Quantum Cellular Automata (QCA)

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Presentation Schedule

• Quantum Observations (5 min)

• Cellular Automata (5 min)

• Quantum Cellular Automata (10 min)
• Pythagoras of Samos
• Pythagorean Theorem
• Pythagoreanism
Flatland: A Romance of Many Dimensions

• Edwin Abbot Abbot (1884)
• women are line-segments
• men are polygons (n-sided)
• for he cannot conceive of any other except himself – and plumes himself upon the variety of Its Thought as an instance of creative Power. Let us leave this God of Pointland to the ignorant fruition of his omnipresence and omniscience. ~ (the Sphere)
The Quantum Mind

- **Hyperspace**: A Scientific Odyssey Through Parallel Universes, Time Warps, and the 10th Dimension
  - Michio Kako

- Human’s see in 3 dimensions
  - No advantage to see in other dimensions
  - Evolutionary survival

- Must our minds work in 3 dimensions?

- **Quantum Probability in Cognition**
  - Dr. MacLennan

- Theories of Cognition
  - Logic
    - Aristotle’s time
  - Classical Probability
    - 1970
  - Quantum Probability
• Collective Unconscious
  • Jung 1916

• dreams, fantasies, and other exceptional states appear natively

• primordial images or archetypes, as I have called them, belong to the basic stock of the unconscious psyche and cannot be explained as personal acquisitions

• Collectively called the collective unconscious
Michelangelo's Creation of Adam
According to a researcher (sic) at Cambridge University, it doesn't matter in what order the letters in a word are, the only important thing is that the first and last letter be at the right place. The rest can be a total mess and you can still read it without problem. This is because the human mind does not read every letter by itself but the word as a whole.
Cellular Automata

• John von Neumann
• “What kind of logical organization is sufficient for an automaton to reproduce itself?”
  • 1903

• Regular Grid of cells
• Finite number of states for each grid
• N-dimensional
• Neighborhood defined for each cell
• Initial state defined for each cell
• A changing function for each cell
  • Based on neighborhood
  • Usually same for each cell
Processing Image of Game of life

Processing Image of Spiernsky Triangle
Game of Life

• John Conway 1970
• Infinite two-dimensional orthogonal grid of cells
• Two possible states
• Rules (function or changing cells)
  • Living cell with fewer than two live neighbours dies
  • living cell with two or three live neighbours lives on
  • Living cell with more than three live neighbours dies
  • Dead cell with exactly three live neighbours becomes a live cell
Cellular Automata?

• Turing Complete

• Edge of Chaos

• Applications to other fields
Rule 110: Cellular Automata

- Turing Complete
  - Simulate a single taped Turing Machine
  - Computable
- Edge of Chaos
  - Lambda value
    - 0 = order
    - 1 = chaos

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<td>1</td>
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<td>1</td>
<td>1</td>
<td>1</td>
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Miller – Urey Experiment

• Create early earth conditions
  • methane (CH₄)
  • water (H₂O)
  • ammonia (NH₃)
  • hydrogen (H₂)
  • Continuous Electric Sparks

• Amino Acids were created
  • Protein
  • 20 +
Quantum Cellular Automata

• Generalization of cellular automata (CA)
• Reversible CA
• No agreed upon definition
Cellular Automata vs Quantum Cellular Automata

• (CA) is a 4-tuple \((L, \Sigma, N, f)\)

Definition (Cellular Automata) A cellular automaton (CA) is a 4-tuple \((L, \Sigma, N, f)\) consisting of
  (1) a \(d\)-dimensional lattice of cells \(I\) indexed \(i \in \mathbb{Z}^d\),
  (2) a finite set of states \(\Sigma\),
  (3) a finite neighborhood scheme \(N \subseteq \mathbb{Z}^d\), and
  (4) a local transition function \(f: \Sigma^N \rightarrow \Sigma\).

• (QCA) is a 3-tuple \((L, H, U)\)

Definition (Grössing–Zeilinger QCA) A Grössing–Zeilinger QCA is a 3-tuple \((L, H, U)\) which consists of
  (1) an infinite 1-dimensional lattice \(I \subseteq \mathbb{Z}\) representing basis states of
  (2) a Hilbert space \(H\) with basis set \(\{|\phi_i\}\), and
  (3) a band-diagonal unitary operator \(U\).

Definition (Partitioned Watrous QCA) A partitioned Watrous QCA is a Watrous QCA with
\(\Sigma = \Sigma_I \times \Sigma_c \times \Sigma_r\) for finite sets \(\Sigma_I, \Sigma_c,\) and \(\Sigma_r,\) and
matrix \(\Lambda\) of size \(\Sigma \times \Sigma\). For any state \(s = (s_I, s_c, s_r) \in \Sigma\)
define transition function \(f\) as
\[
f(s_1, s_2, s_3, s) = \Lambda_{(s_1, s_m, s_r)}(s),
\]  \(m = 1, 2, 3, 4\),
with matrix element \(\Lambda_{s_i, s_j}\).
Partitioned Watrous QCA

Definition (Partitioned Watrous QCA) A partitioned Watrous QCA is a Watrous QCA with \( \Sigma = \Sigma_l \times \Sigma_c \times \Sigma_r \) for finite sets \( \Sigma_l, \Sigma_c, \) and \( \Sigma_r, \) and matrix \( \Lambda \) of size \( \Sigma \times \Sigma \). For any state \( s = (s_l, s_c, s_r) \in \Sigma \) define transition function \( f \) as

\[
 f(s_1, s_2, s_3, s) = \Lambda_{(s_1, s_2), (s_3, s)},
\]

with matrix element \( \Lambda_{s_i, s_j} \).

• First proof of computational universality
• Given any quantum Turing Machine, there exists a PWQCA which simulates the Turing Machine with Constant slowdown
• Given any PWQCA, there exists a Turing Machine which simulates the PWQCA with linear slowdown
Types of QCA

• Reversible
• Local Unitary
  • 1 dimensional
  • Universal Simulate any quantum circuit
  • Can be simulated using a family of quantum circuits
• Block Partitioned
• Continuous Time
• Hamiltonian
QCA Exploration

• Logical Devices using QCA
  • Quantum Dots
    • Xor
    • adder
  
• Quantum Turing Machines are still theoretical
  • Confined to a very finite number of elements

• Optical lattices
  • Artificial crystals of light and consists of microtraps
  • Crossed laser beams,
    • creating polarization pattern
  • Atoms cool and congregate in the minima
    • Resembles a crystal lattice
  • Quantum tunneling can occur
Quantum Lattice Gas Automata

• partial differential equation can be the evolution of some CA
• CA will mimic the partial differentiation equation
  • Continuous time limit
  • Continuous space limit
• QLGA has demonstrated the equivalence
  • Evolution of set of quantum lattice Boltzmann equations
• Explored to model physical systems
Quantum Tunneling

• Wave-Particle Duality
• Particle
  • Classical mechanics
  • Sum of energy
• Wave
  • Quantum mechanics
Quantum Tunneling

• The matter rides the wave
Quantum Dot Automata

• Quantum Dots
  • Consist of nanoparticles
  • 5 quantum dots
  • Two electrons
    • Dots hop among the five states
    • Electrons find farthest distance
Works Cited

• Available upon request