist from several hours to several months
- Pheromone has mean lifetime of 30-60 min.
- But remains detectable for many times this
- Long persistence of pheromone prevents switching to shorter trail
- Artificial ant colony systems rely more heavily on evaporation

# Resnick's Ants

# Environment

- Nest emits *nest-scent*, which
  - diffuses uniformly
  - decays slowly
  - provides general orientation signal
  - by diffusing around barriers, shows possible paths around barriers
- Trail pheromone
  - emitted by ants carrying food
  - diffuses uniformly
  - decays quickly
- Food detected only by contact

# Resnick Ant Behavior

1. Looking for food:

   **if** trail pheromone weak **then** wander

   **else** move toward increasing concentration

2. Acquiring food:

   **if** at food **then**

         pick it up, turn around, & begin depositing pheromone

3. Returning to nest:

   deposit pheromone & decrease amount available

   move toward increasing nest-scent

4. Depositing food:

   **if** at nest **then**

         deposit food, stop depositing pheromone, & turn around

5. Repeat forever

# Demonstration of Resnick Ants

Run Ants.nlogo

# Ant Colony Optimization (ACO)

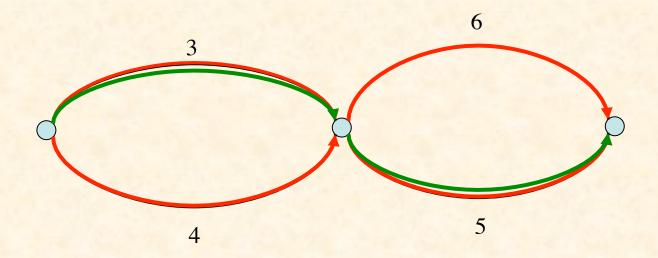Developed in 1991 by Dorigo (PhD dissertation) in collaboration with Colorni & Maniezzo

# Basis of all Ant-Based Algorithms

- Positive feedback

- Negative feedback

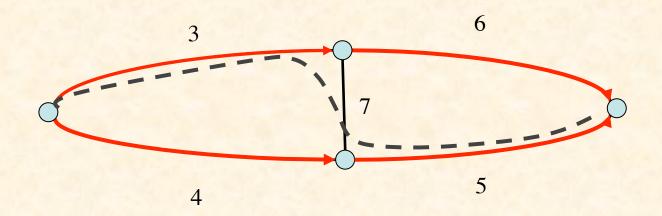- Cooperation

# Positive Feedback

- To reinforce portions of good solutions that contribute to their goodness

- To reinforce good solutions directly

- Accomplished by *pheromone accumulation*

# Reinforcement of Solution Components



Parts of good solutions *may* produce better solutions

# Negative Reinforcement of Non-solution Components



Parts not in good solutions *tend* to be forgotten

# Negative Feedback

- To avoid premature convergence (*stagnation*)

- Accomplished by *pheromone evaporation*

# Cooperation

- For simultaneous exploration of different solutions

- Accomplished by:
  - *multiple ants* exploring solution space
  - *pheromone trail* reflecting multiple perspectives on solution space

# Traveling Salesman Problem

- Given the travel distances between $N$ cities
  - may be symmetric or not
- Find the shortest route visiting each city exactly once and returning to the starting point
- NP-hard
- Typical combinatorial optimization problem

# Ant System for Traveling Salesman Problem (AS-TSP)

- During each iteration, each ant completes a tour

- During each tour, each ant maintains *tabu list* of cities already visited

- Each ant has access to
  - distance of current city to other cities
  - intensity of local pheromone trail

- Probability of next city depends on both

# Transition Rule

- Let $\eta_{ij} = 1/d_{ij} =$ "nearness" of city $j$ to current city $i$
- Let $\tau_{ij} =$ strength of trail from $i$ to $j$
- Let $J_i^k =$ list of cities ant $k$ still has to visit after city $i$ in current tour

- Then transition probability for ant $k$ going from $i$ to $j \in J_i^k$ in tour $t$ is:

$$p_{ij}^k = \frac{\left[\tau_{ij}(t)\right]^\alpha \left[\eta_{ij}\right]^\beta}{\displaystyle\sum_{l \in J_i^k}\left[\tau_{il}(t)\right]^\alpha \left[\eta_{il}\right]^\beta}$$

# Pheromone Deposition

- Let $T^k(t)$ be tour $t$ of ant $k$
- Let $L^k(t)$ be the length of this tour
- After completion of a tour, each ant $k$ contributes:

$$\Delta\tau_{ij}^{k} = \begin{cases} Q\!\!\Big/\!L^k(t) & \text{if } (i,j) \in T^k(t) \\ 0 & \text{if } (i,j) \notin T^k(t) \end{cases}$$

# Pheromone Decay

- Define total pheromone deposition for tour $t$:

$$\Delta\tau_{ij}(t) = \sum_{k=1}^{m} \Delta\tau_{ij}^{k}(t)$$

- Let $\rho$ be decay coefficient
- Define trail intensity for next round of tours:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \Delta\tau_{ij}(t)$$

# Number of Ants is Critical

- Too many:
  - suboptimal trails quickly reinforced
  - $\therefore$ early convergence to suboptimal solution
- Too few:
  - don't get cooperation before pheromone decays
- Good tradeoff:
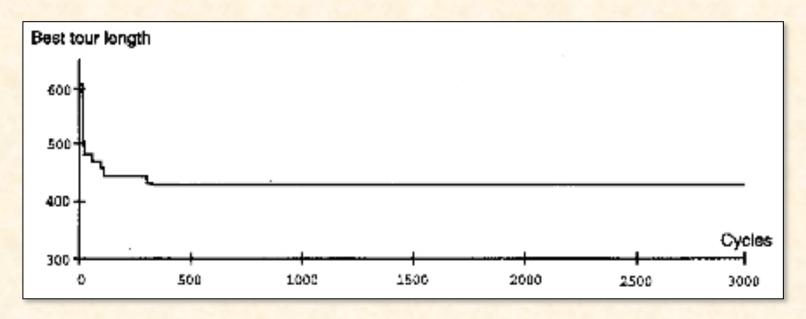  number of ants = number of cities
  $(m = n)$

# Improvement: "Elitist" Ants

- Add a few ($e \approx 5$) "elitist" ants to population
- Let $T^+$ be best tour so far
- Let $L^+$ be its length
- Each "elitist" ant reinforces edges in $T^+$ by $Q/L^+$
- Add $e$ more "elitist" ants
- This applies accelerating positive feedback to best tour

# Time Complexity

- Let $t$ be number of tours
- Time is $\mathcal{O}(tn^2m)$
- If $m = n$ then $\mathcal{O}(tn^3)$
  - that is, cubic in number of cities

# Convergence



- 30 cities ("Oliver30")
- Best tour length
- Converged to optimum in 300 cycles

fig. < Dorigo et al. (1996)

# Evaluation

- Both "very interesting and disappointing"
- For 30-cities:
  - beat genetic algorithm
  - matched or beat tabu search & simulated annealing
- For 50 & 75 cities and 3000 iterations
  - did not achieve optimum
  - but quickly found good solutions
- I.e., does not scale up well
- Like all general-purpose algorithms, it is out-performed by special purpose algorithms

# Improving Network Routing

1. Nodes periodically send *forward ants* to some recently recorded destinations
2. Collect information on way
3. Die if reach already visited node
4. When reaches destination, estimates time and turns into *backward ant*
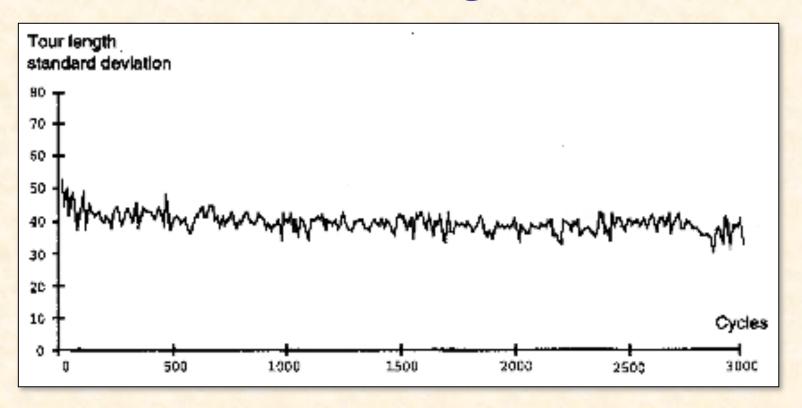5. Returns by same route, updating routing tables

# Some Applications of ACO

- Routing in telephone networks
- Vehicle routing
- Job-shop scheduling
- Constructing evolutionary trees from nucleotide sequences
- Various classic NP-hard problems
  - shortest common supersequence, graph coloring, quadratic assignment, …
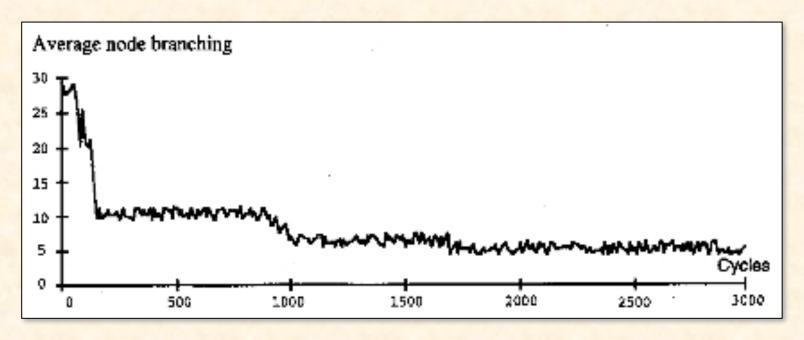
# Improvements as Optimizer

- Can be improved in many ways
- E.g., combine local search with ant-based methods
- As method of stochastic combinatorial optimization, performance is promising, comparable with best heuristic methods
- Much ongoing research in ACO
- But optimization is not a principal topic of this course

# Nonconvergence



- Standard deviation of tour lengths
- Optimum = 420

fig. < Dorigo et al. (1996)

# Average Node Branching Number



Average node branching

- Branching number = number of edges leaving a node with pheromone > threshold
- Branching number = 2 for fully converged solution

fig. < Dorigo et al. (1996)

# The Nonconvergence Issue

- AS often does not converge to single solution
- Population maintains high diversity
- A bug or a feature?
- Potential advantages of nonconvergence:
  - avoids getting trapped in local optima
  - promising for dynamic applications
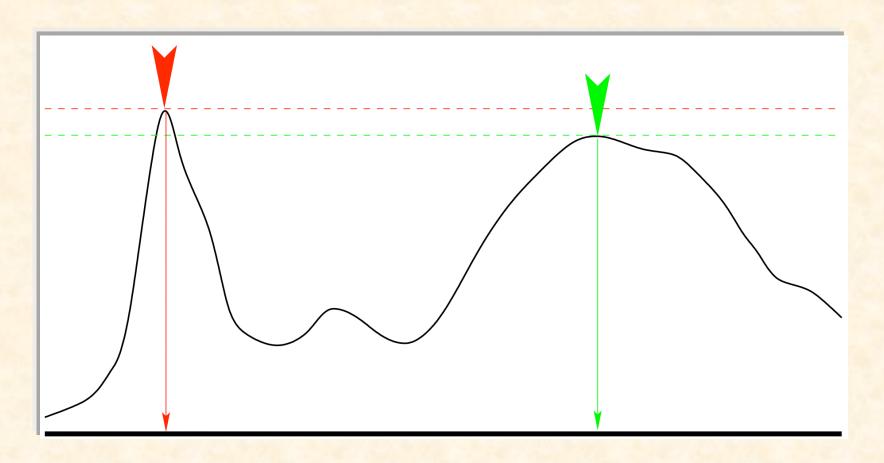- Flexibility & robustness are more important than optimality in natural computation

# Natural Computation

*Natural computation* is computation that occurs in nature or is inspired by computation occurring in nature

# Optimization
# in Natural Computation

- Good, but suboptimal solutions may be preferable to optima if:
  - suboptima can be obtained more quickly
  - suboptima can be adapted more quickly
  - suboptima are more robust
  - an ill-defined suboptimum may be better than a sharp optimum
- "The best is often the enemy of the good"

# Robust Optima

# Effect of Error/Noise

3C